

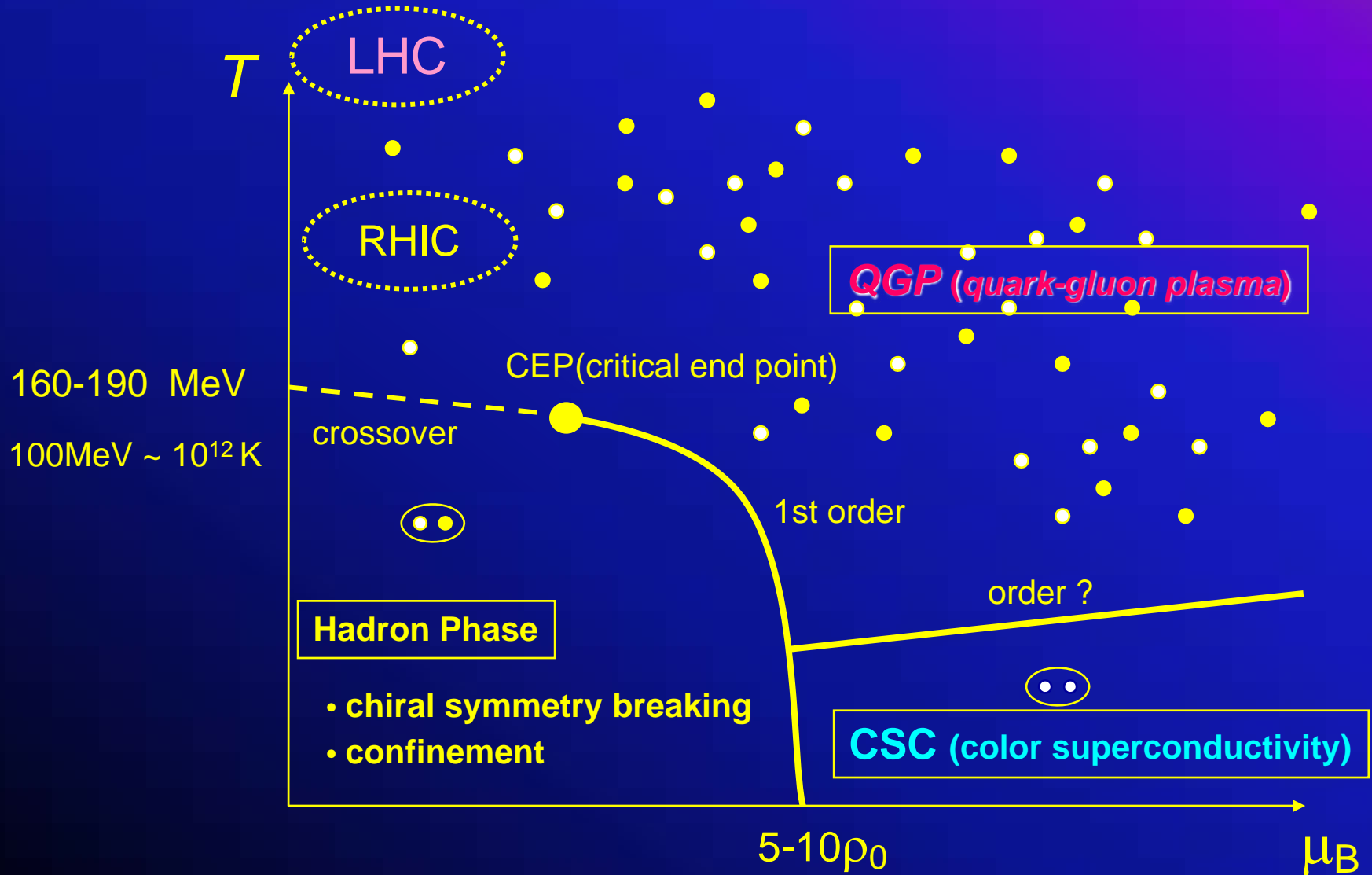
QCD Critical Point and Its Effect on Physical Observables

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in collaboration with C. Nonaka, B. Müller, S.A. Bass

