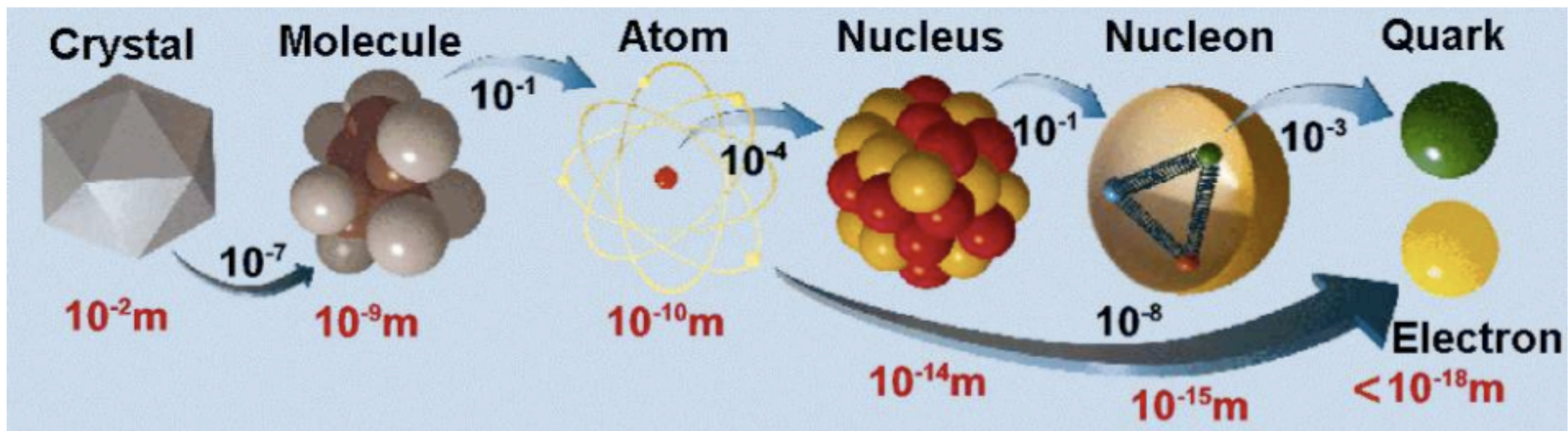
Chiral Phase Boundary from Quark-Gluon Dynamics

Jens Braun
TRIUMF
- Canada's National Laboratory for Particle and Nuclear Physics -

