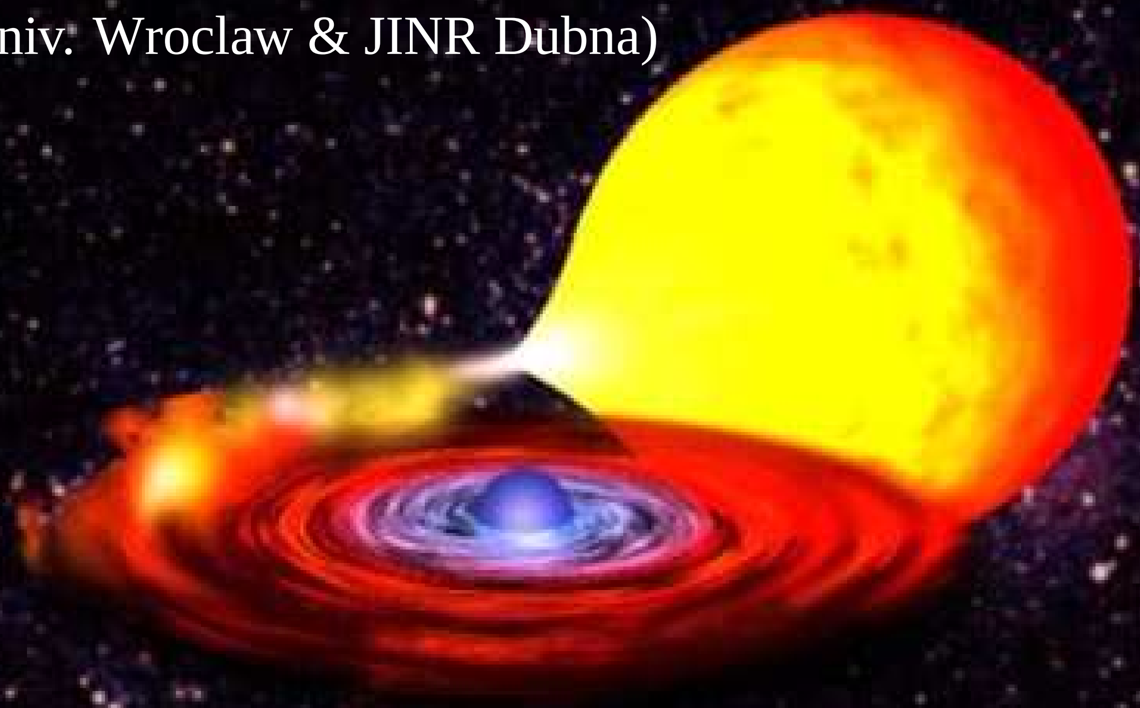
Nonlocal Chiral Quark Models & Critical (End-)Point

David Blaschke (Univ. Wroclaw & JINR Dubna)



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INT Workshop, Seattle, August 15th, 2008

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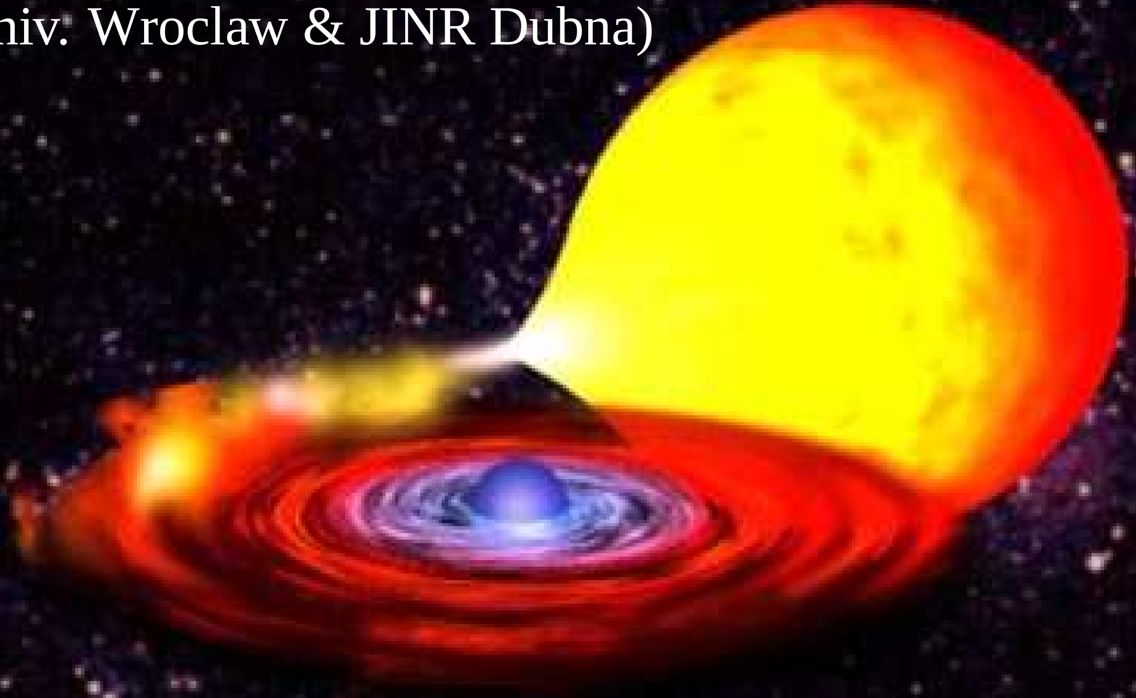


- NCQM as a tool for strong $\mathrm{QCD}|_{T,\mu}$
- Mean-field: order params. & phase diagram
- Flucuations: mesons & Mott effect
- Gibbs constr: compact stars vs. CBM et al.

INT Workshop, Seattle, August 15^{th}, 2008

Quantum Field Theory for Chiral Quark Matter

- Partition function for chiral Quark Field theory

$$Z[T,V,\mu] = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left\{-\int^{\beta} d\tau \int_V d^3x[\bar{\psi}(i\gamma^\mu\partial_\mu - m - \gamma^0\mu)\psi - \mathcal{L}_{\rm int}]\right\}$$

- Current-current coupling (4-fermion interaction)

$$\mathcal{L}_{\rm int} = \sum_{M=\pi,\sigma,\ldots} G_M(\bar{\psi}\Gamma_M\psi)^2 + \sum_D G_D(\bar{\psi}^C\Gamma_D\psi)^2$$

- Bosonisation (Hubbard-Stratonovich Transformation)

$$Z[T,V,\mu] = \int \mathcal{D}\phi_M\mathcal{D}\Delta_D^\dagger\mathcal{D}\Delta_D \exp\left\{-\sum_M \frac{\phi_M^2}{4G_M} - \sum_D \frac{|\Delta_D|^2}{4G_D} + \frac{1}{2}\mathrm{Tr}\,\ln S^{-1}[\{M_M\},\{\Delta_D\}]\right\}$$

- Collective (stochastic) Fields: Mesons (ϕ_M) and Diquarks (Δ_D)
- Systematic Evaluation: Mean field + Fluctuations
  - Mean-field Approximation: Order parameter for Phase transitions (Gap equations)
  - Fluctuations (2. Order): Hadronic Correlations (Bound- & Scattering states)
  - Fluctuations of higher Order: Hadron-Hadron Interaction

Phase diagram for nonlocal chiral quark model

Euclidean action at $T, \mu = 0$

$$S_E = \int d^4x \left\{ \bar{\psi}(x)\left(-i\partial\!\!\!/ + m\right)\psi(x) - \frac{G_M}{2} j^f_M(x) j^f_M(x) - \frac{G_D}{2}\left[j^a_D(x)\right]^\dagger j^a_D(x) \right\}$$

Two alternatives to introduce nonlocality

Model I
Instanton Liquid
Model inspired

$$j^f_M(x) = \int d^4y\, d^4z\, r(y-x)\, r(x-z)\, \bar{\psi}(y)\, \Gamma_f\, \psi(z)\,,$$

$$j^a_D(x) = \int d^4y\, d^4z\, r(y-x)\, r(x-z)\, \bar{\psi}_C(y)\, i\gamma_5\tau_2\lambda_a\, \psi(z)$$

Model II
One-Gluon-
Exchange
inspired

$$j^f_M(x) = \int d^4z\, g(z)\, \bar{\psi}(x+\frac{z}{2})\, \Gamma_f\, \psi(x-\frac{z}{2})\,,$$

$$j^a_D(x) = \int d^4z\, g(z)\, \bar{\psi}_C(x+\frac{z}{2})\, i\gamma_5\tau_2\lambda_a\, \psi(x-\frac{z}{2})$$

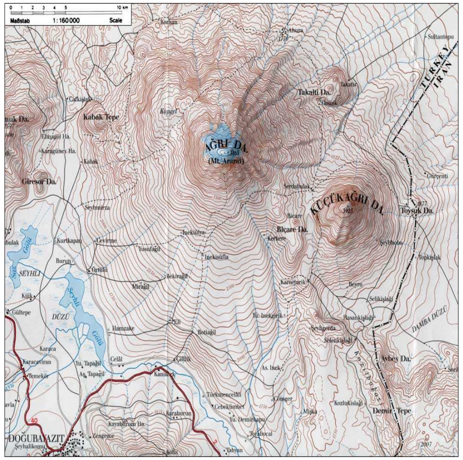
$r(x-y)$ and $g(z)$ are nonlocal regulators, $\psi_C(x) = \gamma_2\gamma_4\,\bar{\psi}^T(x)$, $\Gamma_f = (\mathbf{1}, i\gamma_5\vec{\tau})$

Gomez-Dumm, D.B., Grunfeld, Scoccola, arXiv:hep-ph/0512218; Phys. Rev. D 73 (2006).

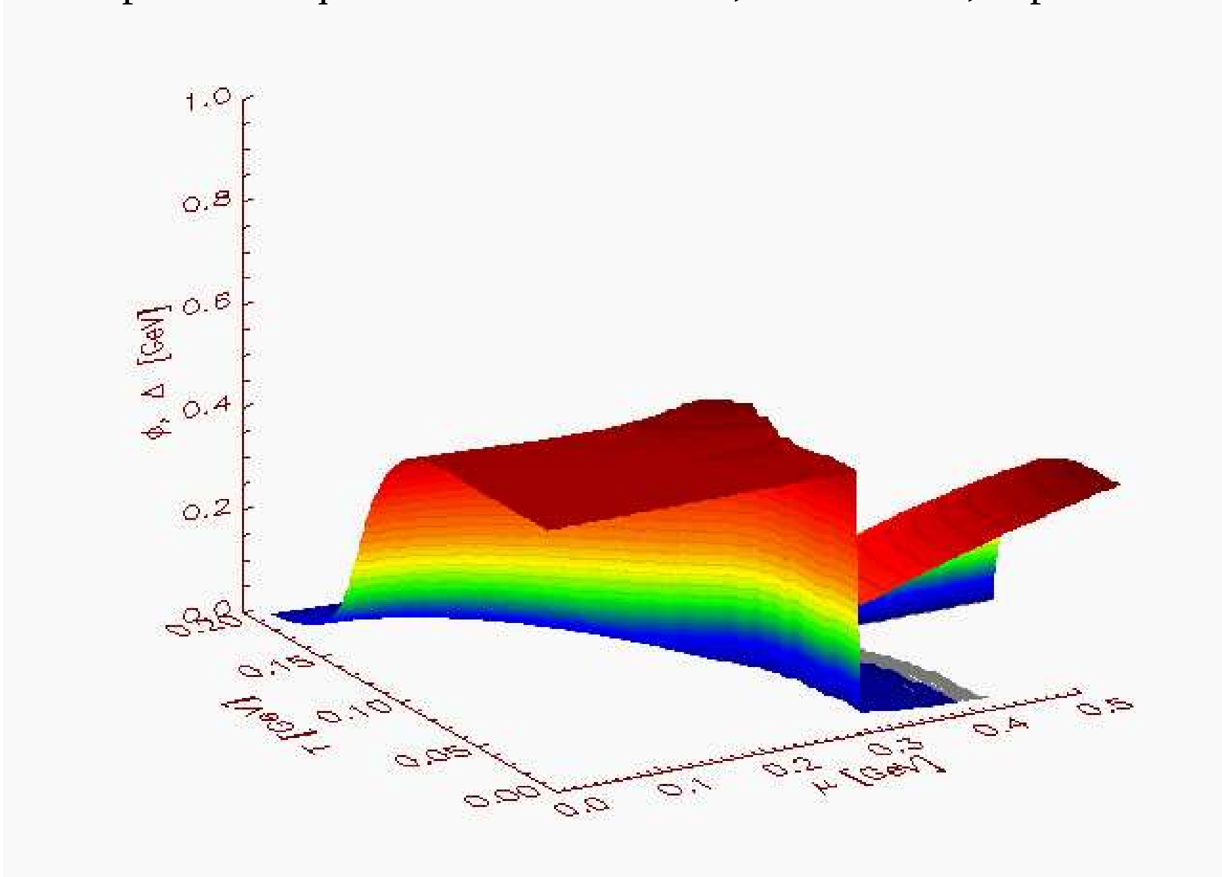
The T-mu plane: order parameter landscape

Map of Armenia:

Ararat



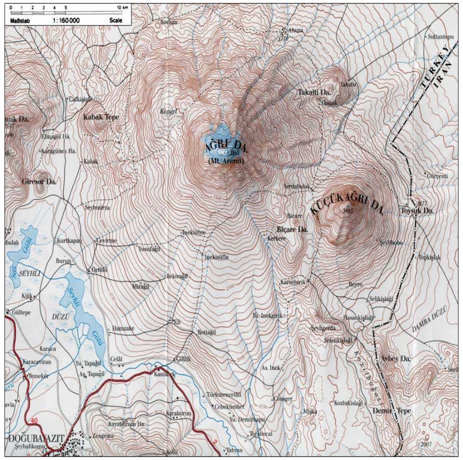
Three phases of quark matter: confined, deconfined, superconducting



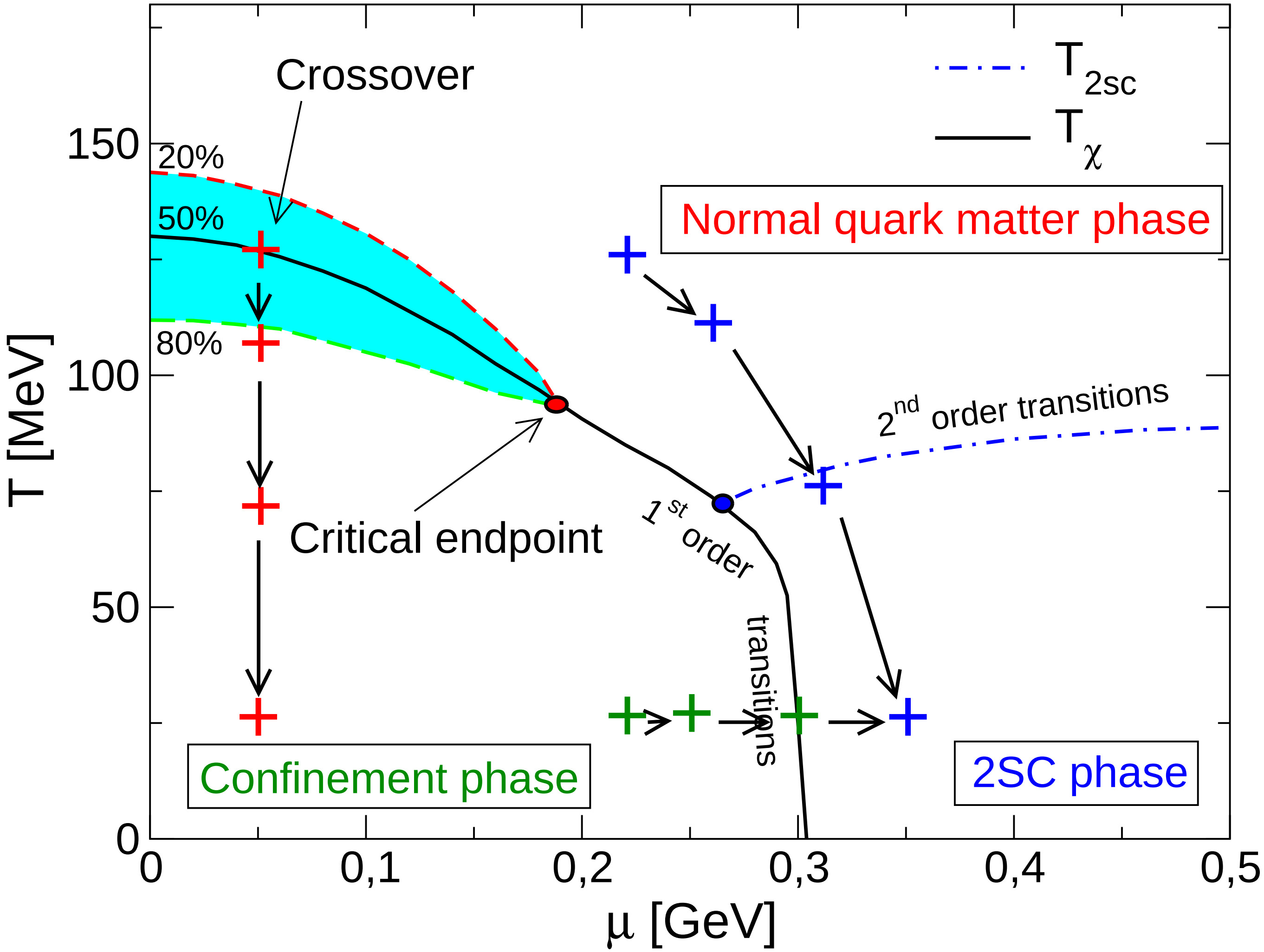
The T-mu plane: walking on the map

Armenia in Europe:

Ararat

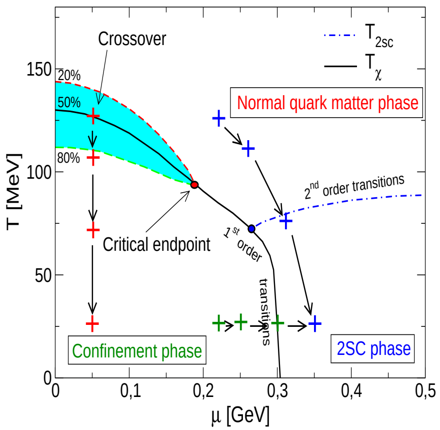


Three phases of quark matter: confi ned, deconfi ned, superconducting

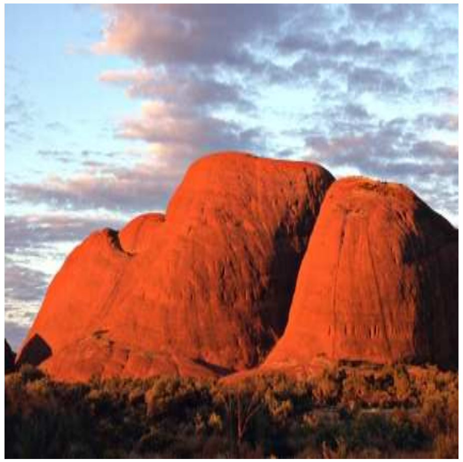


The T-mu plane: walking the routes (I)

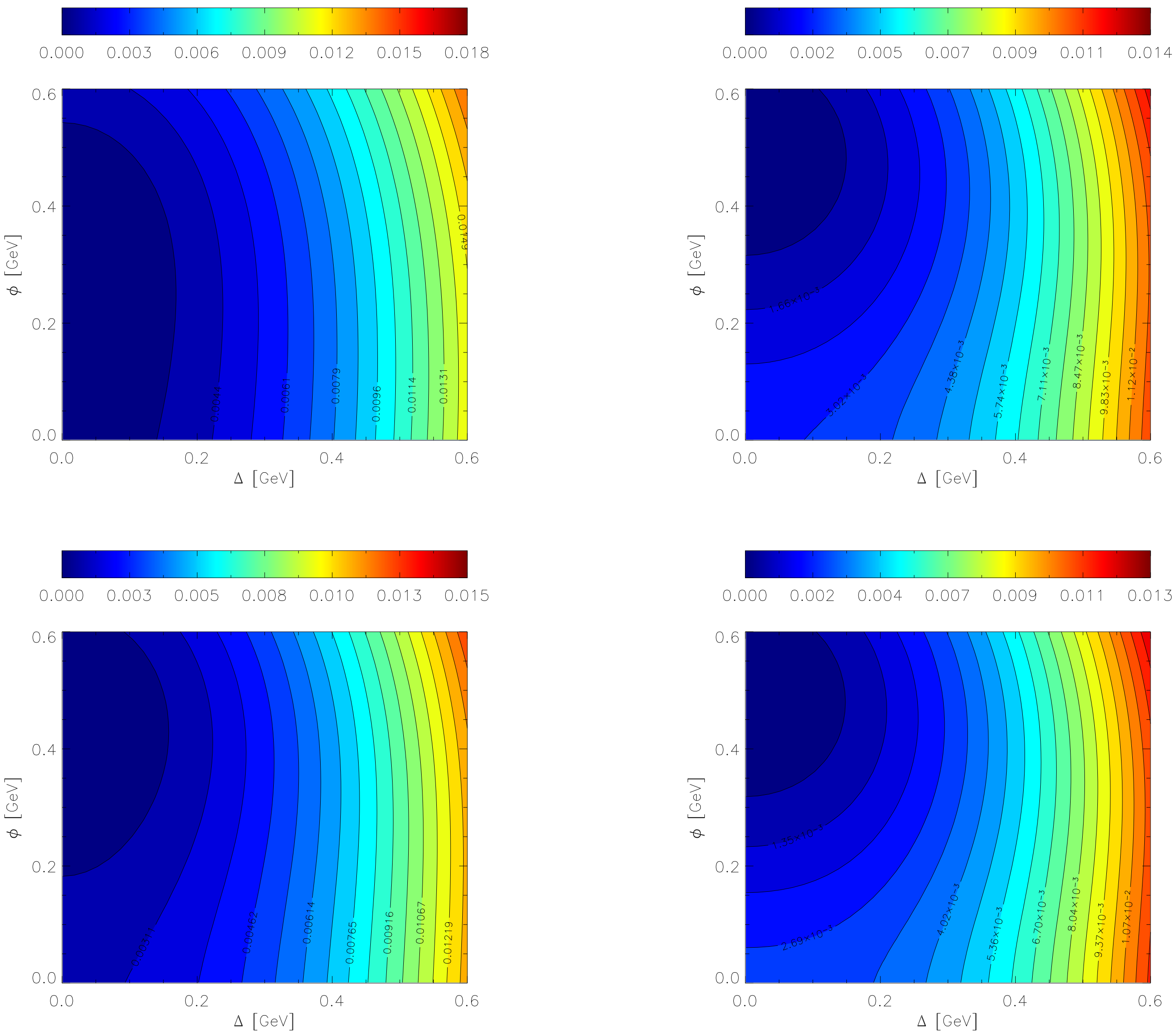
Map of routes:



The Olgas



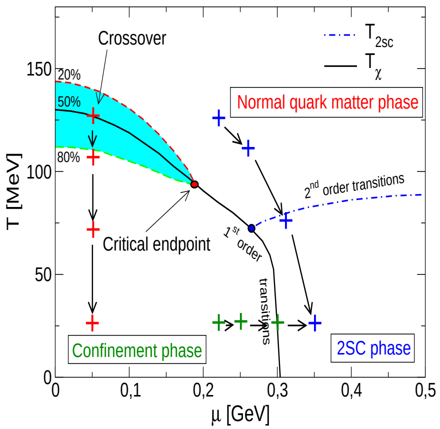
Route 1: deconfined $\longrightarrow$ confined



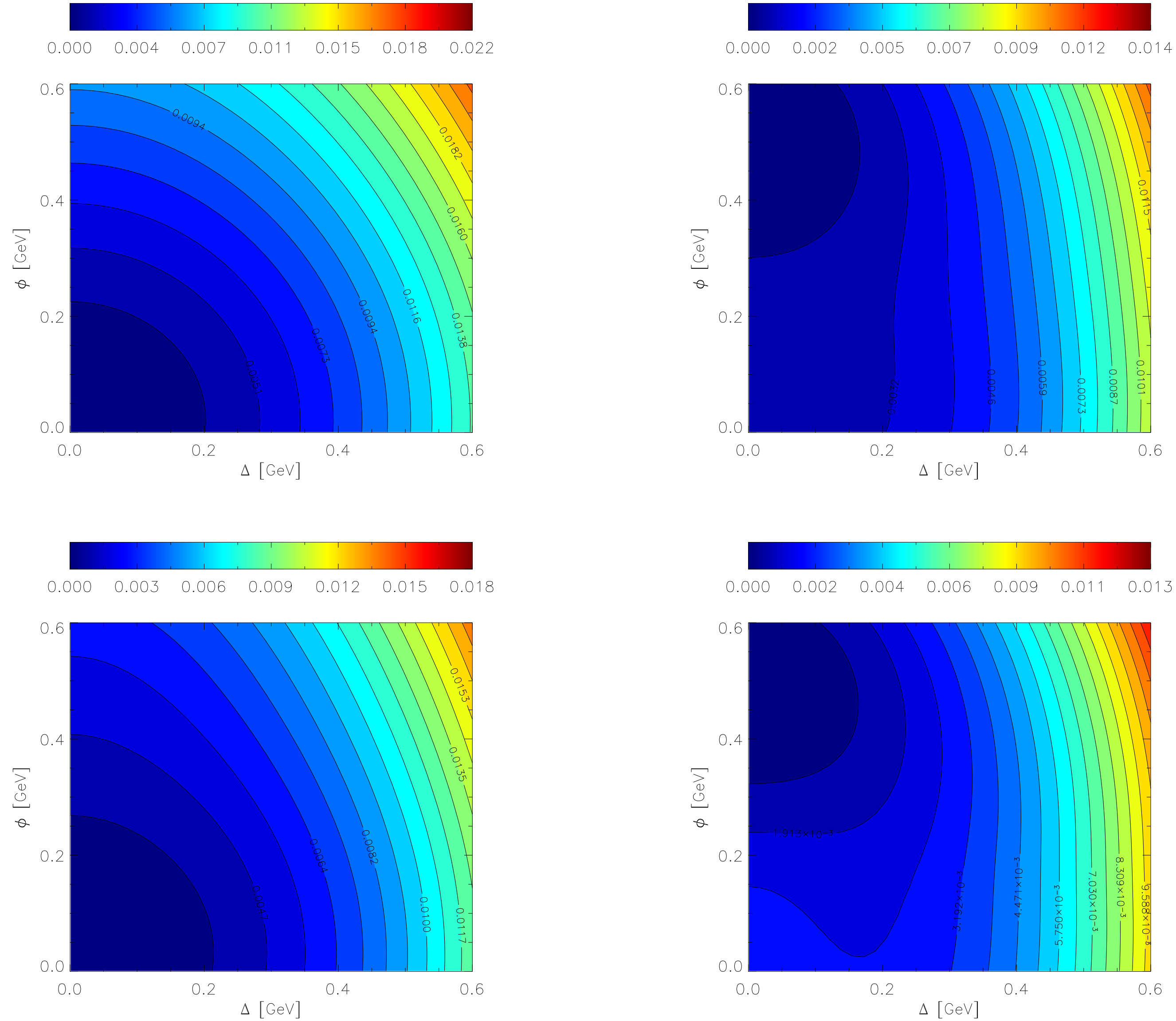
The T-mu plane: walking the routes (II)

Route 1’: deconfi ned $\longrightarrow$ confi ned

Map of routes:

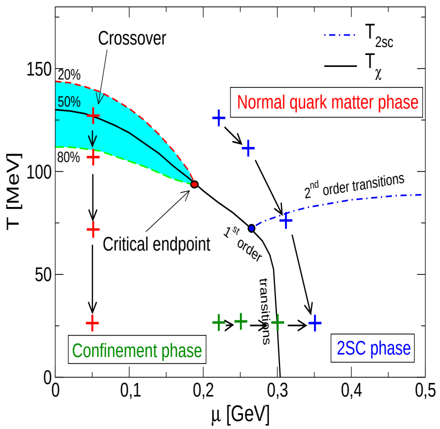


Mount Rainier



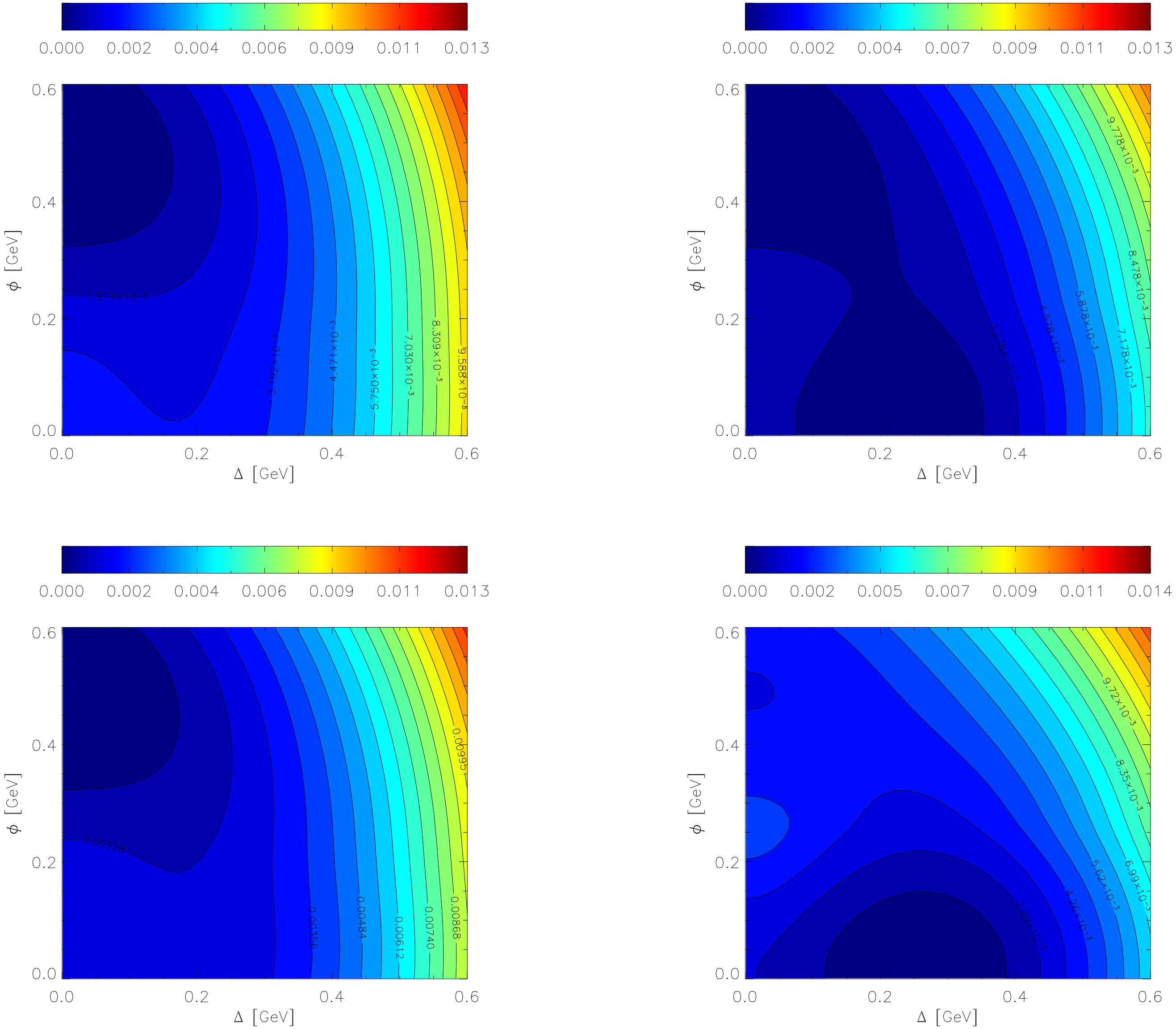
The T-mu plane: walking the routes (III)