QCD Phase Diagram



20th Anniversary of CEP in QCD

Nuclear Physics **A504** (1989) 668–684
North-Holland, Amsterdam

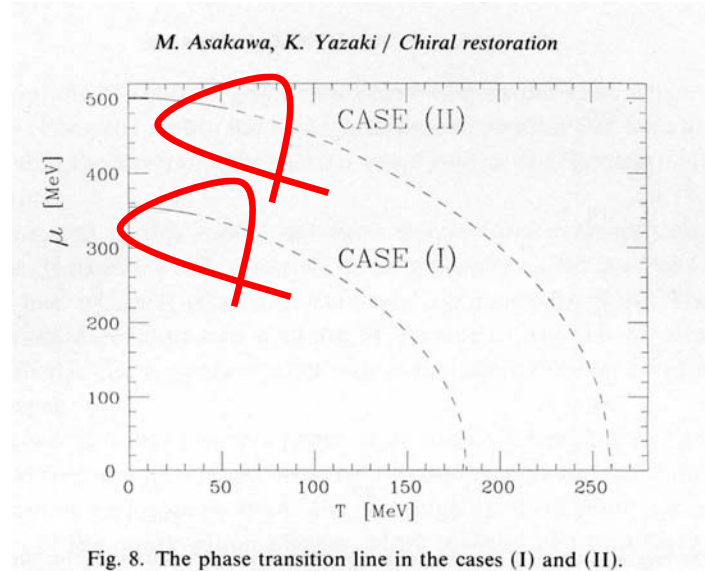
CHIRAL RESTORATION AT FINITE DENSITY AND TEMPERATURE

Masayuki ASAKAWA and Koichi YAZAKI

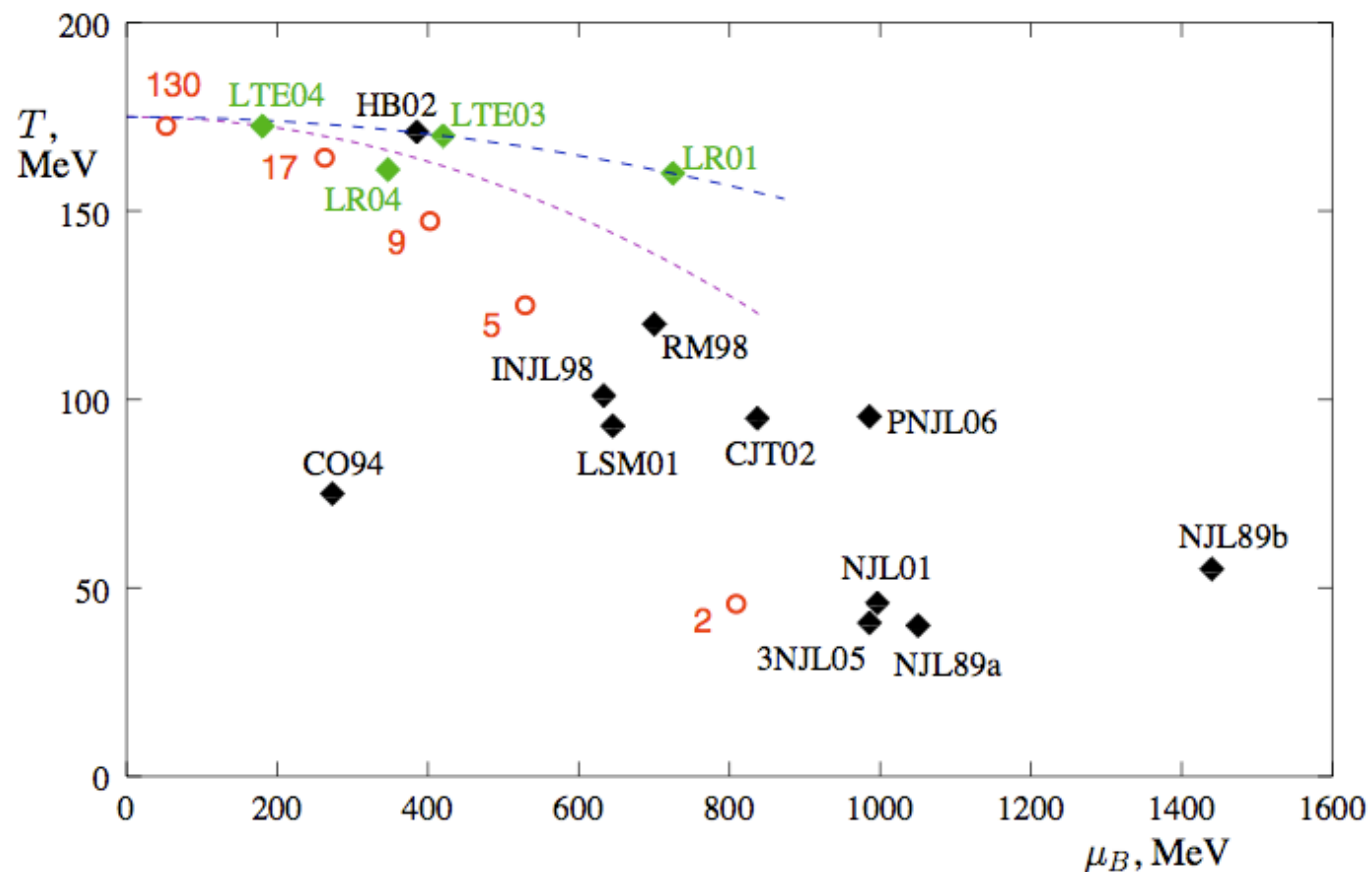
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Received 2 May 1988

(Revised 24 April 1989)



Where is CEP, if any?



Stephanov, hep-lat/0701002

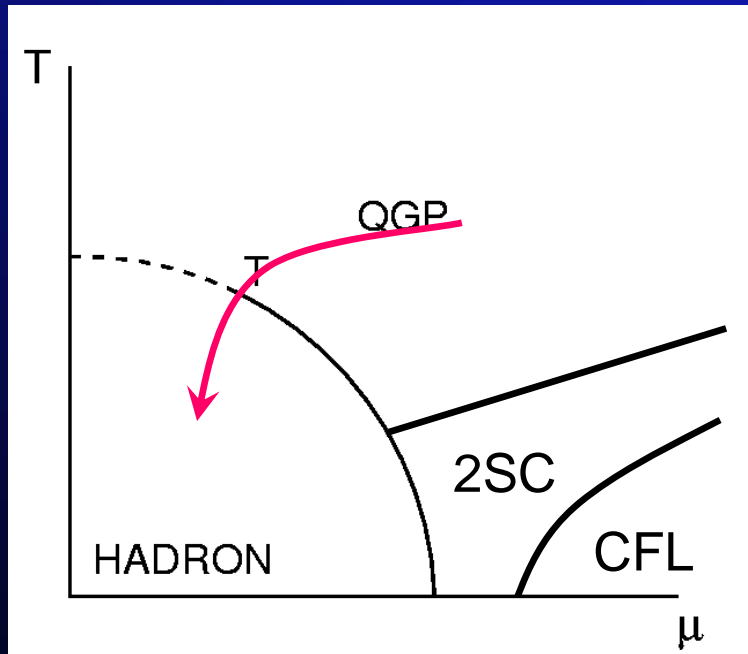
CEP = 2nd order phase transition, but...

CEP =
2nd Order Phase Transition Point



Divergence of Fluctuation
Correlation Length
Specific Heat ?

*If expansion
is adiabatic*



even if the system goes right through
the critical end point...

There is no conservation law that slows down the change of those quantities !

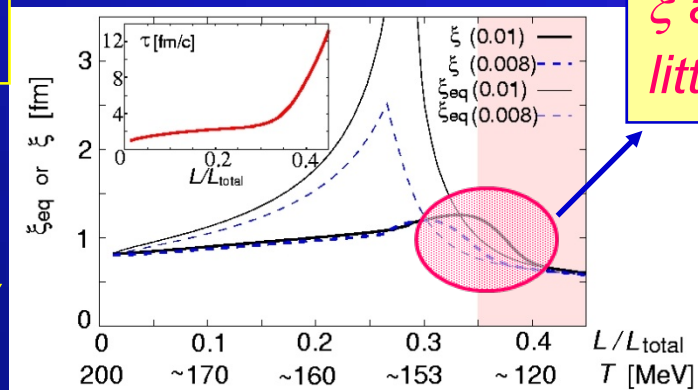
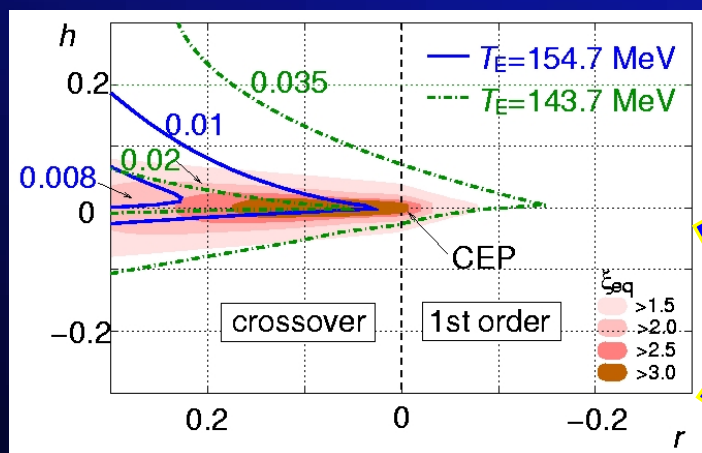