INT Program: The QCD Critical Point
31/07/2008

From Microscopic Degrees to Macroscopic DoF



large length scales

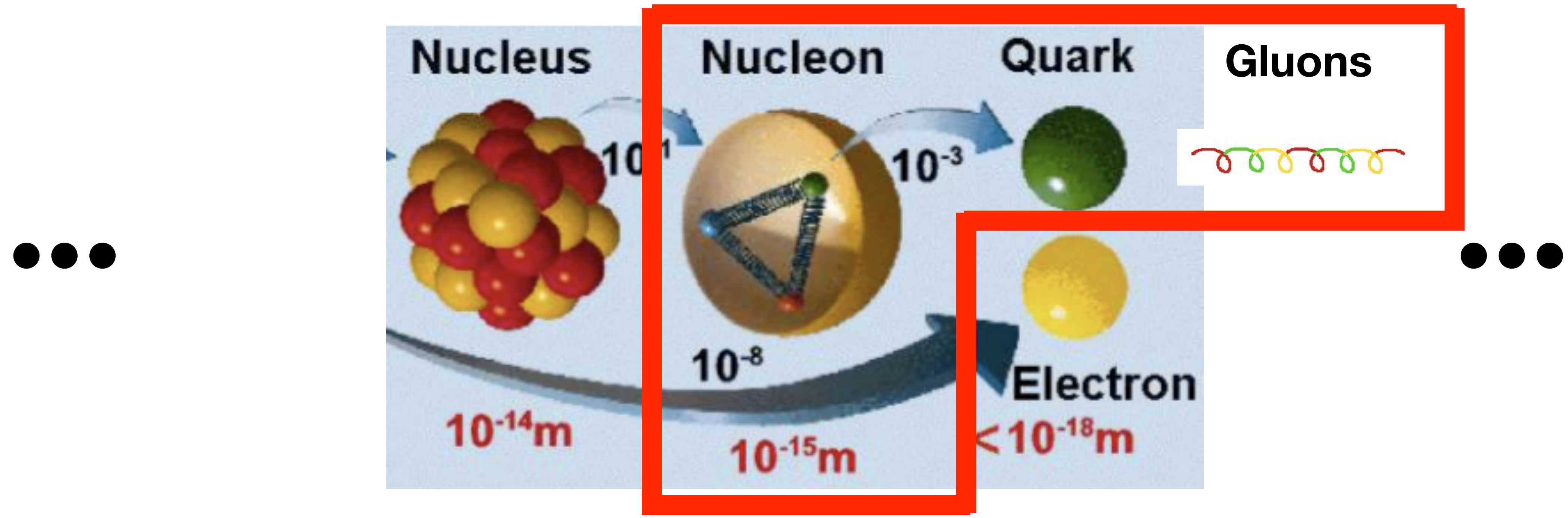
small length scales

small momentum scales

Renormalization Group

large momentum scales

From Microscopic Degrees to Macroscopic DoF