Map of routes:



Schneekoppe

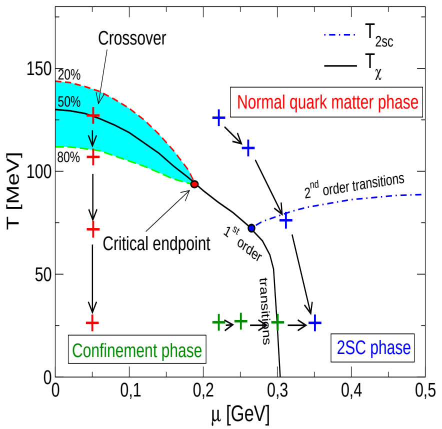
Route 2: confined ⟶ superconducting

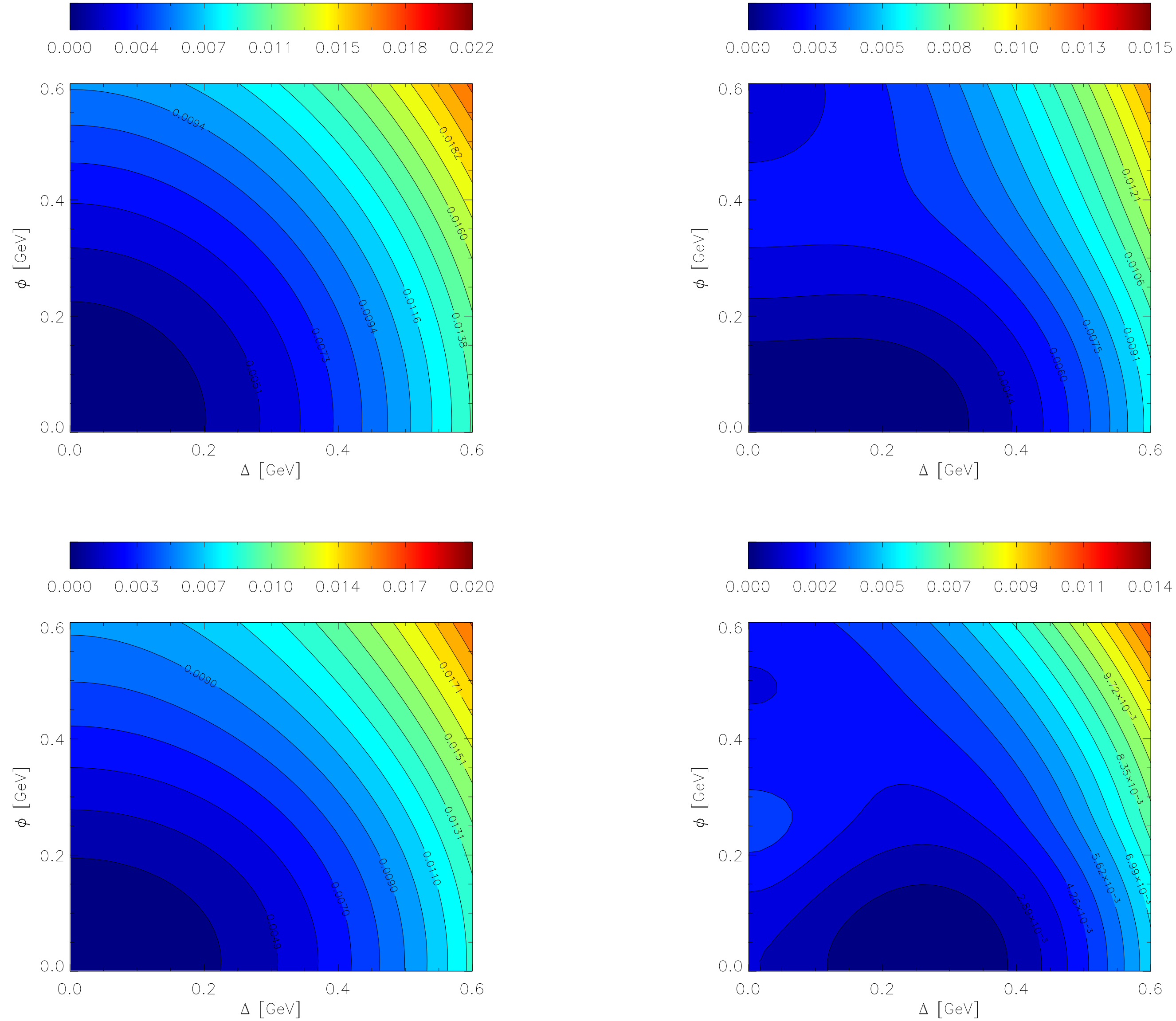


The T-mu plane: walking the routes (IV)

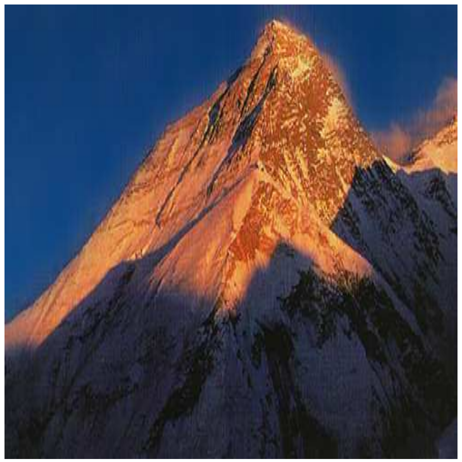
Route 3: deconfi ned ⟶ superconducting

Map of routes:



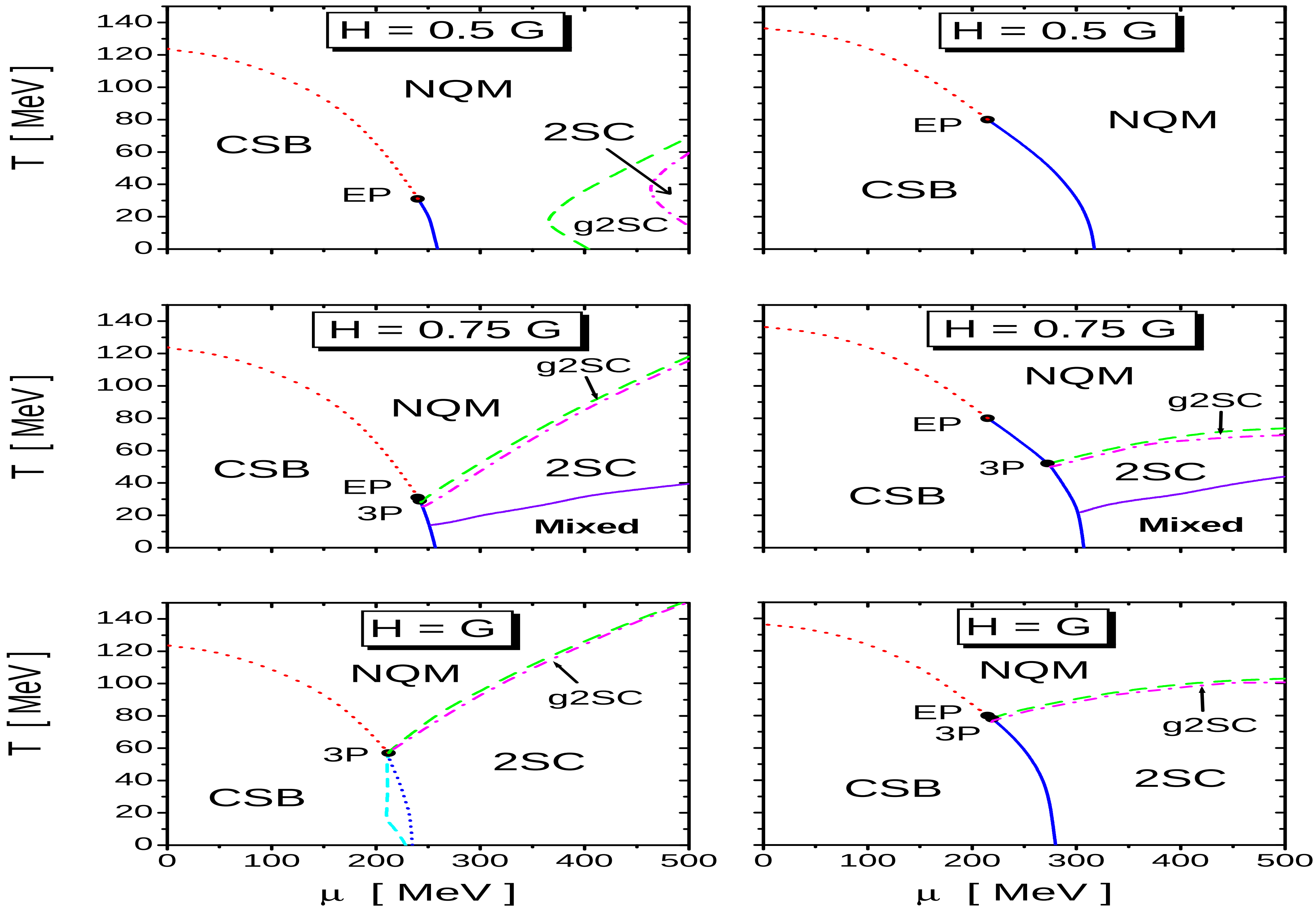


Mount Everest

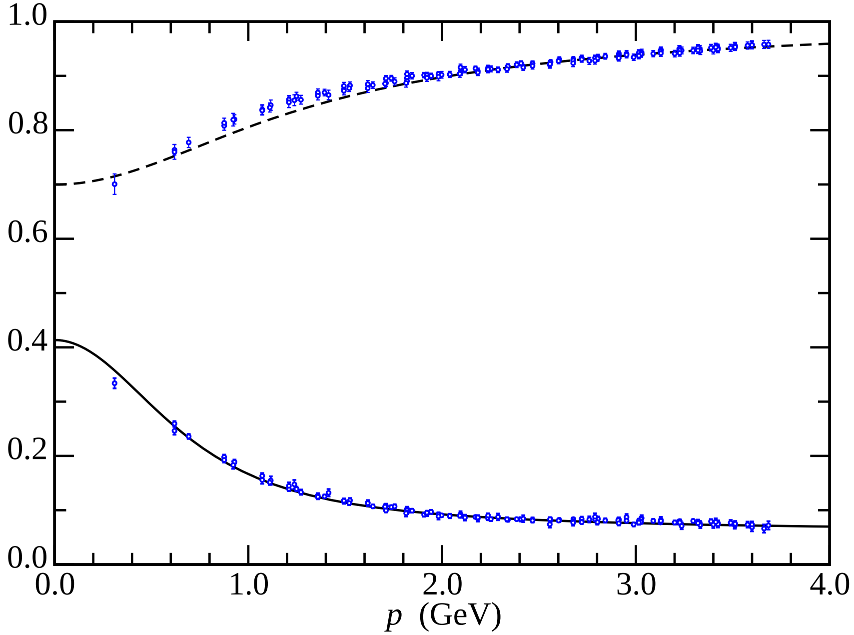


Phase diagram for nonlocal chiral quark model (II)

Instanton model (left) vs. One-Gluon-Exchange model (right)



Complex mass pole fit to Lattice propagator



BHAGWAT, PICHOWSKY, ROBERTS, TANDY, PHYS. REV. **C68** (2003) 015203

$$S(p)^{-1} = i\not p A(p^2) + B(p^2) \,,$$

$$M(p^2) = B(p^2)/A(p^2)$$

$$Z(p^2) = 1/A(p^2)$$

$S(p)$ sum of N pairs of complex conj. mass poles

$$S(p) = \sum_{i=1}^{N} \frac{1}{Z_2} \left\{ \frac{z_i}{i\not p + m_i} + \frac{z_i^*}{i\not p + m_i^*} \right\} = -i\not p \sigma_V(p^2) + \sigma_S(p^2)$$

Representation of the scalar amplitude

$$\sigma_S(p^2) = \sum_{i=1}^{N} Z_2^{-1} \left\{ \frac{z_i\, m_i}{p^2 + m_i^2} + \frac{z_i^*\, m_i^*}{p^2 + m_i^{*2}} \right\}$$

"Derivation" of the equivalent separable model (in Feynman-like gauge) $D_{\mu\nu}(p-q) = \delta_{\mu\nu}\, D(p,q)$ and

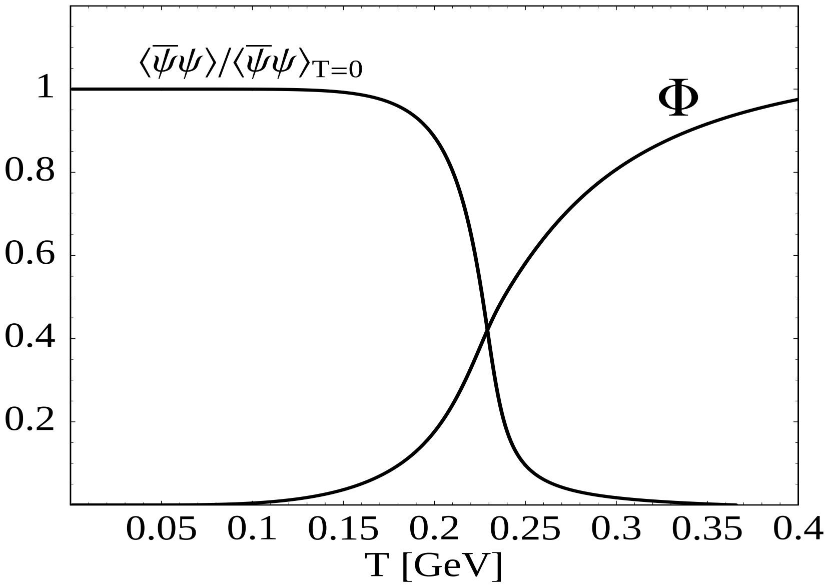
$$D(p,q) = f_0(p^2)\, f_0(q^2) + f_1(p^2)\, p\cdot q\, f_1(q^2)$$

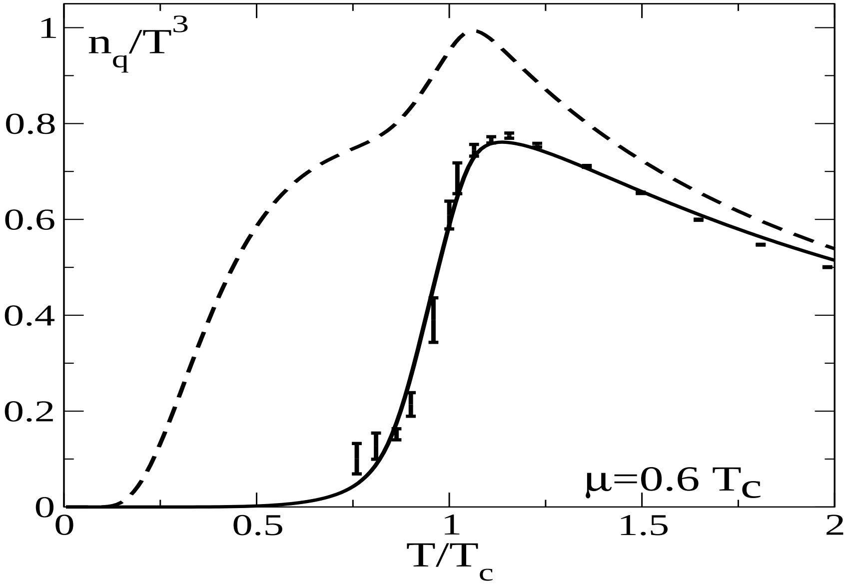
$$f_1(p^2) = \frac{A(p^2)-1}{a} \quad ; \quad f_0(p^2) = \frac{B(p^2)-m_c}{b}$$

$$b^2 = \frac{16}{3} \int_q^\Lambda [B(q^2) - m_c] \sigma_s(q^2)$$

$$a^2 = \frac{8}{3} \int_q^\Lambda [A(q^2) - 1] \frac{q^2}{4} \sigma_v(q^2)$$

Confinement: Polyakov Loop Chiral Quark Model





Grand canonical thermodynamical potential

$$\begin{aligned}\Omega(T,\mu;\Phi,m) &= \frac{\sigma^2}{2G} - 6N_f \int \frac{d^3p}{(2\pi)^3} E_p\,\theta\left(\Lambda^2 - \vec{p}^{\,2}\right) \\ &- 2N_f\,T \int \frac{d^3p}{(2\pi)^3} \left\{ \mathrm{Tr}_c \ln\left[1 + L\,e^{-\left(E_p-\mu\right)/T}\right] \right. \\ &+ \left. \mathrm{Tr}_c \ln\left[1 + L^\dagger\,e^{-\left(E_p+\mu\right)/T}\right] \right\} + \mathcal{U}\left(\Phi,\bar{\Phi},T\right)\end{aligned}$$