Subject to Final State Interactions

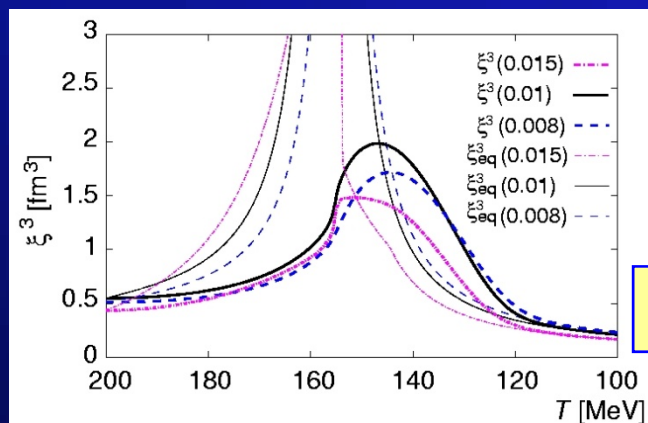
Furthermore...

Furthermore,
critical slowing down limits the size of fluctuation, correlation length !

Time Evolution along given
isentropic trajectories (n_B/s : fixed)



ξ and ξ_{eq}
little difference



fluctuation $\sim \xi^3$

$$\frac{d}{d\tau} m_\sigma(\tau) = -\Gamma[m_\sigma(\tau)] \left(m_\sigma(\tau) - \frac{1}{\xi_{eq}(\tau)} \right)$$

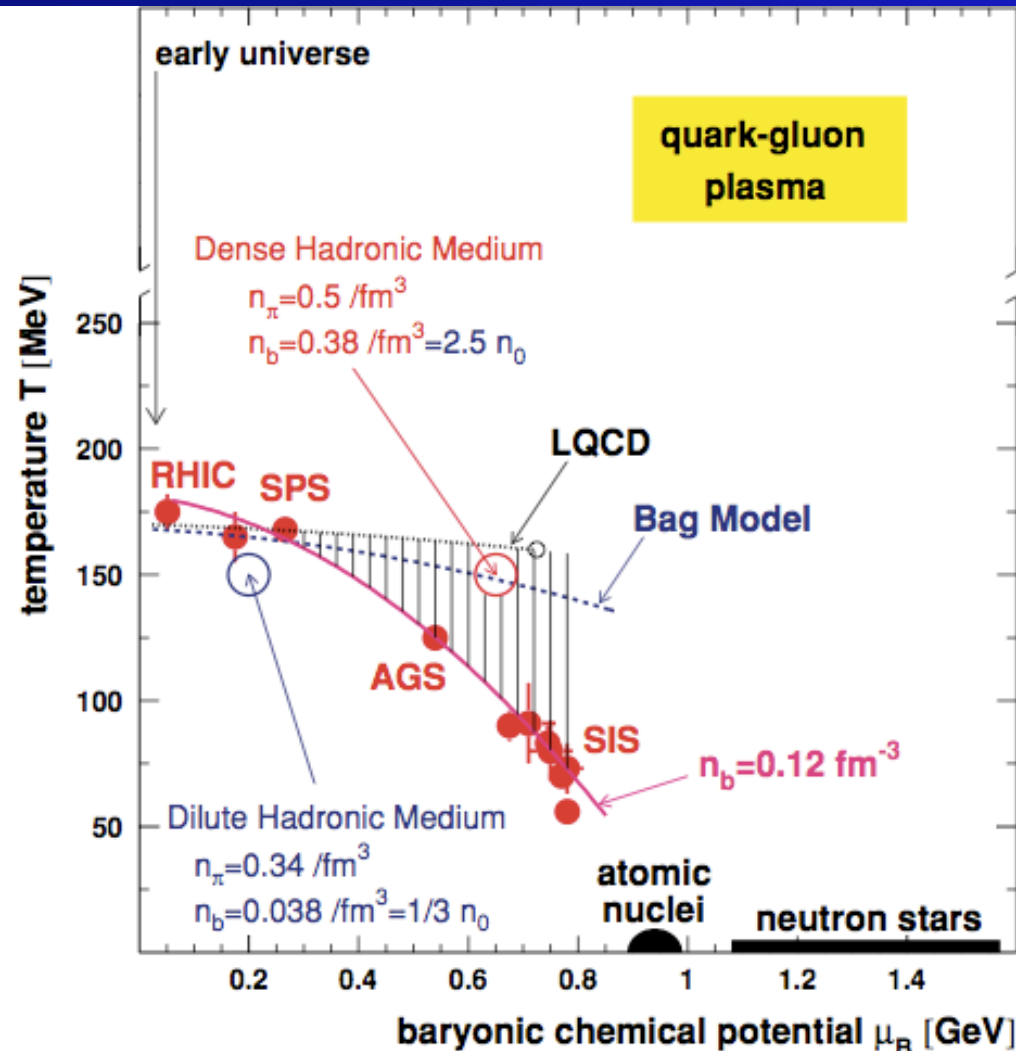
$$\Gamma[m_\sigma(\tau)] = \frac{A}{\xi_0} (m_\sigma(\tau) \xi_0)^z, \quad m_\sigma(\tau) = \frac{1}{\xi(\tau)}$$

$z \approx 3$ Model H (Hohenberg and Halperin RMP49(77)435)