large length scales

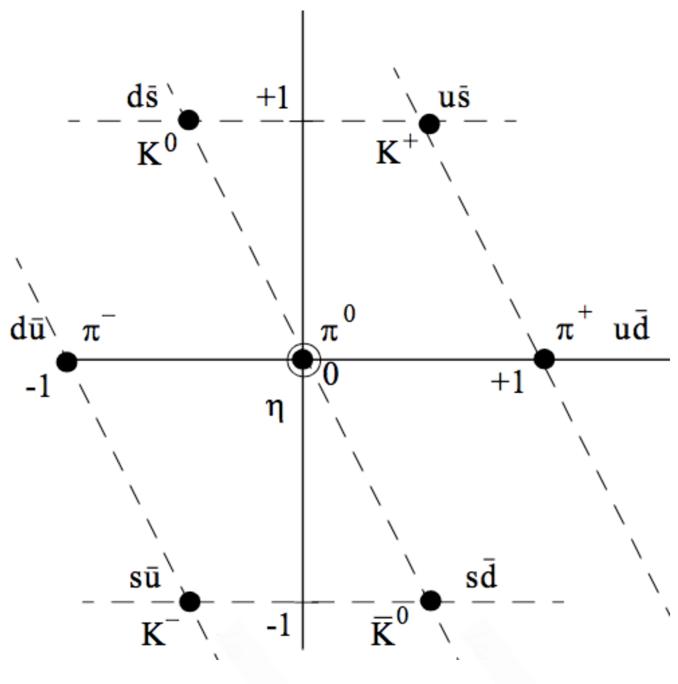
small length scales

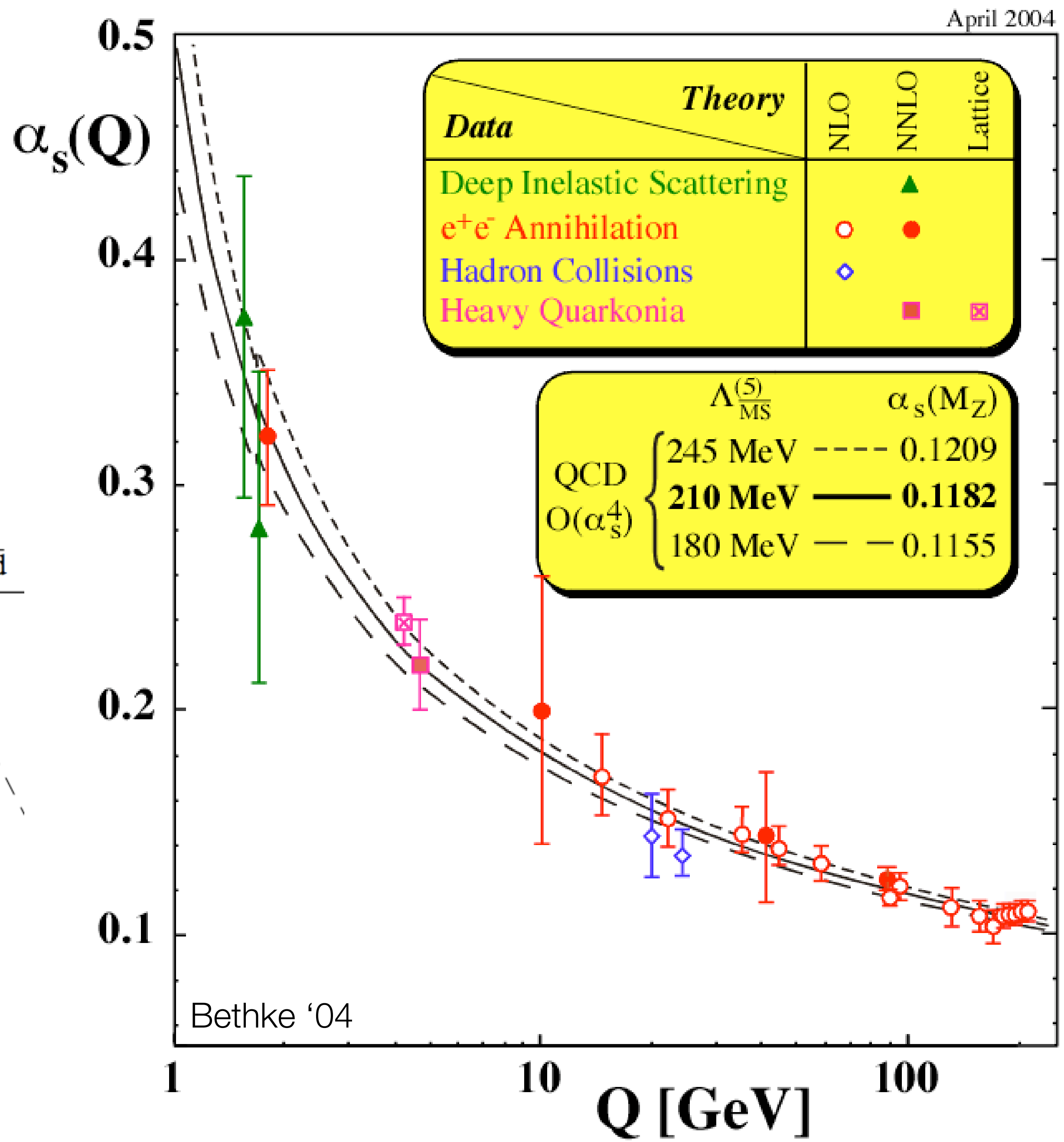
small momentum scales

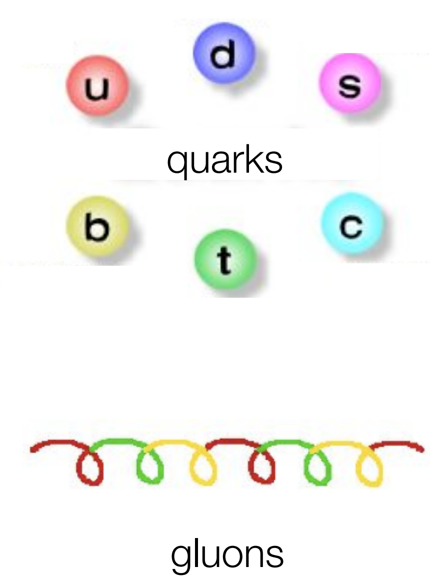
large momentum scales

Functional Renormalization Group flows (JB, H. Gies, J. M. Pawlowski)