Appearance of quarks below T_c largely suppressed:

$$\begin{aligned}&\ln\det\left[1 + L\,e^{-\left(E_p-\mu\right)/T}\right] + \ln\det\left[1 + L^\dagger\,e^{-\left(E_p+\mu\right)/T}\right] \\ &= \ln\left[1 + 3\left(\Phi + \bar{\Phi}e^{-\left(E_p-\mu\right)/T}\right) e^{-\left(E_p-\mu\right)/T} + e^{-3\left(E_p-\mu\right)/T}\right] \\ &+ \ln\left[1 + 3\left(\bar{\Phi} + \Phi e^{-\left(E_p+\mu\right)/T}\right) e^{-\left(E_p+\mu\right)/T} + e^{-3\left(E_p+\mu\right)/T}\right].\end{aligned}$$

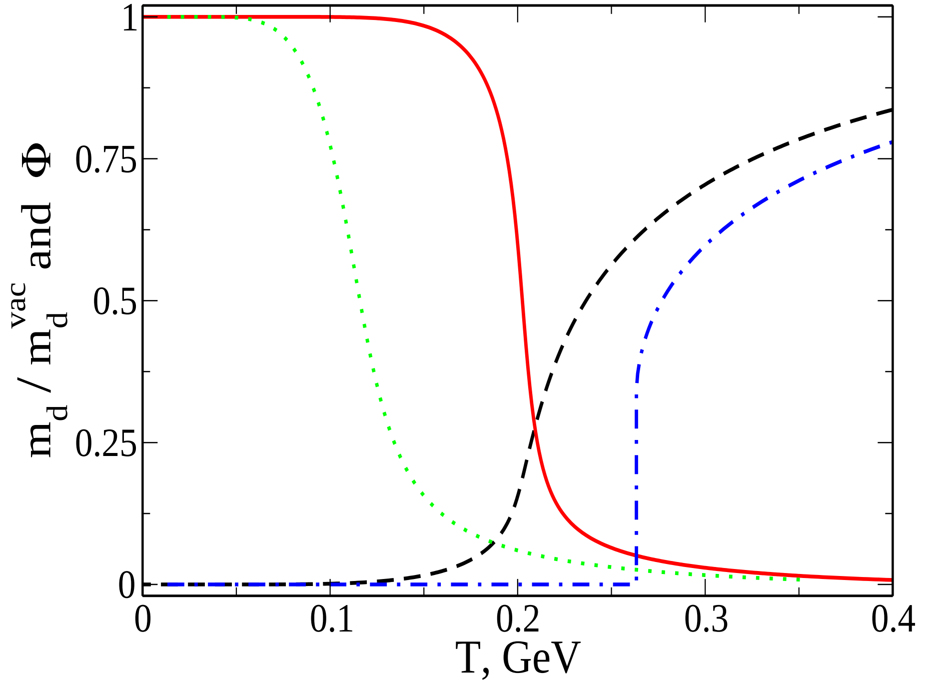
Accordance with QCD lattice susceptibilities! Example:

$$\frac{n_q\left(T,\mu\right)}{T^3} = -\frac{1}{T^3}\frac{\partial\Omega\left(T,\mu\right)}{\partial\mu},$$

Ratti, Thaler, Weise, PRD 73 (2006) 014019.

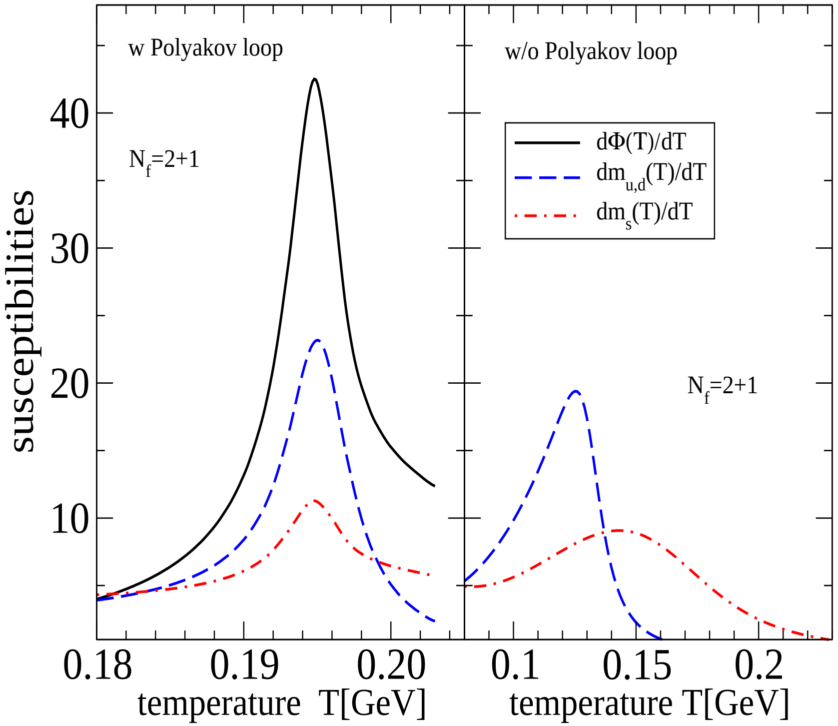
Nonlocal Polyakov Loop Chiral Quark Model

rank-1 separable, two-flavor, Instanton model



D.B., Buballa, Radzhabov, Volkov, arXiv:0705.0384
Yad. Fiz. (2008) in press.

rank-2 separable, three-flavor, OGE model



D.B., Horvatic, Klabucar, Radzhabov, in prep.

Nonlocal Polyakov Loop Chiral Quark Model

$$\Omega(T) \;=\; = \mathcal{U}(\Phi,\bar{\Phi}) - T\mathrm{Tr}_{\vec{p},n,\alpha,f,D}\left[\ln\{S_f^{-1}(p_n^\alpha,T)\} - \frac{1}{2}\Sigma_f(p_n^\alpha,T)\cdot S_f(p_n^\alpha,T)\right] \;, \tag{1}$$

where the full quark propagator for the flavor $f = u, d, s$,

$$\begin{aligned} S_f^{-1}(p_n^\alpha,T) &= S_{f,0}^{-1}(p_n^\alpha,T) - \Sigma_f^{-1}(p_n^\alpha,T) \\ &= i\vec{\gamma}\cdot\vec{p}\,A_f((p_n^\alpha)^2,T) + i\gamma_4\omega_n\,C_f((p_n^\alpha)^2,T) + B_f((p_n^\alpha)^2,T)\;, \end{aligned} \tag{2}$$

is defined by the DSE for the quark selfenergy Σ, see below. The Polyakov-loop potential is of the form:

$$\frac{\mathcal{U}(\Phi,\bar{\Phi})}{T^4} = -\frac{1}{2}a(T)\Phi^*\Phi + b(T)\ln\left[1 - 6\Phi^*\Phi + 4(\Phi^{*3}+\Phi^3) - 3(\Phi^*\Phi)^2\right]\;. \tag{3}$$

The Matsubara 4-momenta are defined as $(p_n^\alpha)^2 = (\omega_n^\alpha)^2 + \vec{p}^2$, $\quad \omega_n^\alpha = \omega_n + \alpha\phi_3$, $\quad \alpha = -1, 0, +1$, and are coupled to the Polyakov-loop variable $\Phi = \bar{\Phi} = \frac{1}{N_c}\left(1 + e^{i\frac{\phi_3}{T}} + e^{-i\frac{\phi_3}{T}}\right) = \frac{1}{N_c}\left(1 + 2\cos\left(\frac{\phi_3}{T}\right)\right)$. via the parameter ϕ_3.

Employing for the effective gluon propagator in a Feynman-like gauge, $g^2 D^{\mathrm{eff}}_{\mu\nu}(p-q) = \delta_{\mu\nu}D(p^2,q^2,p\cdot q)$, a rank-2 separable ansatz

$$D(p^2,q^2,p\cdot q) = D_0\mathcal{F}_0(p^2)\mathcal{F}_0(q^2) + D_1\mathcal{F}_1(p^2)(p\cdot q)\mathcal{F}_1(q^2)\;, \tag{4}$$

the propagator amplitudes are given by

$$B_f(p_n^2,T) = \widetilde{m}_f + b_f(T)\mathcal{F}_0(p_n^2)\;, \tag{5}$$

$$A_f(p_n^2,T) = 1 + a_f(T)\mathcal{F}_1(p_n^2)\;, \tag{6}$$

$$C_f(p_n^2,T) = 1 + c_f(T)\mathcal{F}_1(p_n^2)\;, \tag{7}$$

Phase diagram for 3-Flavor Quark Matter

Thermodynamic Potential $\Omega(T,\mu) = -T \ln Z[T,\mu]$

$$\Omega(T,\mu) \;=\; \frac{\phi_u^2+\phi_d^2+\phi_s^2}{8G_S} + \frac{|\Delta_{ud}|^2+|\Delta_{us}|^2+|\Delta_{ds}|^2}{4G_D} - T\sum_n \int \frac{d^3p}{(2\pi)^3}\frac{1}{2}\mathrm{Tr}\ln\left(\frac{1}{T}S^{-1}(i\omega_n,\vec{p})\right) + \Omega_e - \Omega_0 .$$

$$\mathrm{InverseNambu-GorkovPropagator} \quad S^{-1}(i\omega_n,\vec{p}) = \begin{bmatrix} \gamma_\mu p^\mu - M(\vec{p}) + \mu\gamma^0 & \widehat{\Delta}(\vec{p}) \\ \widehat{\Delta}^\dagger(\vec{p}) & \gamma_\mu p^\mu - M(\vec{p}) - \mu\gamma^0 \end{bmatrix},$$

$$\Delta_{k\gamma} = 2G_D\langle \bar{q}_{i\alpha} i\gamma_5 \epsilon_{\alpha\beta\gamma}\epsilon_{ijk} g(\vec{q}) q^C_{j\beta}\rangle . \quad \widehat{\Delta}(\vec{p}) = i\gamma_5\epsilon_{\alpha\beta\gamma}\epsilon_{ijk}\Delta_{k\gamma}g(\vec{p}).$$

Fermion Determinant (Tr ln D = ln det D)

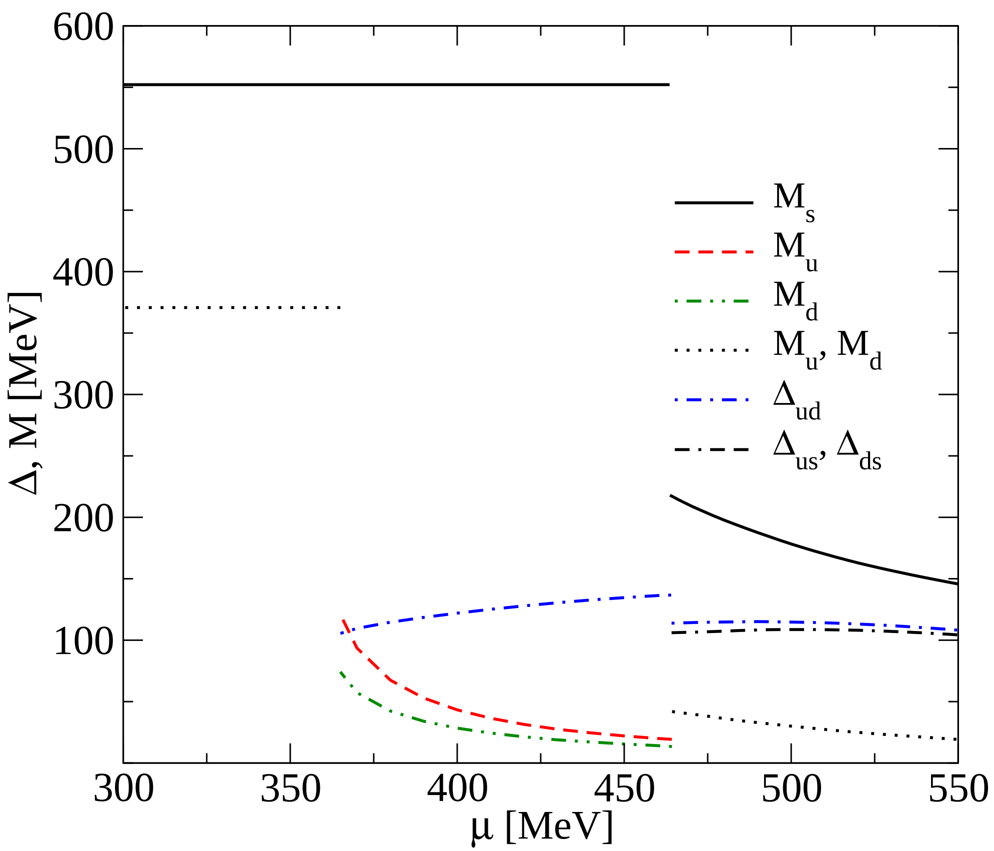
$$\mathrm{lndet}\left(\frac{1}{T}S^{-1}(i\omega_n,\vec{p})\right) = 2\sum_{a=1}^{18}\ln\left(\frac{\omega_n^2+\lambda_a(\vec{p})^2}{T^2}\right).$$

Result for the thermodynamic Potential (Meanfi eld approximation)

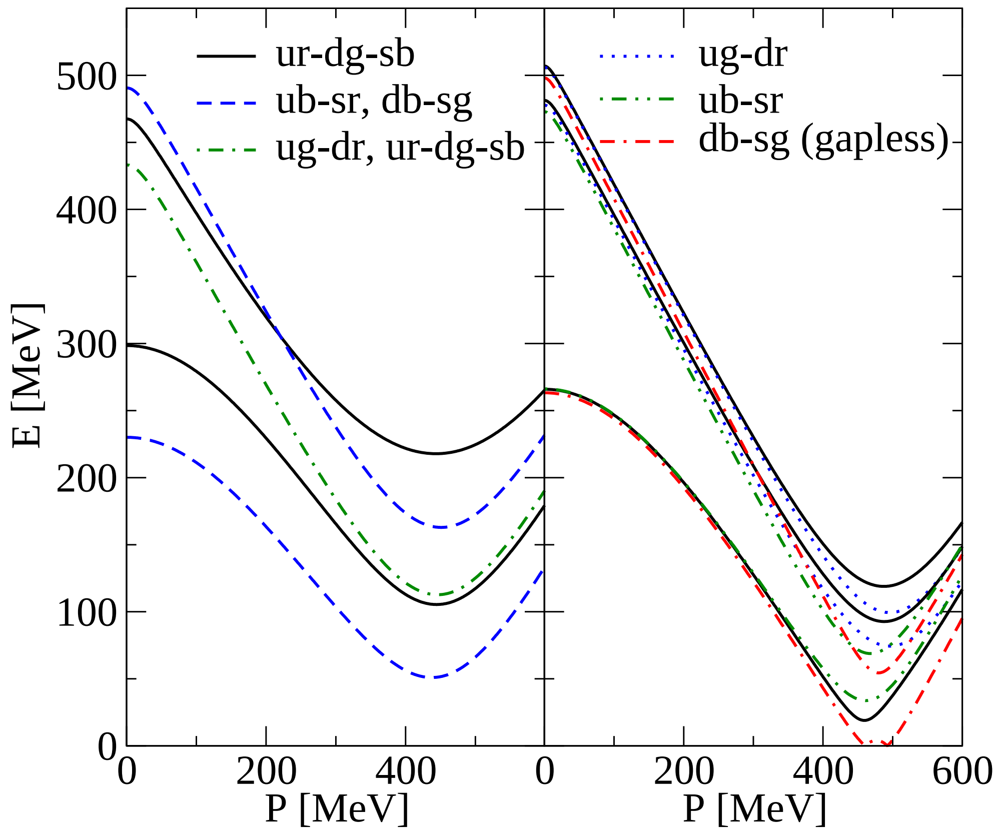
$$\Omega(T,\mu) \;=\; \frac{\phi_u^2+\phi_d^2+\phi_s^2}{8G_S} + \frac{|\Delta_{ud}|^2+|\Delta_{us}|^2+|\Delta_{ds}|^2}{4G_D} - \int \frac{d^3p}{(2\pi)^3}\sum_{a=1}^{18}\left[\lambda_a + 2T\ln\left(1+e^{-\lambda_a/T}\right)\right] + \Omega_e - \Omega_0 .$$

Neutrality constraints: $n_Q = n_8 = n_3 = 0$, $n_i = -\partial\Omega/\partial\mu_i = 0$,
Equations of state: $P = -\Omega$, etc.