Principles to Look for Other Observables

- We are in need of observables that are not subject to final state interactions

➡ After Freezeout, no effect of final state interactions



Chemical Freezeout

- usually assumed momentum independent
- but this is not right

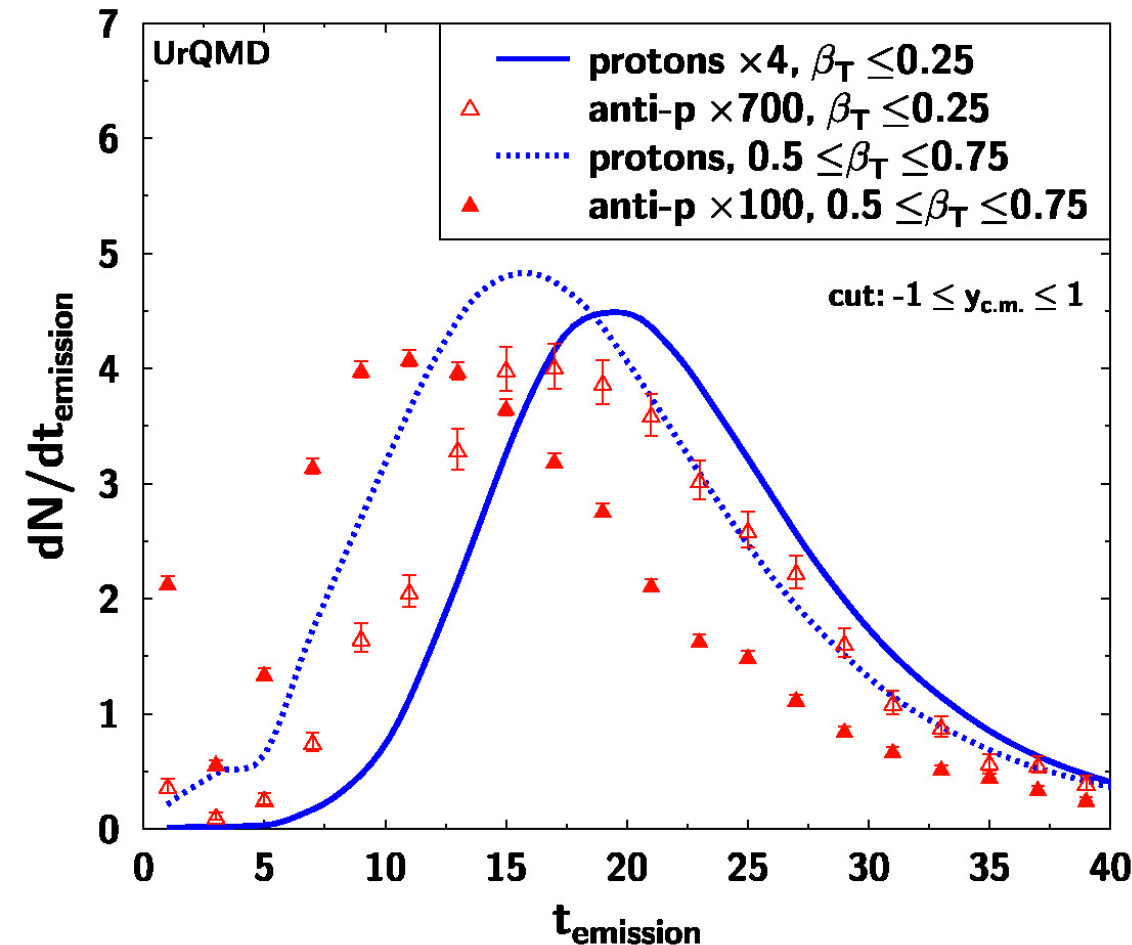
chemical freezeout time:
 p_T (or y_T) dependent

- ✓ Larger p_T (or y_T), earlier ch. freezeout

➡ **Principle I**

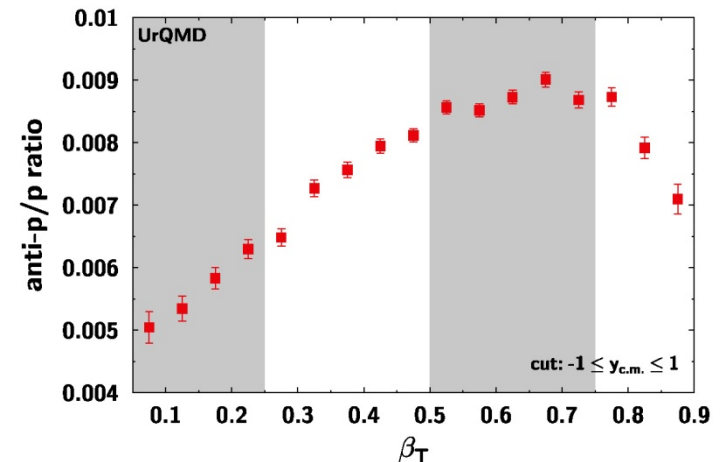
Emission Time Distribution

Au+Au, $E_{\text{lab}}=40$ GeV/A



Emission Time

- Larger β_T , earlier emission
- No CEP effect (UrQMD)



Principle II

Universality:

QCD CEP belongs to the same universality class as 3d Ising Model

Lattice QCD at finite density: still in its infancy

For critical behavior: need to carry out $V \rightarrow \infty$ limit

same universality class

QCD Critical End Point

T and μ_B



3d Ising Model End Point

$(T, \mu_B) \longleftrightarrow (r, h)$

$$r = \frac{T - T_c}{T_c}$$

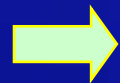
h : external magnetic field

What is not universal

■ Further Assumptions

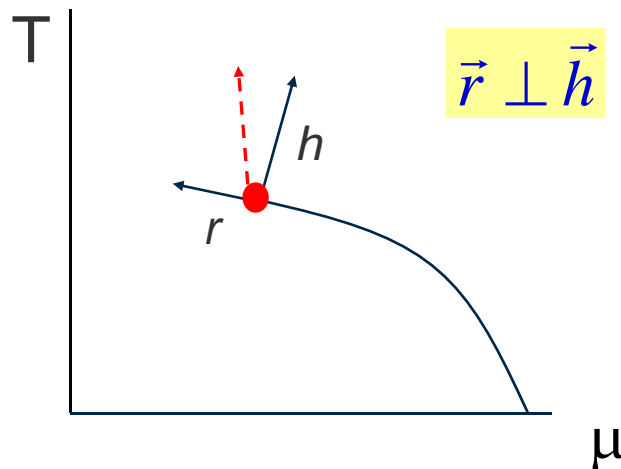
- Size of Critical Region

- No general universality
- Lattice calculation: not yet $V \rightarrow \infty$ limit
- Renormalization group analysis in Effective Models ?



need to be treated as an input, at the moment

- Mapping



$\vec{r} \parallel$ 1st order PT line
is *not* an assumption

EOS on Ising Side

■ Critical Behavior on Ising Side

- parametric representation

$$M = M_0 R^\beta \theta$$

$$r = R(1 - \theta^2)$$

$$h = h_0 R^{\beta\delta} h(\theta) = h_0 R^{\beta\delta} (\theta - 0.76201\theta^3 + 0.00804\theta^5)$$