Challenges in QCD

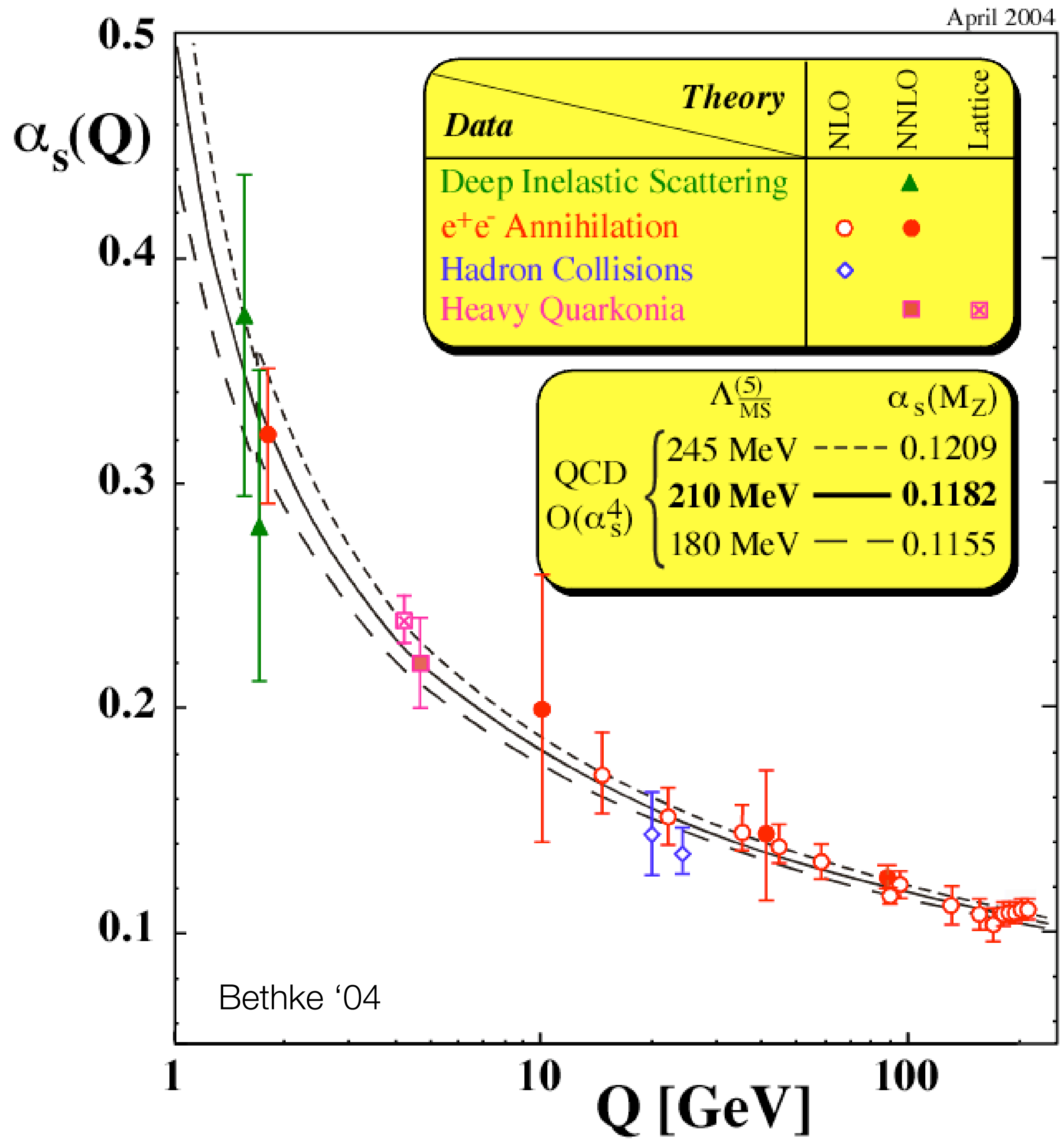




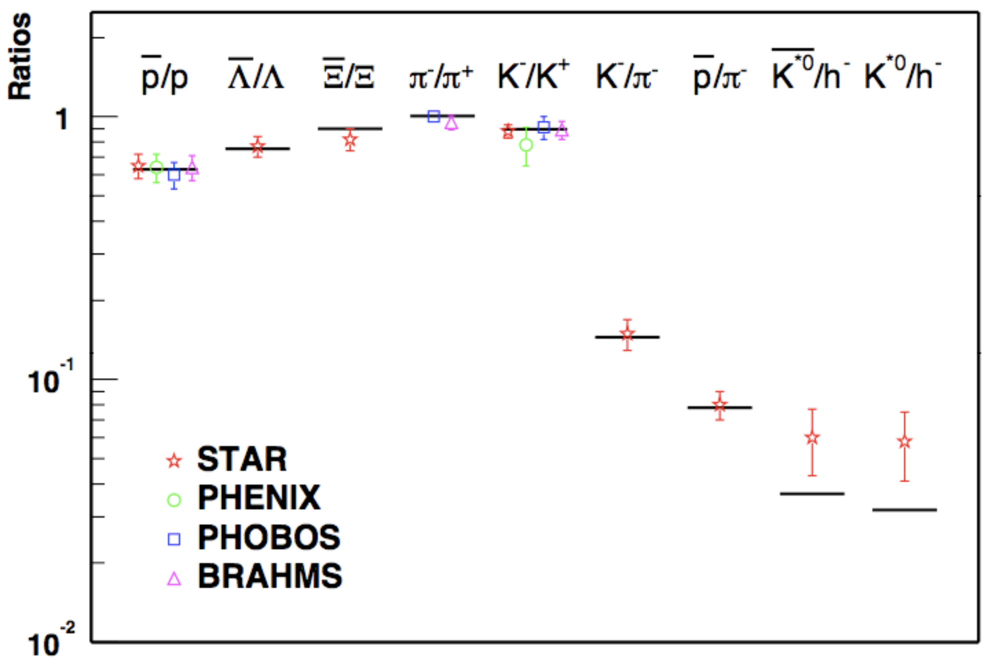


Challenges in QCD

- Asymptotic freedom at high momenta (Gross & Wilczek ’73, Politzer ’73)
- running coupling exhibits Landau pole at small momenta
  $\longrightarrow$ pQCD fails
- Understanding of QCD in the mid-momentum regime is needed to study confinement & chiral symmetry breaking