Quark Masses, Diquark Gaps, Gapless Modes



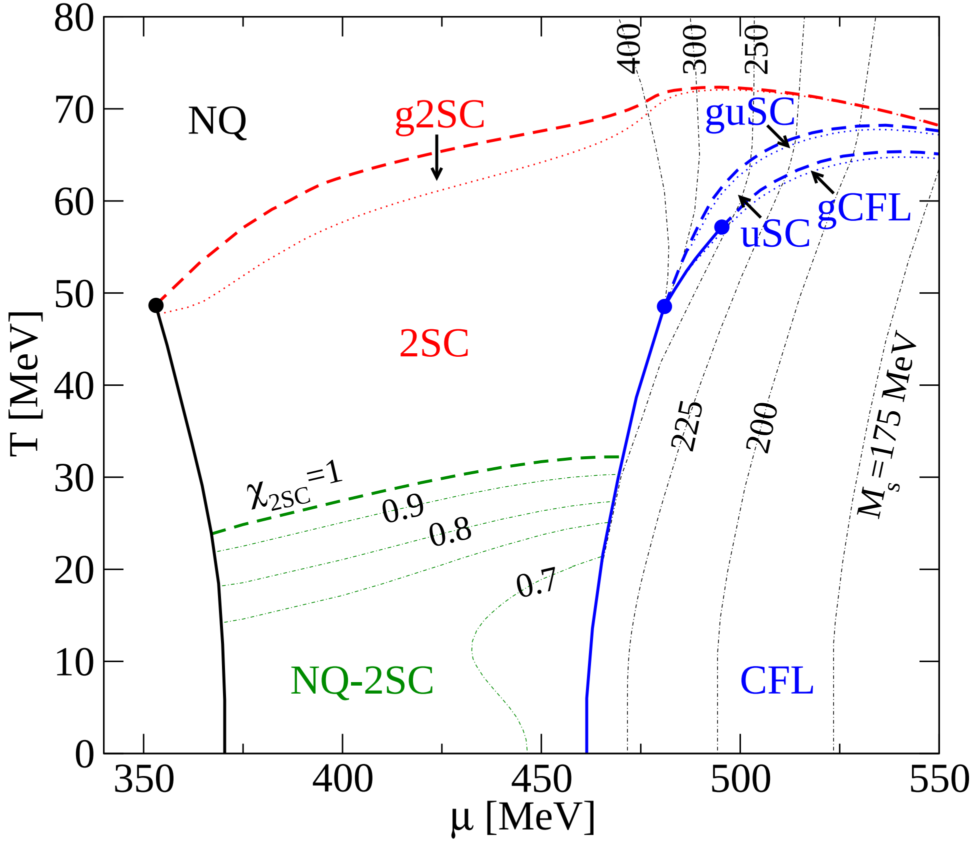
Dynamical quark masses and diquark gaps at $T = 0$ for intermediate diquark coupling $G_D = 0.75\ G_S$



Dispersion relations for $G_D = 0.75\ G_S$, $T = 0$, $\mu = 465$ MeV (left), $G_D = 1.0\ G_S$, $T = 59$ MeV, $\mu = 500$ MeV (right)

Rüster et al, PRD 72 (2005) 034004; Blaschke et al, PRD 72 (2005) 065020

Three-flavor Quark Matter Phase Diagram



Rüster et al, PRD 72 (2005) 034004;
Blaschke et al, PRD 72 (2005) 065020;
Abuki, Kunihiro, NPA768 (2006) 118;
Warringa et al, PRD 72 (2005) 014015

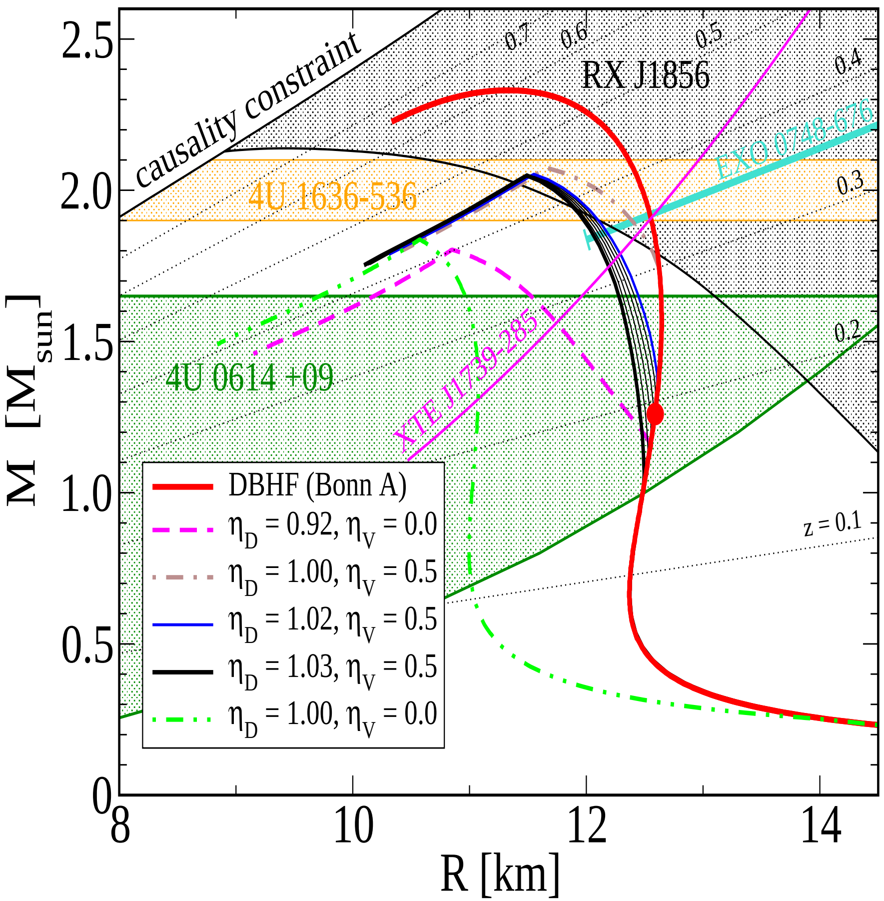
The phases are:

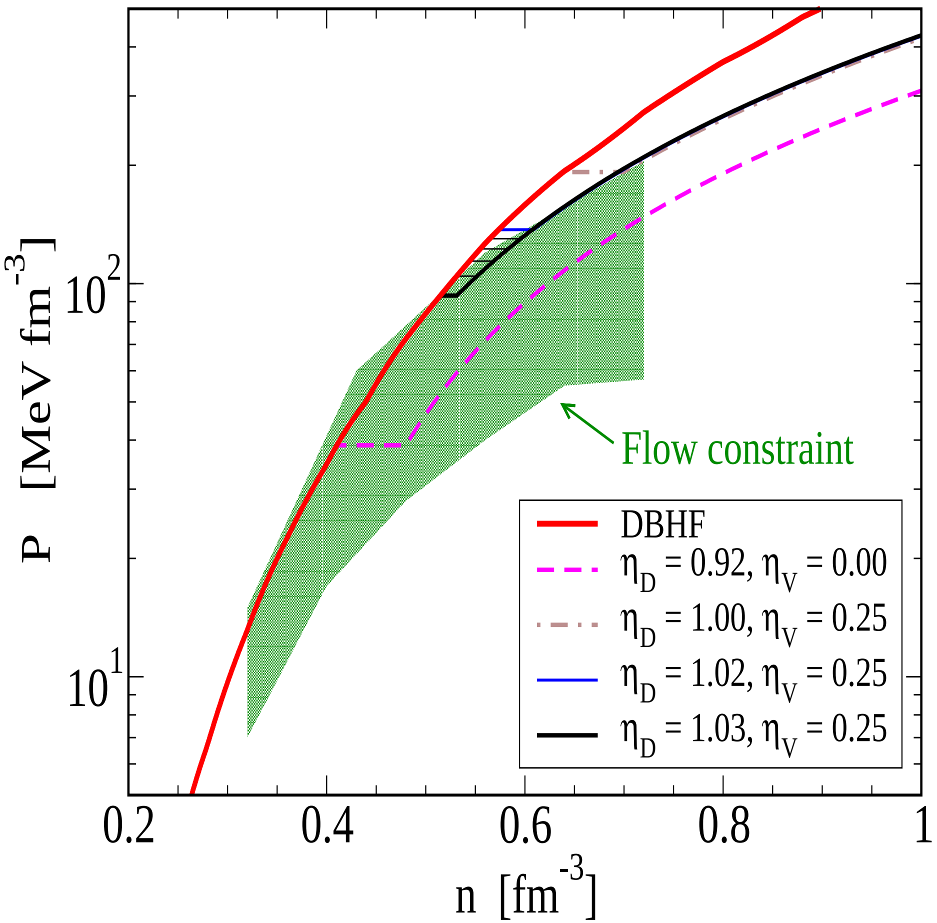
- NQ: $\Delta_{ud} = \Delta_{us} = \Delta_{ds} = 0$;
- NQ-2SC: $\Delta_{ud} \neq 0$, $\Delta_{us} = \Delta_{ds} = 0$, $0 \leq \chi_{2SC} \leq 1$;
- 2SC: $\Delta_{ud} \neq 0$, $\Delta_{us} = \Delta_{ds} = 0$;
- uSC: $\Delta_{ud} \neq 0$, $\Delta_{us} \neq 0$, $\Delta_{ds} = 0$;
- CFL: $\Delta_{ud} \neq 0$, $\Delta_{ds} \neq 0$, $\Delta_{us} \neq 0$;

Result:

- Gapless phases only at high T,
- CFL only at high chemical potential,
- At T $\leq$25-30 MeV: mixed NQ-2SC phase,
- Critical point (T_c,μ_c)=(48 MeV, 353 MeV),
- Strong coupling, $G_D = G_S$, similar,
  no NQ-2SC mixed phase.

Mass-Radius constraint and Flow constraint (II)

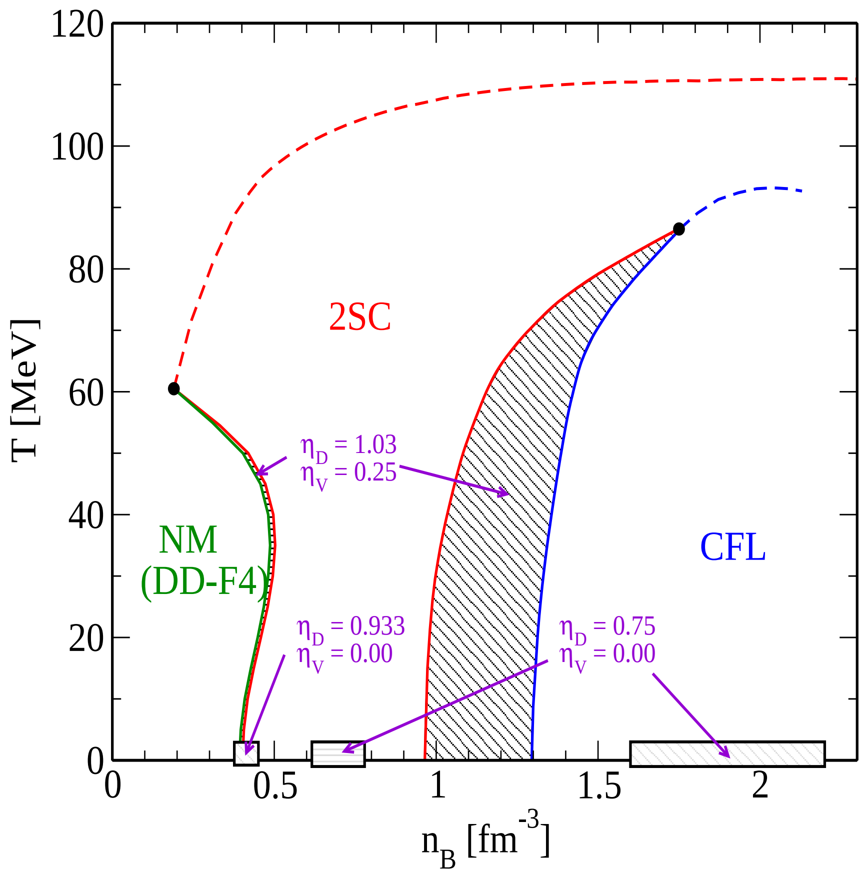


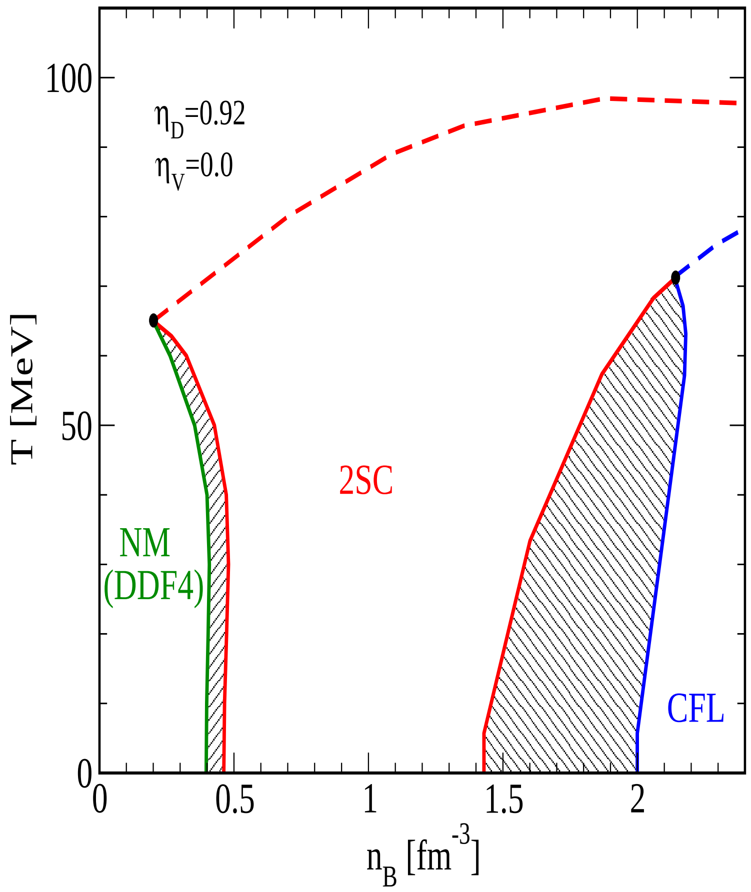


- Large Mass ($\sim 2\ M_\odot$) and radius ($R \geq 12$ km) $\Rightarrow$ stiff quark matter EoS;
  Note: DU problem of DBHF removed by deconfinement! and: CFL core Hybrids unstable!
- Flow in Heavy-Ion Collisions $\Rightarrow$ not too stiff EoS !
  Note: Quark matter removes violation by DBHF at high densities