$$(R \geq 0, -1.154 \leq \theta \leq 1.154)$$

Condition for M_0 and h_0

$$M(r, h) = 1 \quad \text{at} \quad (r, h) = (0, 1)$$

$$M(r, h) = 1 \quad \text{at} \quad (r, h) = (-1, +0)$$

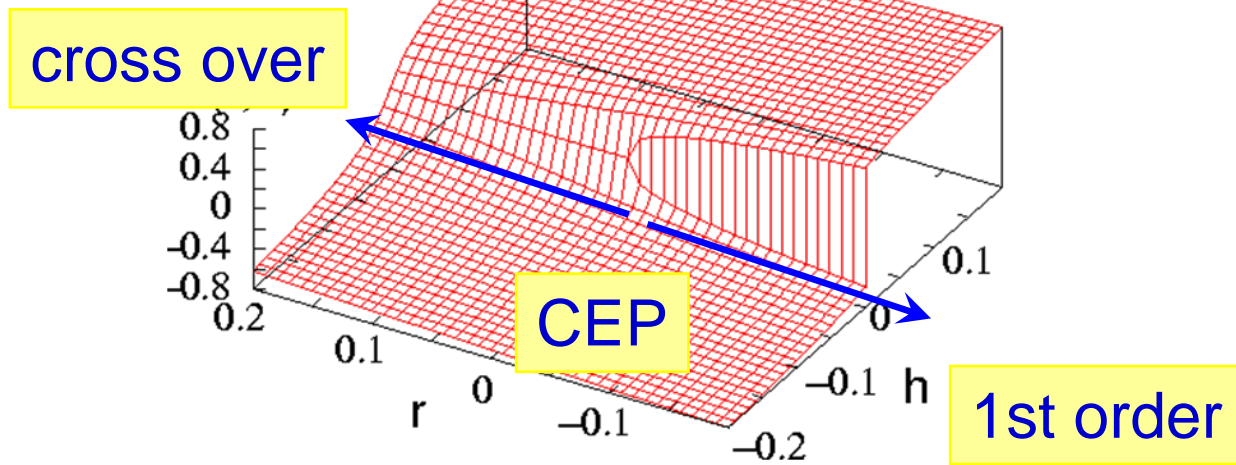
$$r = \frac{T - T_c}{T_c}$$

h : external magnetic field

$$\beta = 0.326$$

$$\delta = 4.8 \quad (\text{Critical Exponents})$$

R. Guida and J. Zinn-Justin,
NPB486 (1997) 626



Singular Part + Non-singular Part

■ Matching between Hadronic and QGP EOS

- Entropy Density consists of *Singular* and *Non-Singular* Parts
- **Only Singular Part** shows universal behavior

■ Requirement:

reproduce both the singular behavior and known asymptotic limits

■ Matched Entropy Density

$$s_{\text{real}}(T, \mu_B) = \frac{1}{2} \left\{ 1 - \tanh[S_c(T, \mu_B)] \right\} s_H(T, \mu_B) + \frac{1}{2} \left\{ 1 + \tanh[S_c(T, \mu_B)] \right\} s_Q(T, \mu_B)$$

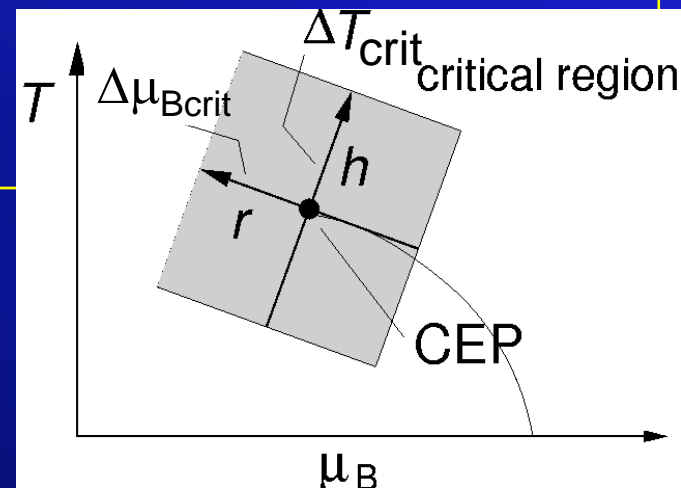
$s_H(T, \mu_B)$: Hadron Phase (excluded volume model)

$s_Q(T, \mu_B)$: QGP phase

- Dimensionless Quantity: S_c

$$S_c(T, \mu_B) = s_c(T, \mu_B) \sqrt{(\Delta T_{\text{crit}})^2 + (\Delta \mu_{\text{crit}})^2} \times D$$

D: related to extent of critical region

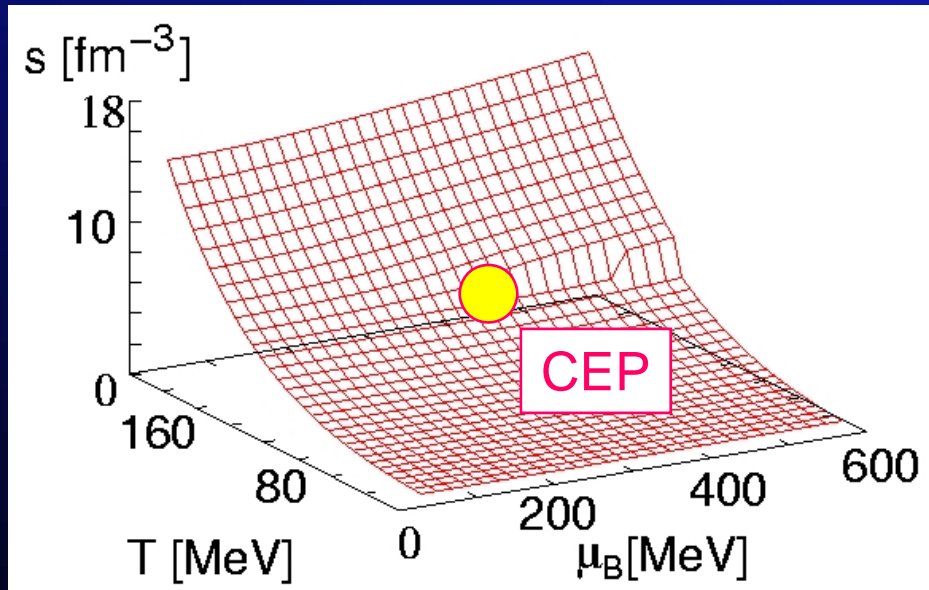


Isentropic Trajectories

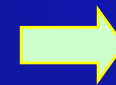
- In each volume element, Entropy (S) and Baryon Number (N_B) are conserved, as long as entropy production can be ignored (= when viscosities are small)

Isentropic Trajectories ($n_B/s = \text{const.}$)

An Example



Near CEP s and n_B change rapidly



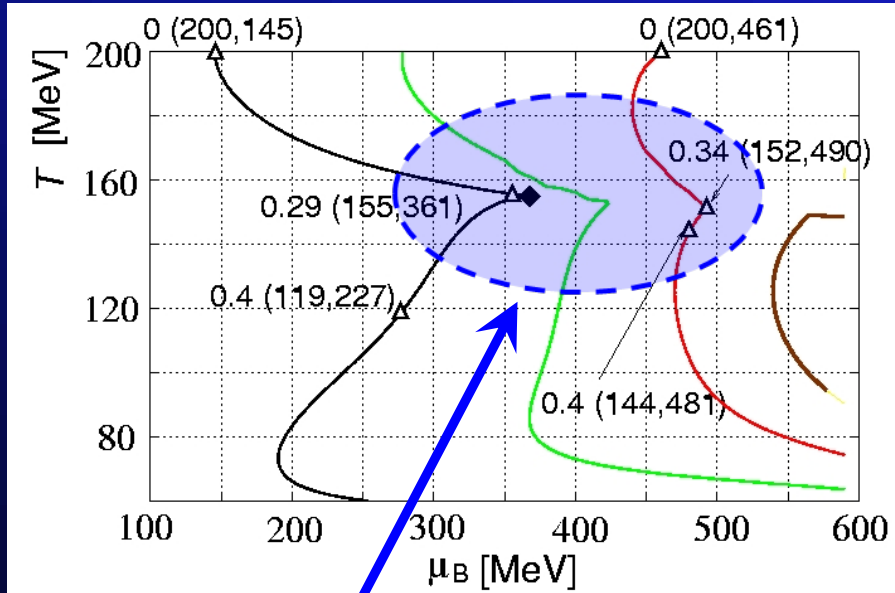
isentropic trajectories show non-trivial behavior