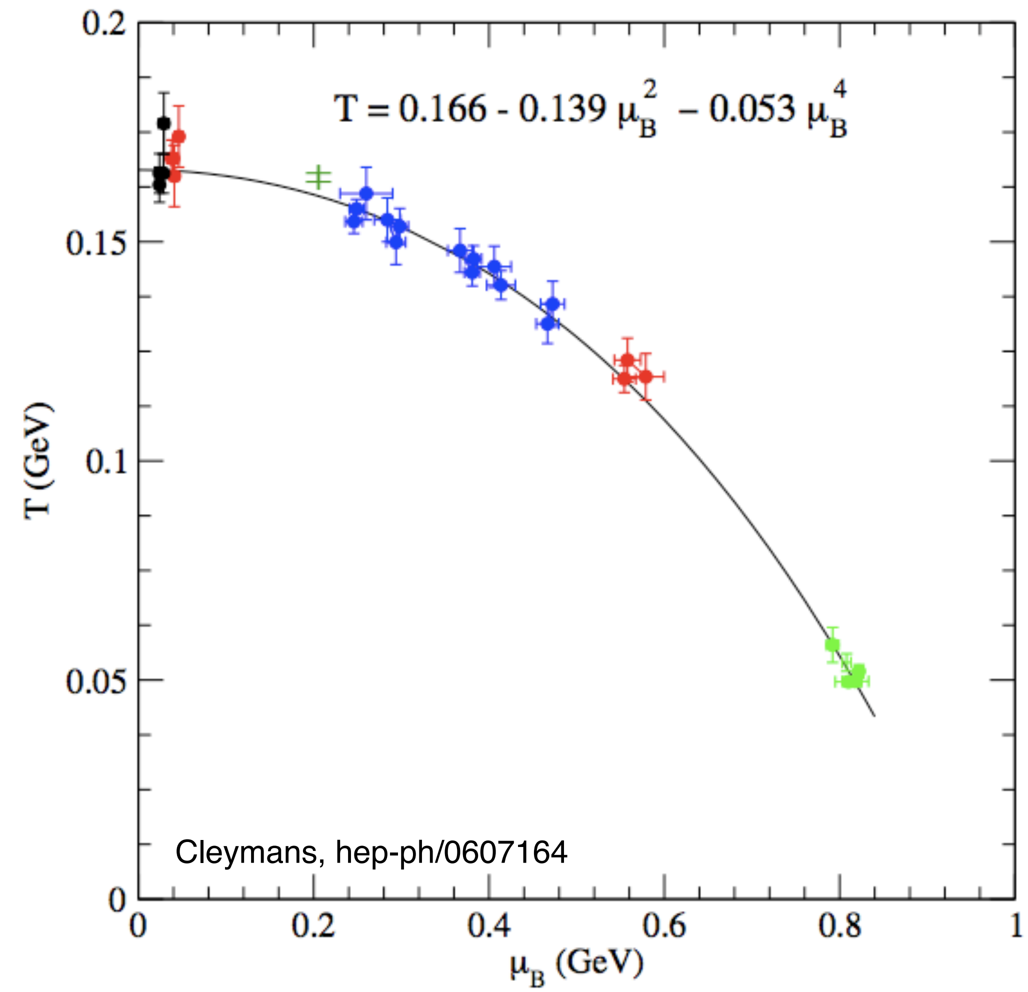
Heavy-Ion Collision Experiments



fitted very well with
Boltzmann-distribution

(T, μ_B) for given $\sqrt{s}$

chemical freeze-out

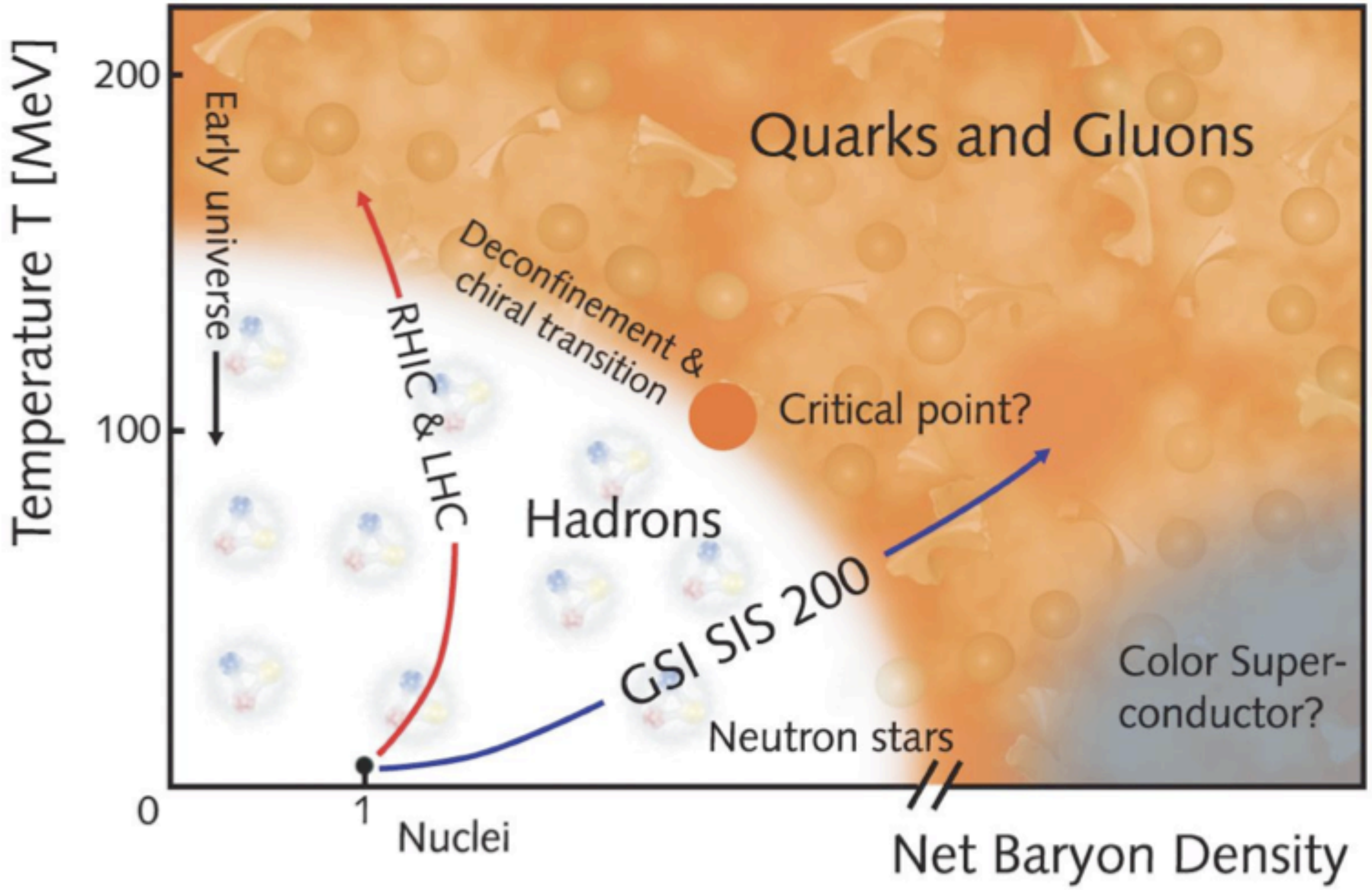


$$T_{\text{exp.}} \leq T_\chi$$

(P. Braun-Munzinger, J. Stachel, C. Wetterich ’03)

QCD phase diagram?