Klähn, D.B., Sandin, Fuchs, Faessler, Grigorian, Röpke, Trümper, Phys. Lett. B567, 160 (2007)

Phase diagrams for the CBM experiment

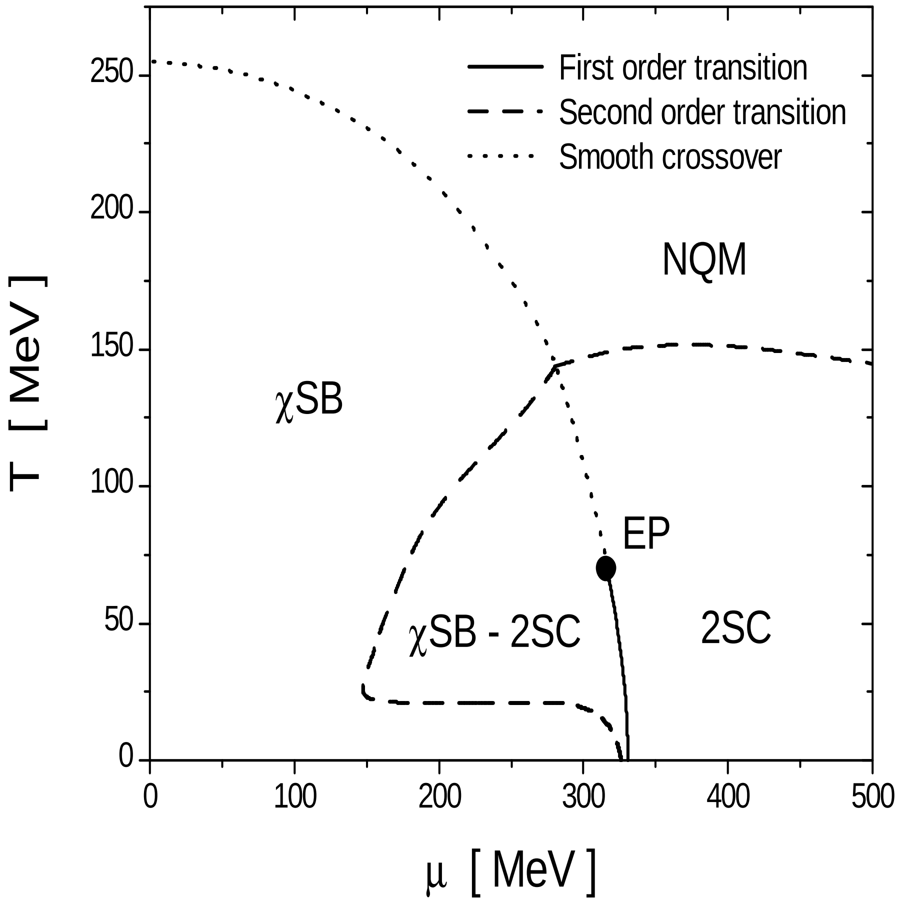


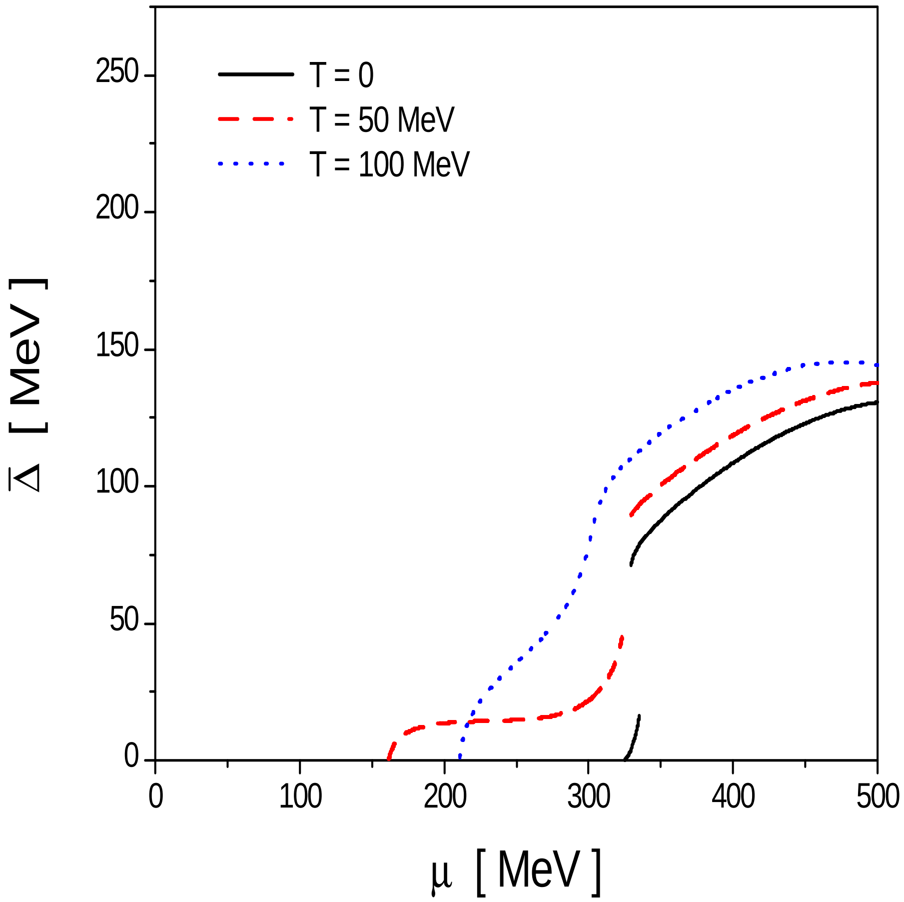


Phase diagrams for isospin-symmetric matter, for hybrid star maximum mass $M_{max} = 2.1\ M_\odot$ (left-hand side) and $M_{max} = 1.7\ M_\odot$ (right-hand side).

D. B., F. Sandin, S. Typel, in preparation.

Phase diagrams for CBM: Surprises?



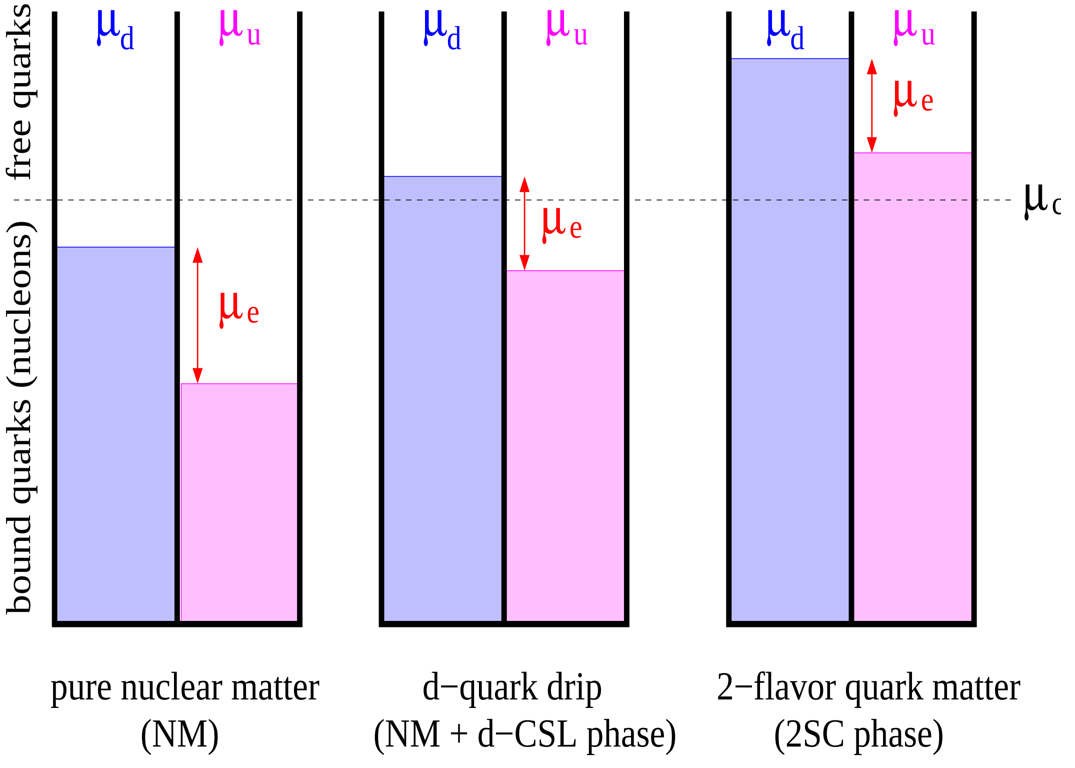


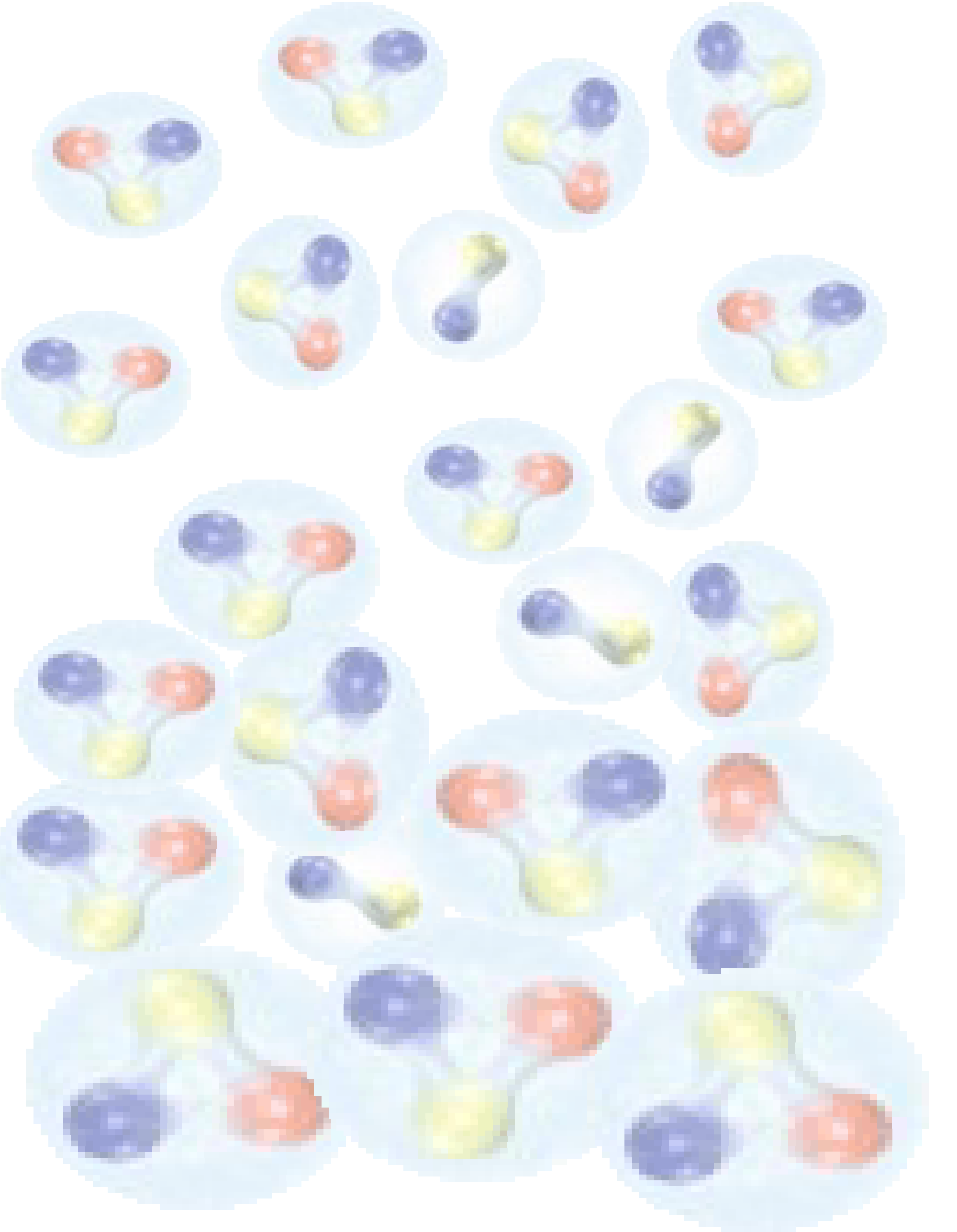
Effect of the color neutrality constraint on the Polyakov-loop NJL phase diagram:
Coexistence of χSB and 2SC phase $\Longrightarrow$ BEC-BCS crossover in the hadronic freezout region!

D. Gomez-Dumm, D. B., G.A. Grunfeld, N.N. Scoccola, arxiv:0807.1660 [hep-ph]

d-quark 'dripline' and single-flavor (d-CSL) phase

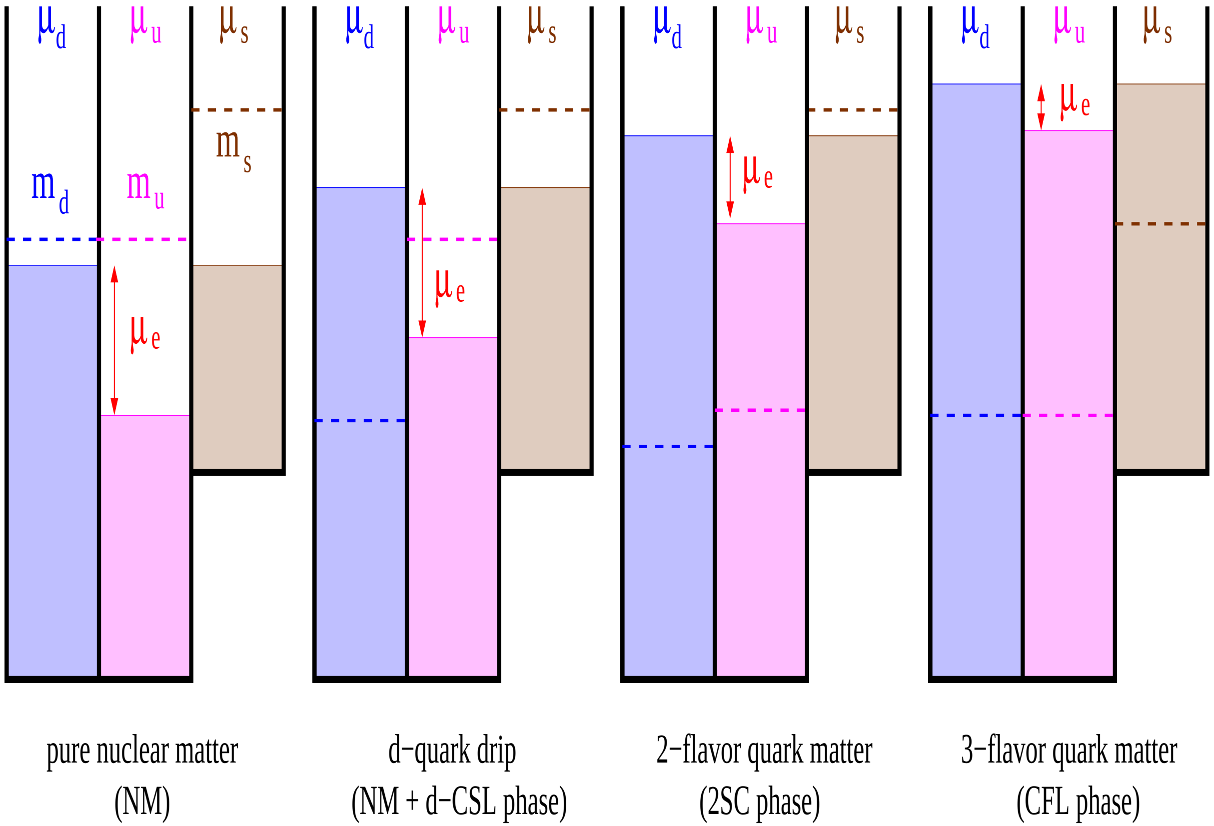
Sequential 'deconfi nement' of quark flavors