Bag Model EOS case

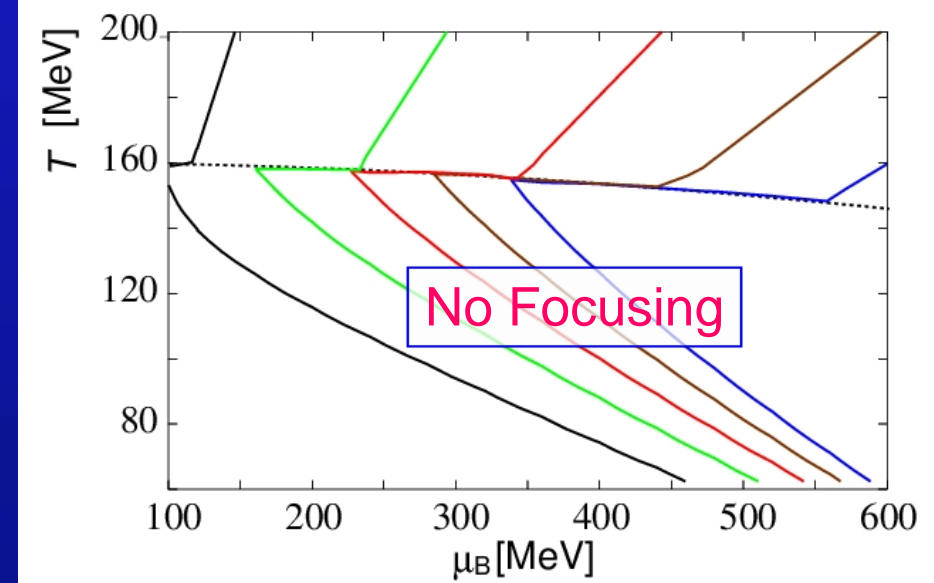
With Large Critical Region

with CEP



Focusing of Isentropic Trajectories

without CEP (EOS in usual hydro calculation)



Excluded Volume Approximation
+ Bag Model EOS

used in most hydro calculations

Consequence

For a given chemical freezeout point, prepare three isentropic trajectories: w/ and w/o CEP

Along isentropic trajectory:

$$\left\{ \begin{array}{ll} \bullet \text{ FO, CO} & \frac{\mu_B}{T} \nearrow \\ \bullet \text{ QCP} & \frac{\mu_B}{T} \rightarrow \end{array} \right.$$