FAIR, www.gsi.de

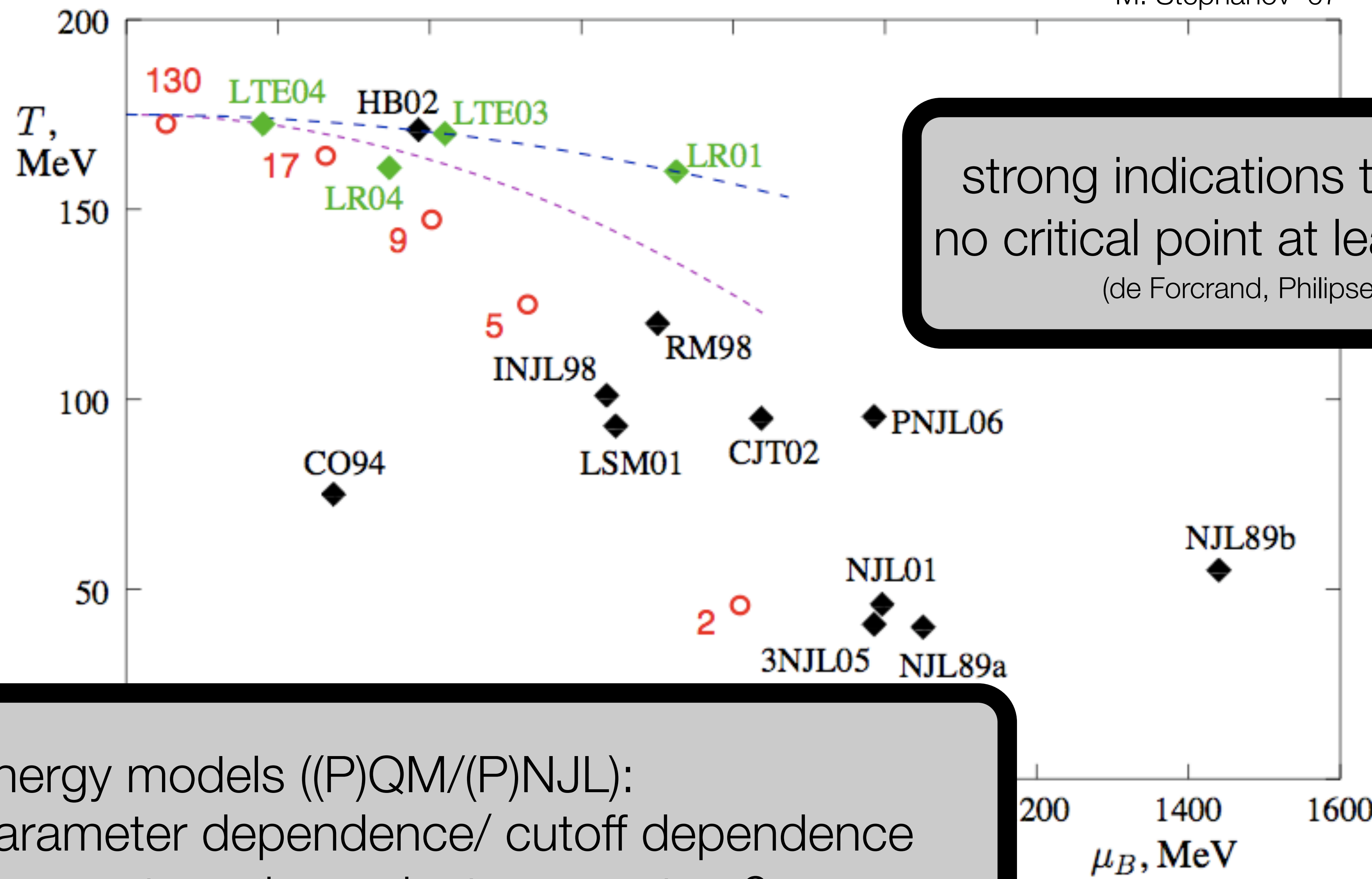


perturbation theory fails:

- not convergent even for very high temperatures
- phase transitions: long-range fluctuations are important

QCD phase diagram? Puzzles ...