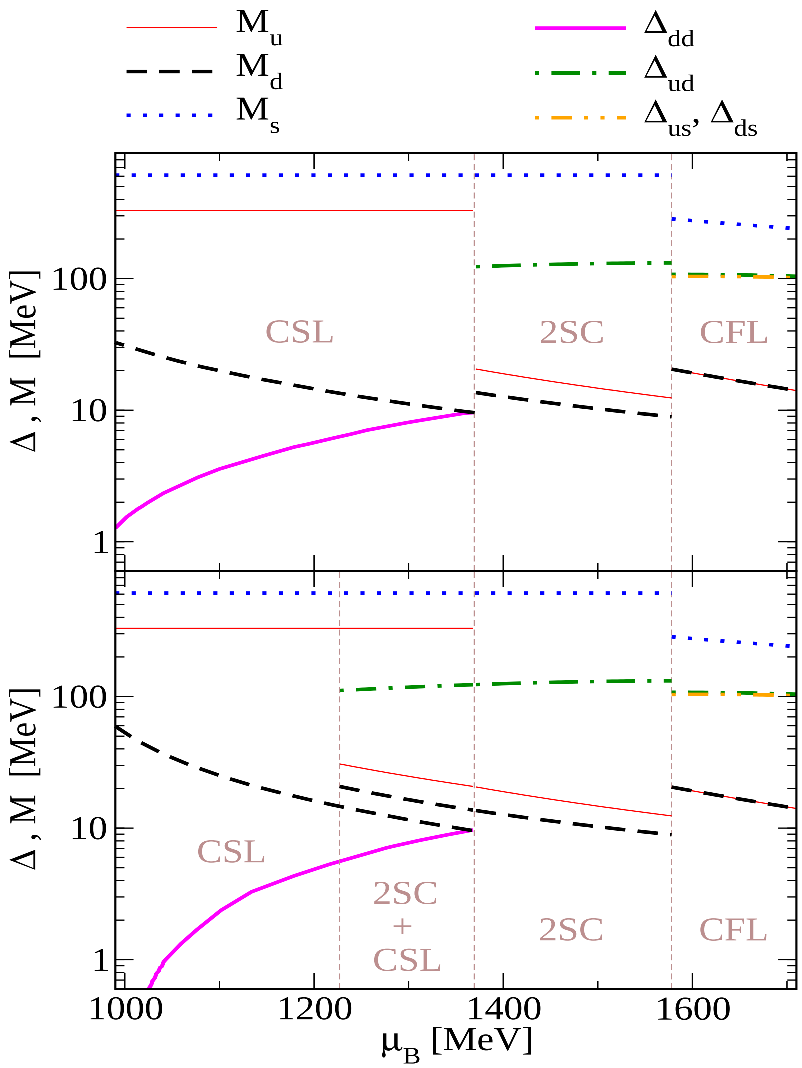




D.B., F. Sandin, T. Klähn, J. Berdermann, arXiv:0807.0414 [nucl-th]

Sequential deconfinement in asymmetric NS matter



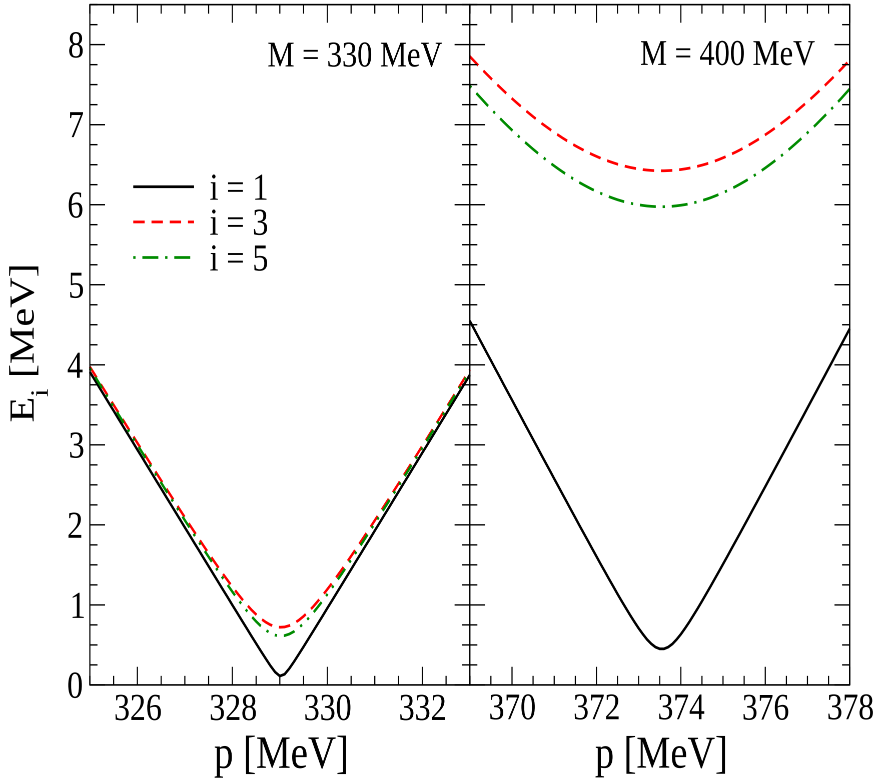


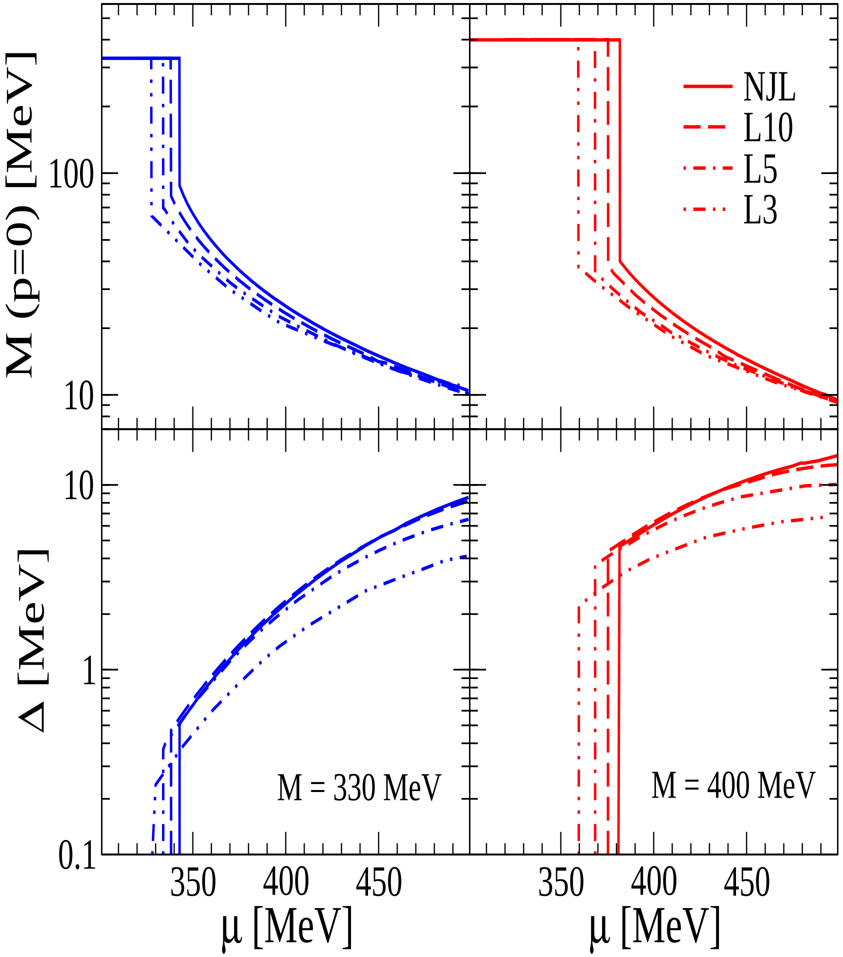
D.B., F. Sandin, T. Klähn, J. Berdermann, arXiv:0807.0414 [nucl-th]

Single-flavor spin-1 superconductor (d-CSL phase)

Ansatz: isotropic Color-spin-locking (CSL)

$$\hat{\Delta} = \Delta(\gamma^3\lambda_2 + \gamma^1\lambda_7 + \gamma^2\lambda_5)$$





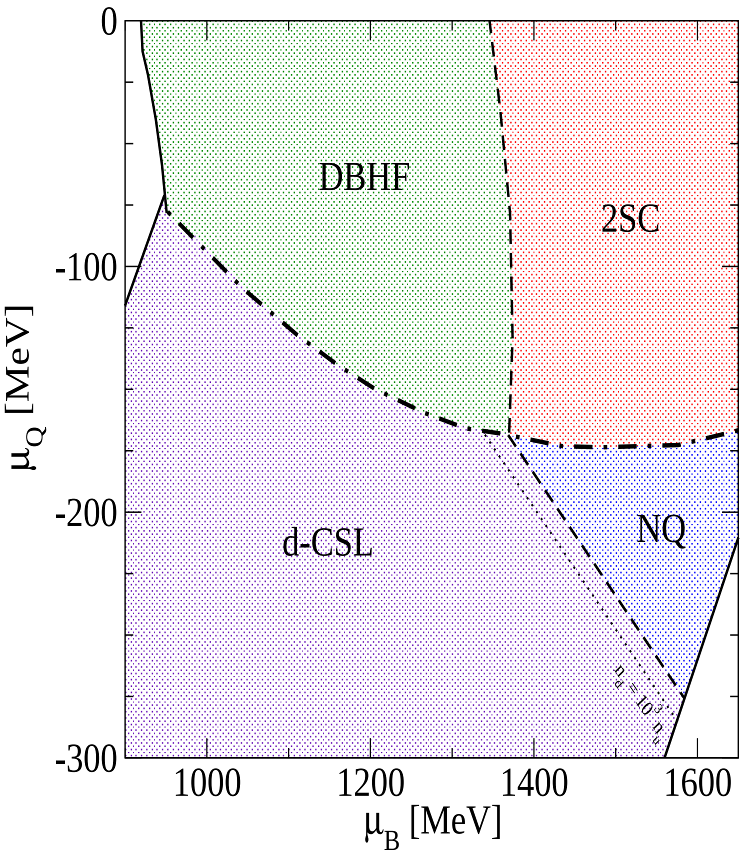
See also:

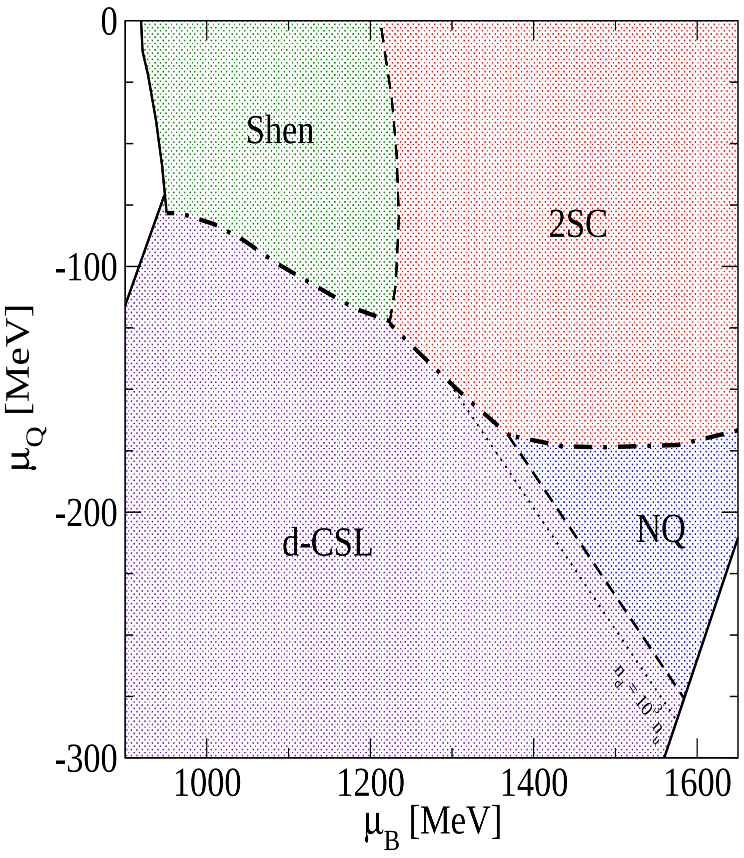
Schmitt, Wang, Rischke, PRD 66, 114010 (2002)

Aguilera et al., PRD 72 (2005) 034008;
PRD 74 (2006) 114005

d-CSL: single-flavor phase in competition

Dash-dotted lines: border between oppositely charged phases

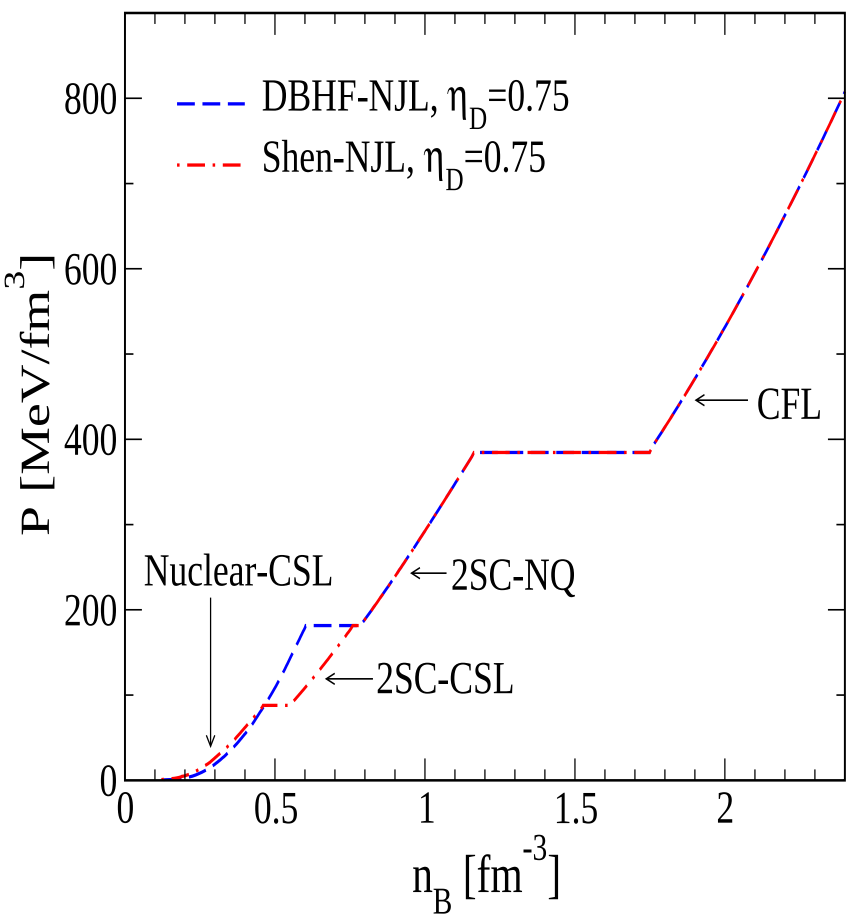




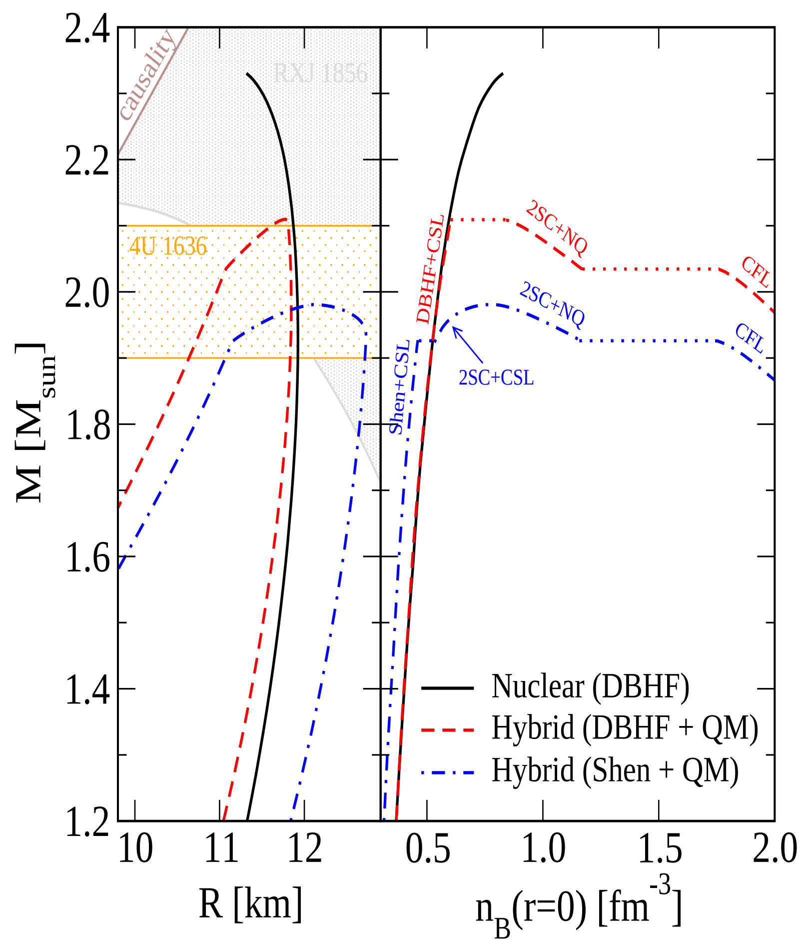
D.B., F. Sandin, T. Klähn, J. Berdermann, in preparation.