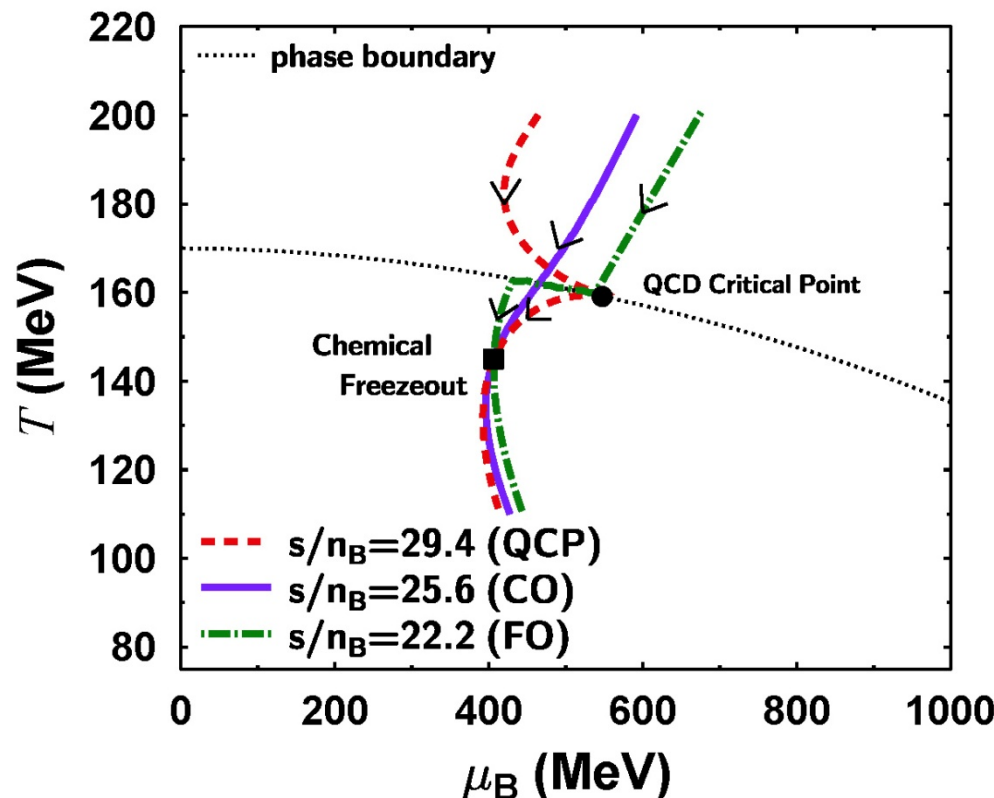


Principle I

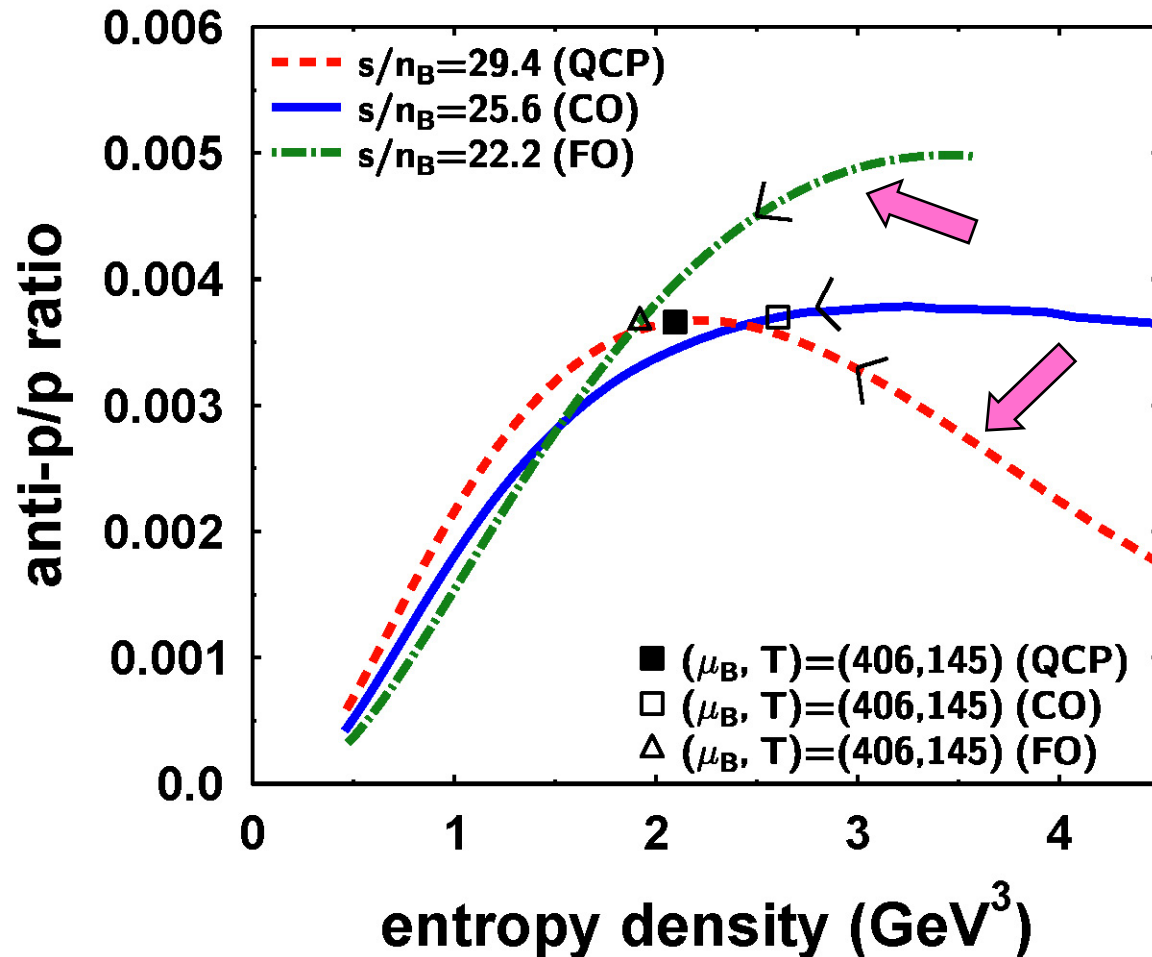
As a function of $p_T(y_T)$:

$$\left\{ \begin{array}{ll} \bullet \text{ FO, CO} & \frac{\mu_B}{T} \searrow \\ \bullet \text{ QCP} & \frac{\mu_B}{T} \rightarrow \end{array} \right.$$

➡ \bar{p}/p ratio : near CEP steeper



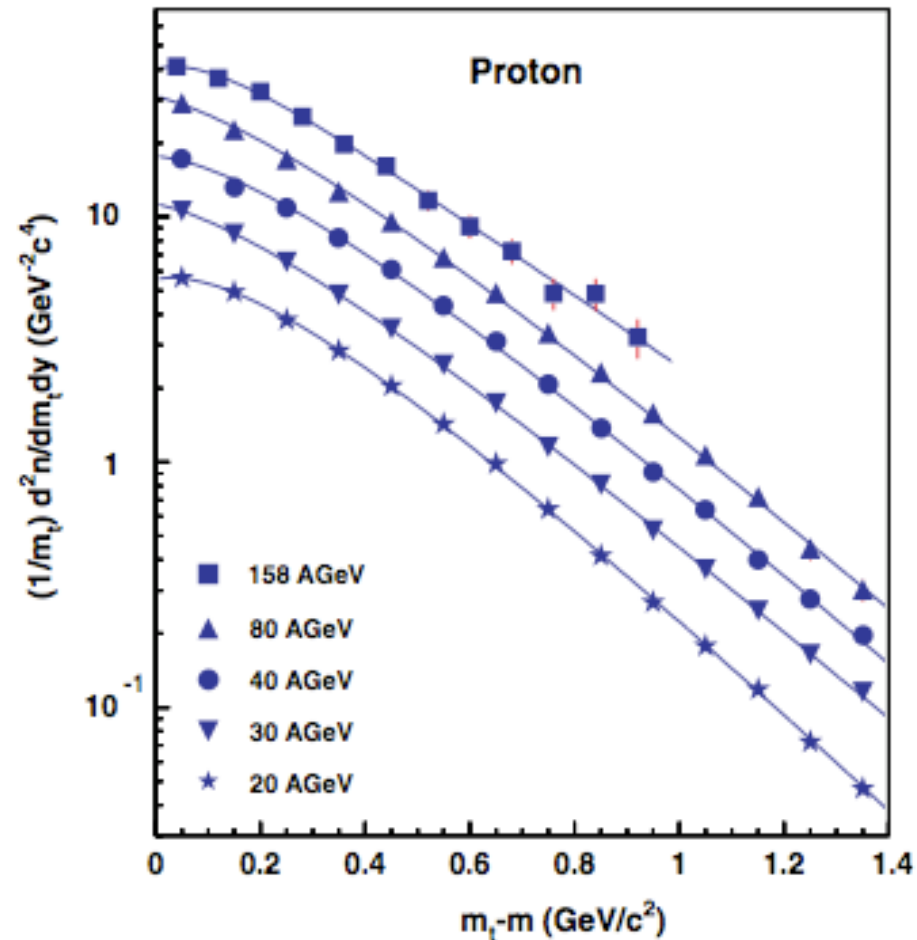
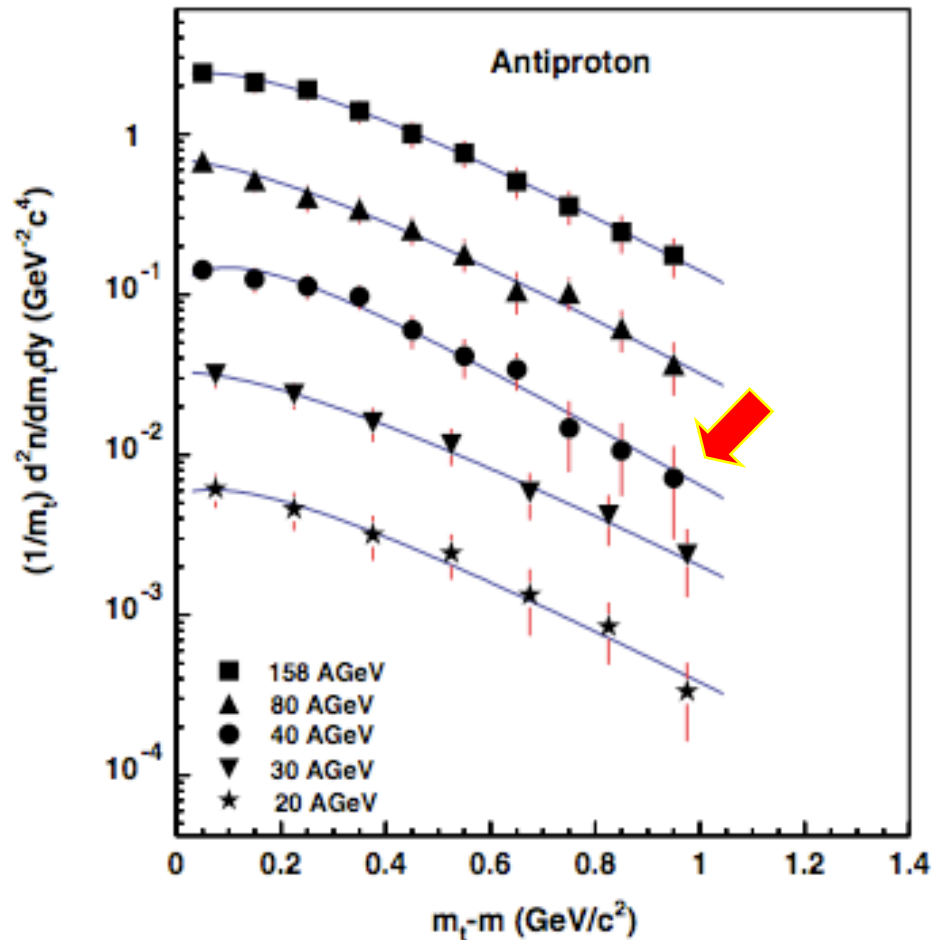
Evolution along Isentropic Trajectory