M. Stephanov ‘07



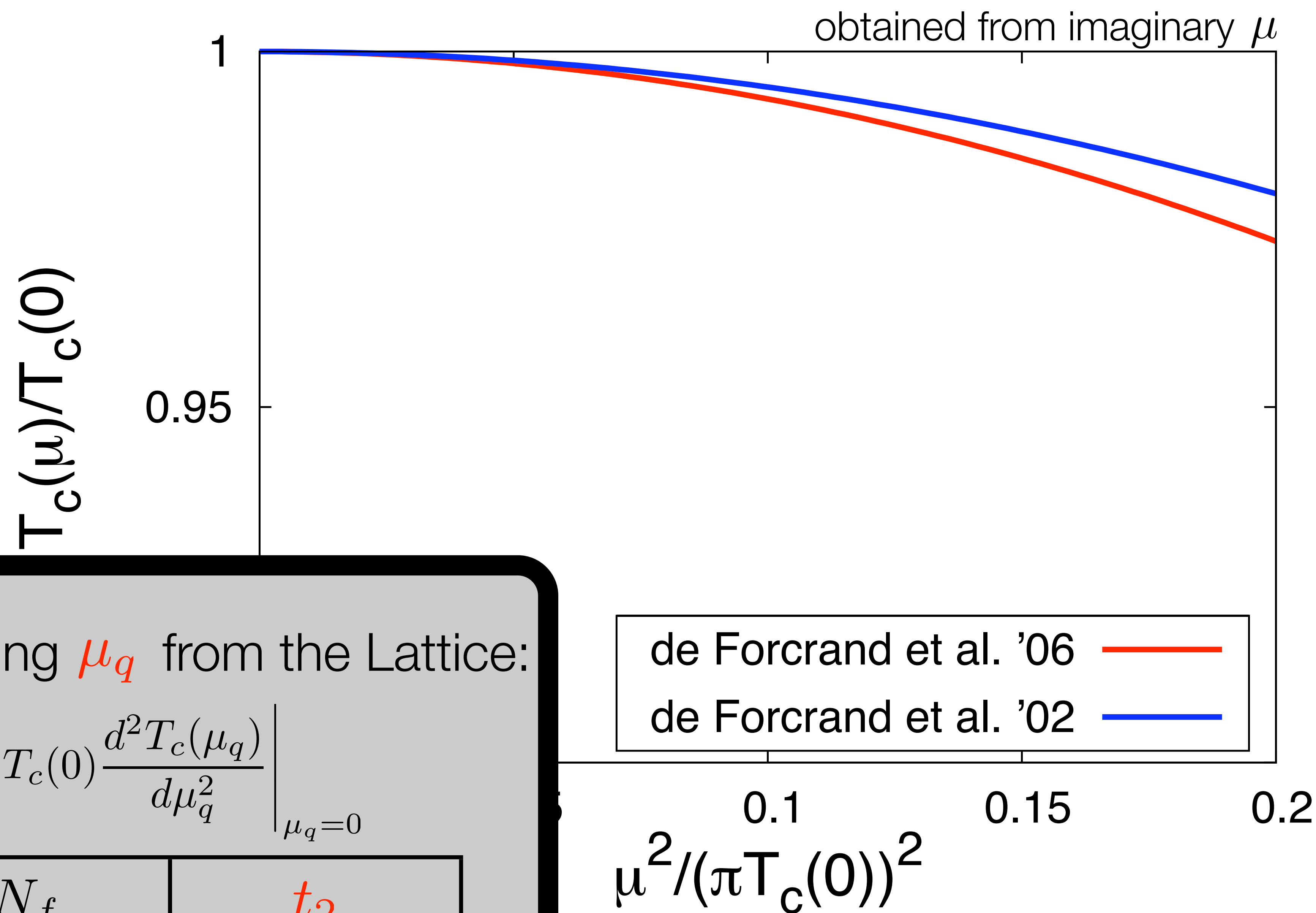
strong indications that there is no critical point at least at small μ

(de Forcrand, Philipsen ’07)

low-energy models ((P)QM/(P)NJL):

- parameter dependence/ cutoff dependence
- temperature-dependent parameters?
- truncated action

QCD phase diagram?