d-CSL: single-flavor phase in neutron stars

Equation of state



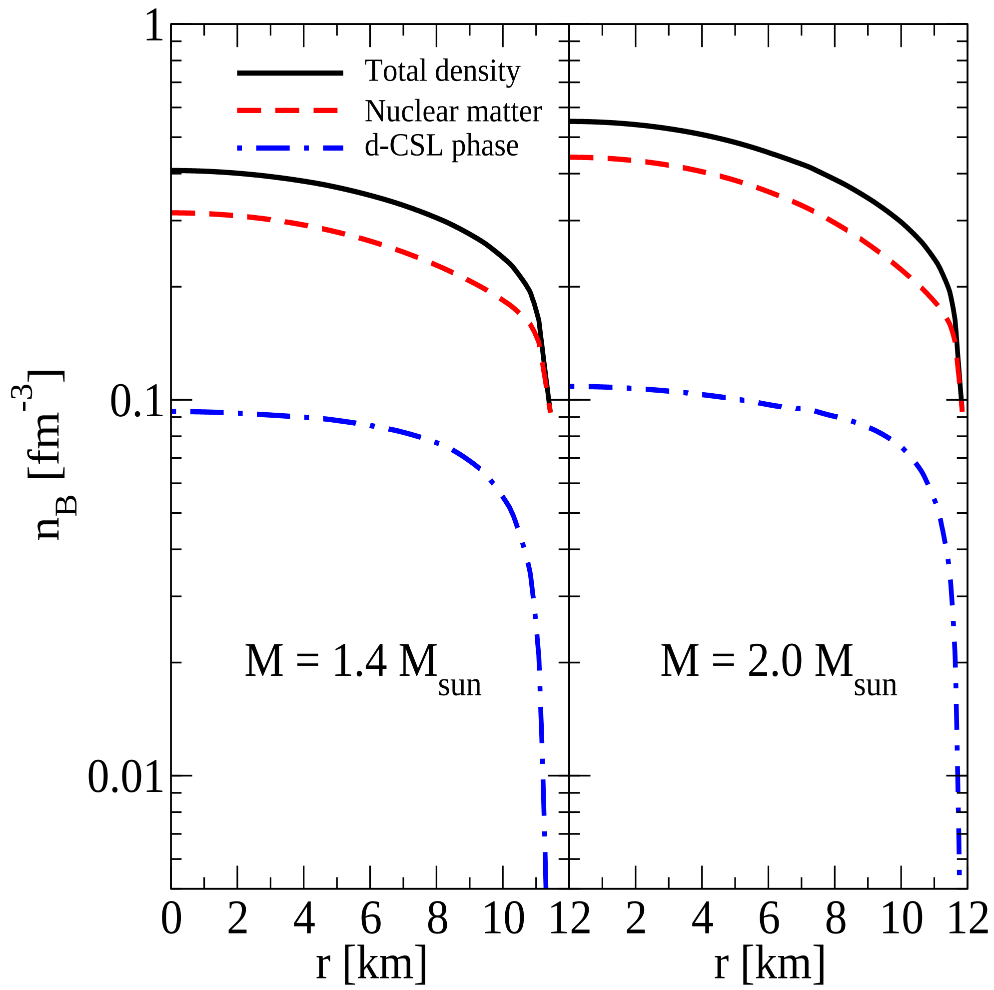
Configuration Sequences



D. B., F. Sandin, T. Klähn, J. Berdermann, arXiv:0807.0414 [nucl-th]

d-CSL: single-flavor phase in neutron stars (II)

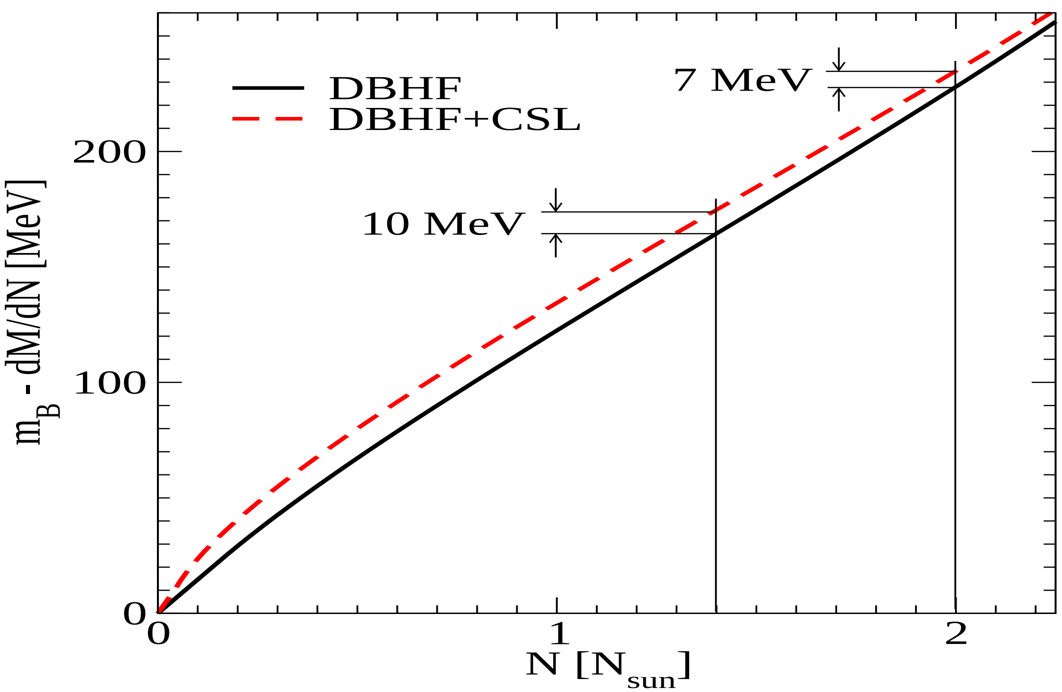
d-quark drip at crust-core boundary: Candidate for "deep crustal heating" (DCH) process?



Haensel and Zdunik,A& A **227**, 431 (1990)
Ushomirsky and Rutledge, MNRAS **325**, 1157 (2001)

Page and Cumming,ApJ **635**, L157 (2005): Superbursts & Strange Stars
Stejner and Madsen,A& A **458**, 523 (2006): SS + Transient Cooling

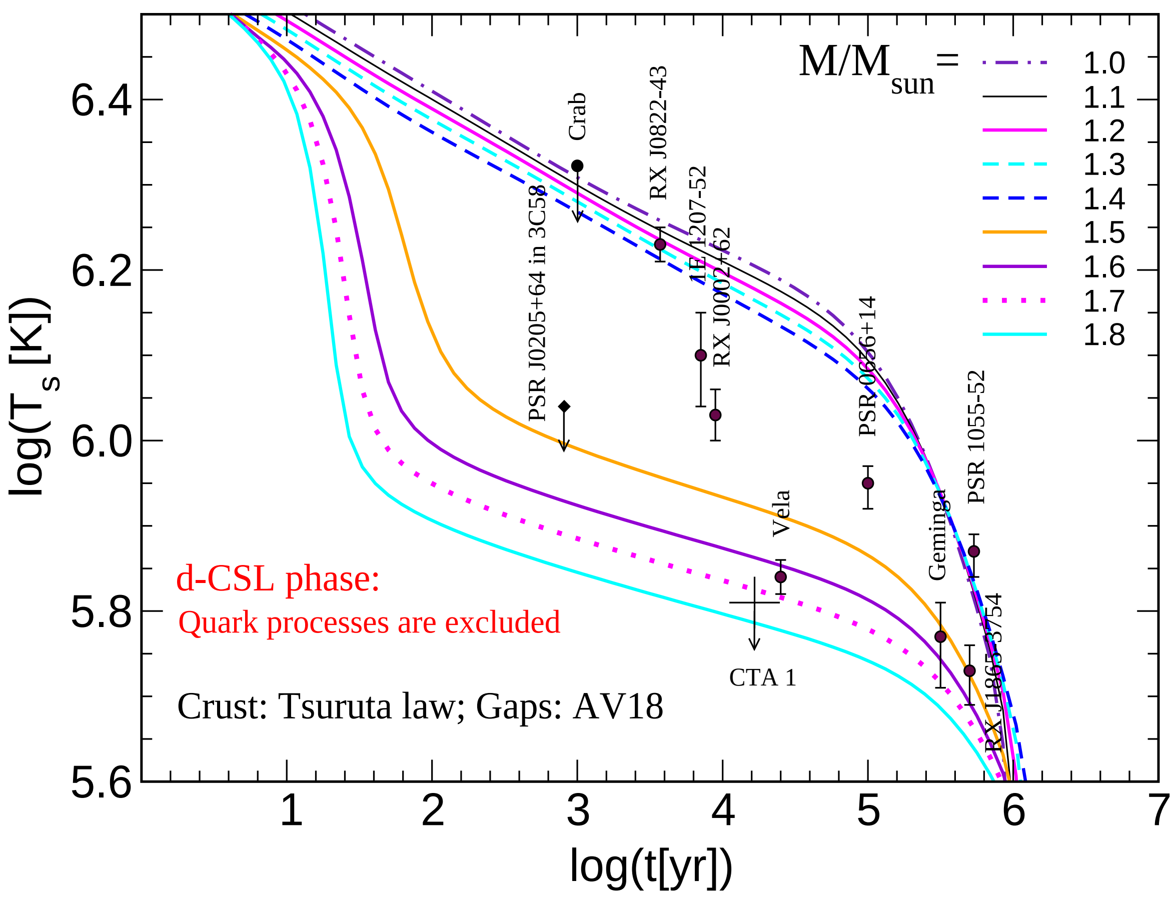
Shternin, Yakovlev, Haensel and Potekhin, MNRAS **382**, L43 (2007): KS1731



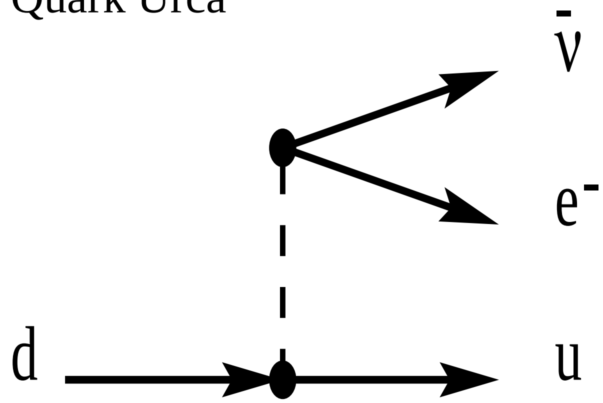
D. B., F. Sandin, T. Klähn, J. Berdermann, arXiv:0807.0414 [nucl-th]

d-CSL: single-flavor phase in neutron stars

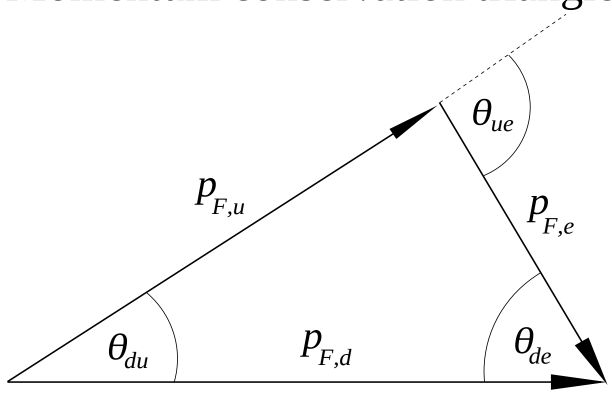
Cooling: processes in single-flavor quark matter are blocked!



Quark Urca



Momentum conservation triangle



not operative since u-quark Fermi sea not populated ($p_{F,u} = 0$)

D. B., F. Sandin, H. Grigorian, in preparation.

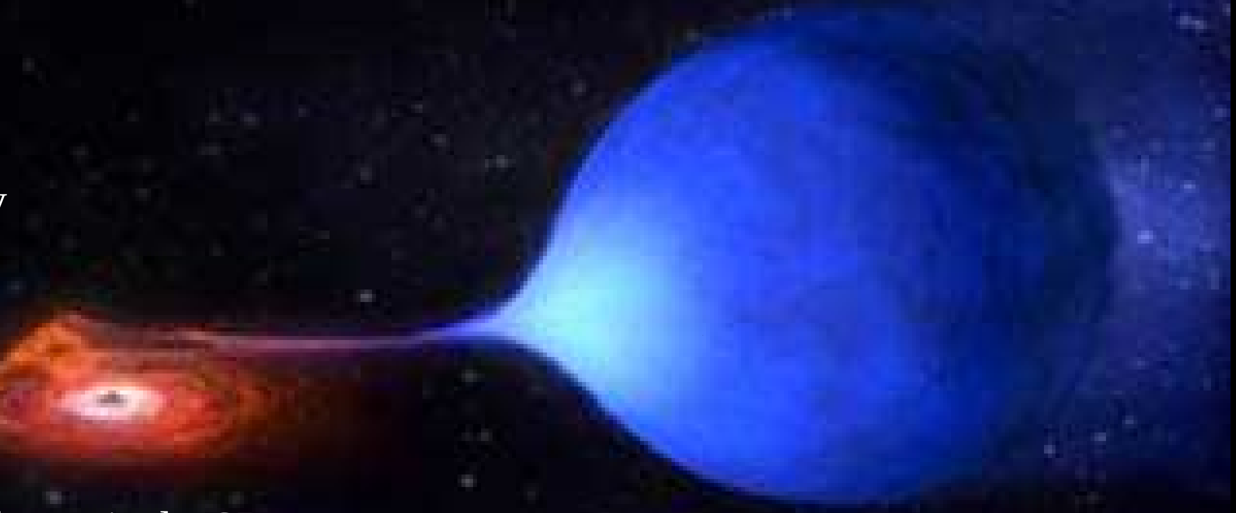
Conclusions

Constraints on the high-density EoS

- Compact star masses $\sim 2\ M_\odot$ require stiff EoS
- Flow data provide upper limits on the stiffness

Local charge neutrality: 2SC + DBHF hybrid

- diquark coupling lowers phase transition density
- vector meanfi eld stiffens quark matter EoS



Global charge neutrality: d-CSL + DBHF hybrid

- single flavor phase (d-CSL) as consequence of dynamical χSR
- no d-CSL in symmetric matter: $x_{p,crit} < 0.2$
- no Urca cooling processes $\rightarrow$ no neutrino trapping?

Next steps

- apply to superbursts, X-ray transients, high-mass supernovae
- extend to inhomogeneous phases: surface tension and Coulomb effects