$$\bar{p}/p \sim \exp\left(-\frac{2\mu_B}{T}\right)$$

with CEP
steeper \bar{p} spectra at high P_T

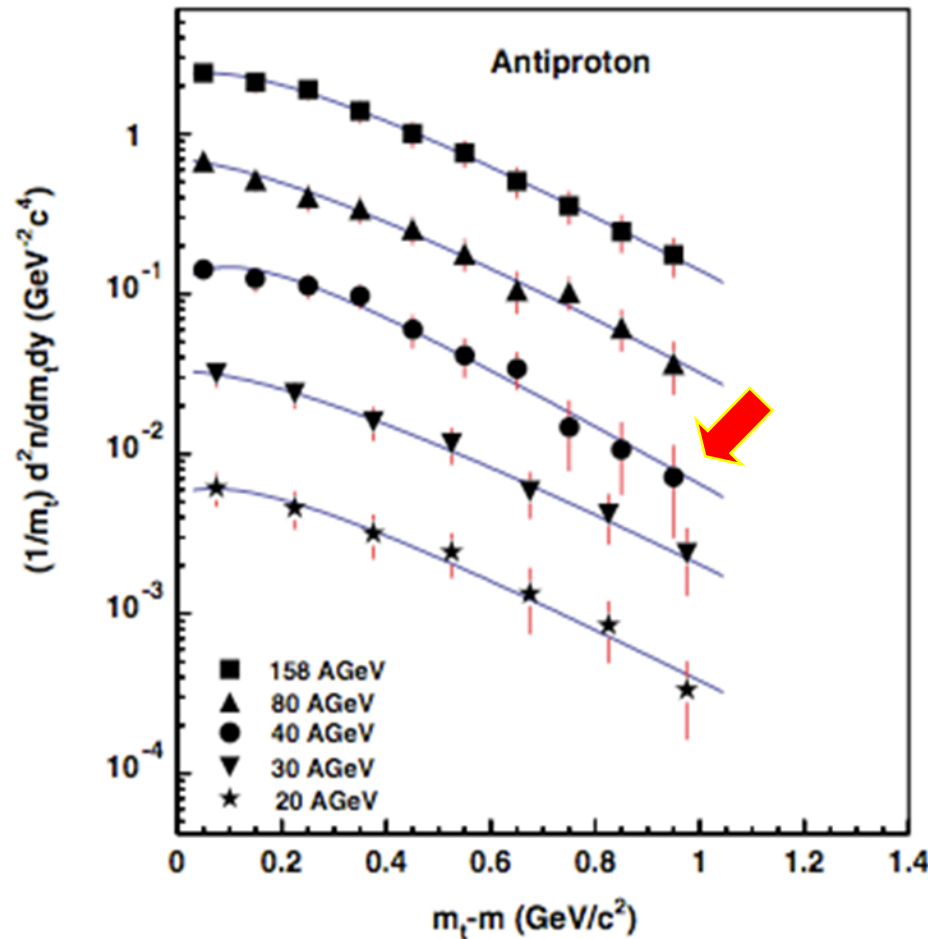
Effect on Spectra ?



steeper \bar{p} spectra at high P_T

NA49, PRC73, 044910(2006)

Result of One Temperature Fit



NA49, PRC73, 044910(2006)

	E_{beam} (A GeV)	dn/dy	T (MeV)	$\langle m_t \rangle - m$ (MeV/c ²)
\bar{p}	158	1.66 ± 0.17	291 ± 15	384 ± 19
	80	0.87 ± 0.07	283 ± 30	385 ± 41
	40	0.32 ± 0.03	246 ± 35	355 ± 51
	30	0.16 ± 0.02	290 ± 45	395 ± 60
	20	0.06 ± 0.01	279 ± 64	394 ± 60
p	158	29.6 ± 0.9	308 ± 9	413 ± 13
	80	30.1 ± 1.0	260 ± 11	364 ± 16
	40	41.3 ± 1.1	257 ± 11	367 ± 16
	30	42.1 ± 2.0	265 ± 10	362 ± 14
	20	46.1 ± 2.1	249 ± 9	352 ± 13

- Only one experimental result for \bar{p} slope
- Still error bar is large

Summary

■ Two Principles:

- i) Chemical Freezeout is $p_T(\beta_T)$ dependent
- ii) Isentropic Trajectory behaves non-trivially near CEP (focusing)

➡ \bar{p}/p ratio behaves non-monotonously near CEP

Information on the QCD critical point:
such as location, size of critical region, existence...

■ We then made a data search

- turned out NA49 \bar{p} data shows non-trivial behavior around 40 GeV/A
- still error bar is large, *finer energy scans at SPS, FAIR, RHIC*: desirable

■ Effect on Flow ?

c_s changes differently from the case with EOS used in usual hydro cal.
(3D hydro cal. with CEP + UrQMD: C. Nonaka in progress)