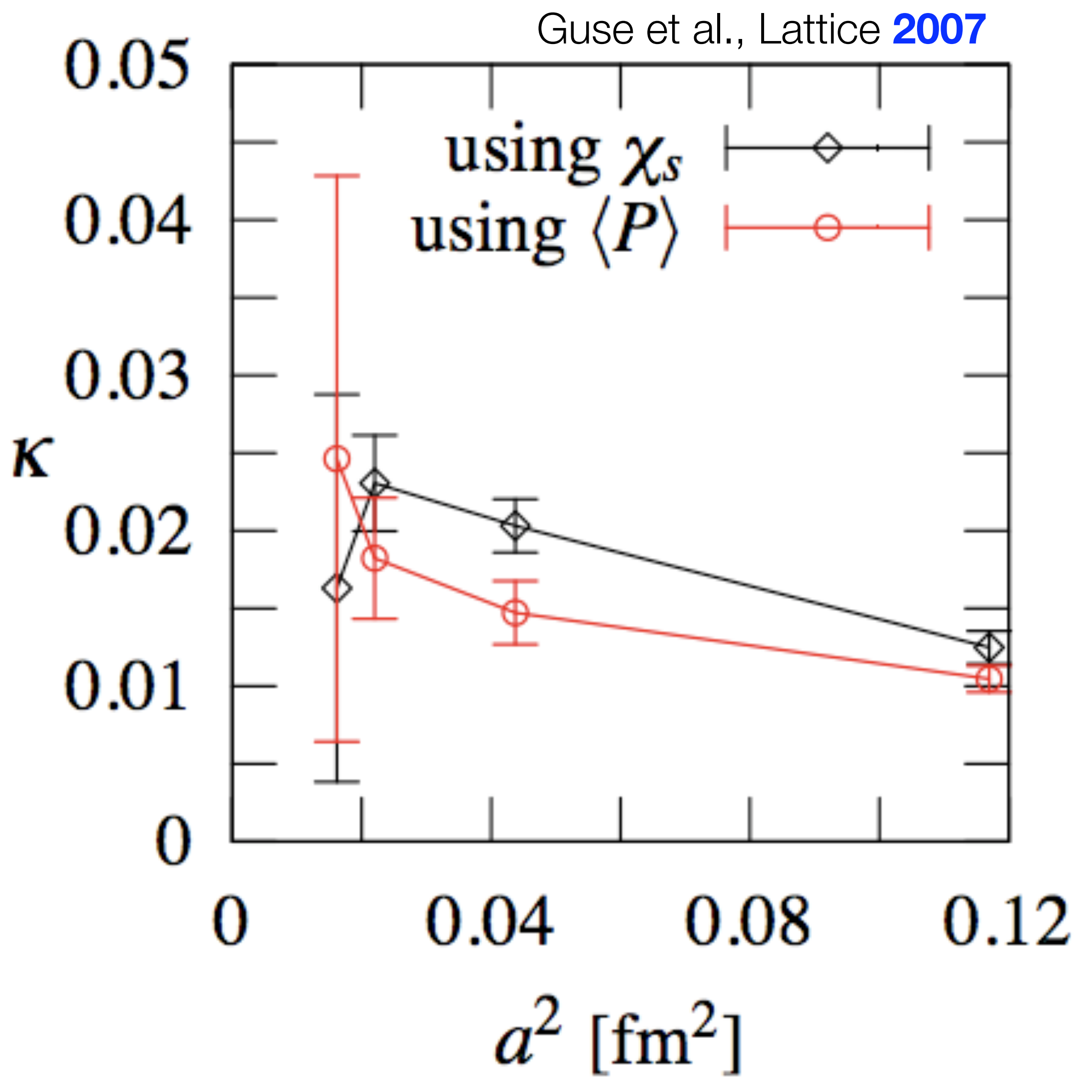


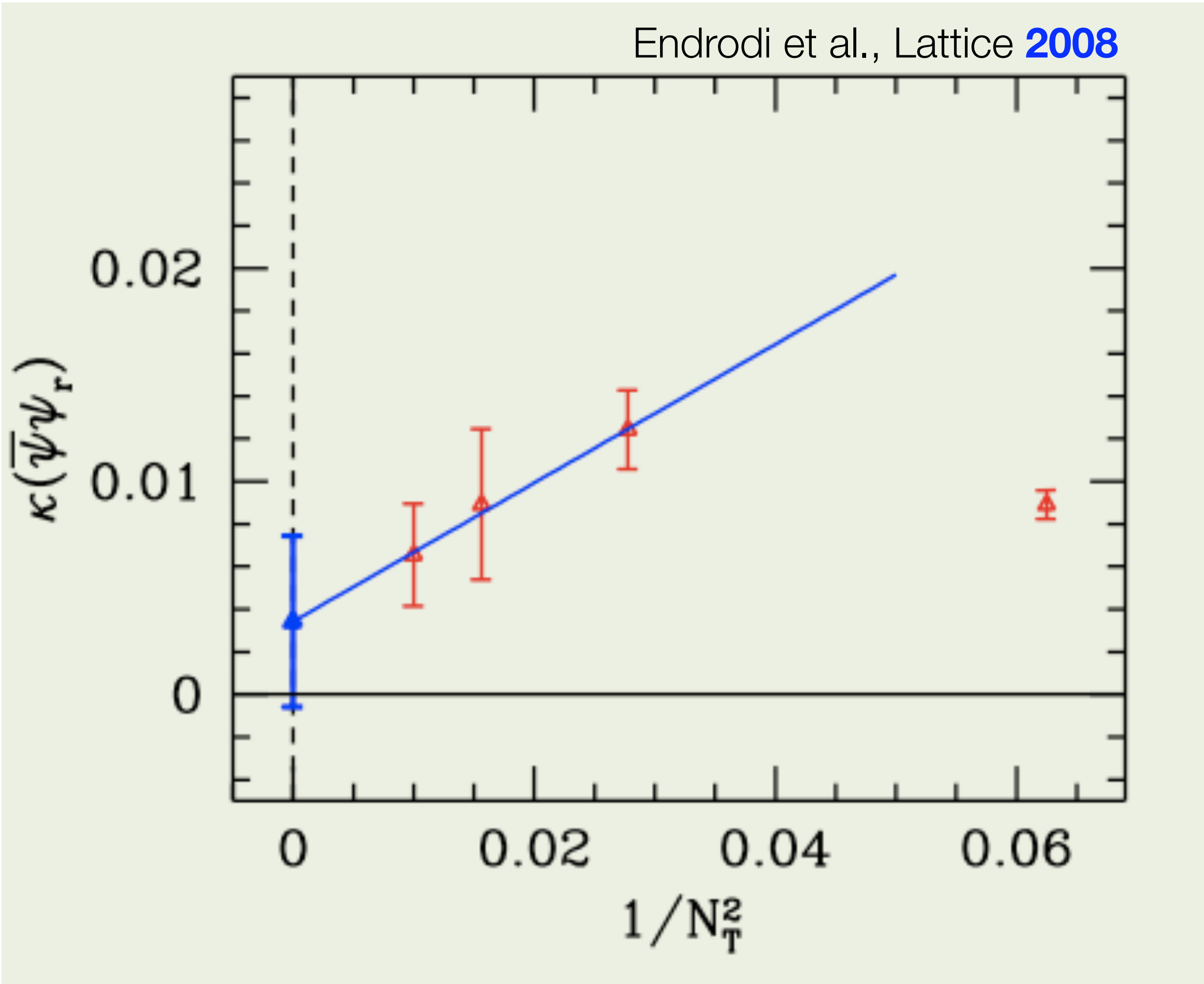
curvature for vanishing μ_q from the Lattice:

$$t_2 = -\pi^2 T_c(0) \frac{d^2 T_c(\mu_q)}{d\mu_q^2}\bigg|_{\mu_q=0}$$

	N_f	t_2
de Forcrand et al. ' 02	2	0.500(54)
de Forcrand et al. '06	3	0.667(6)

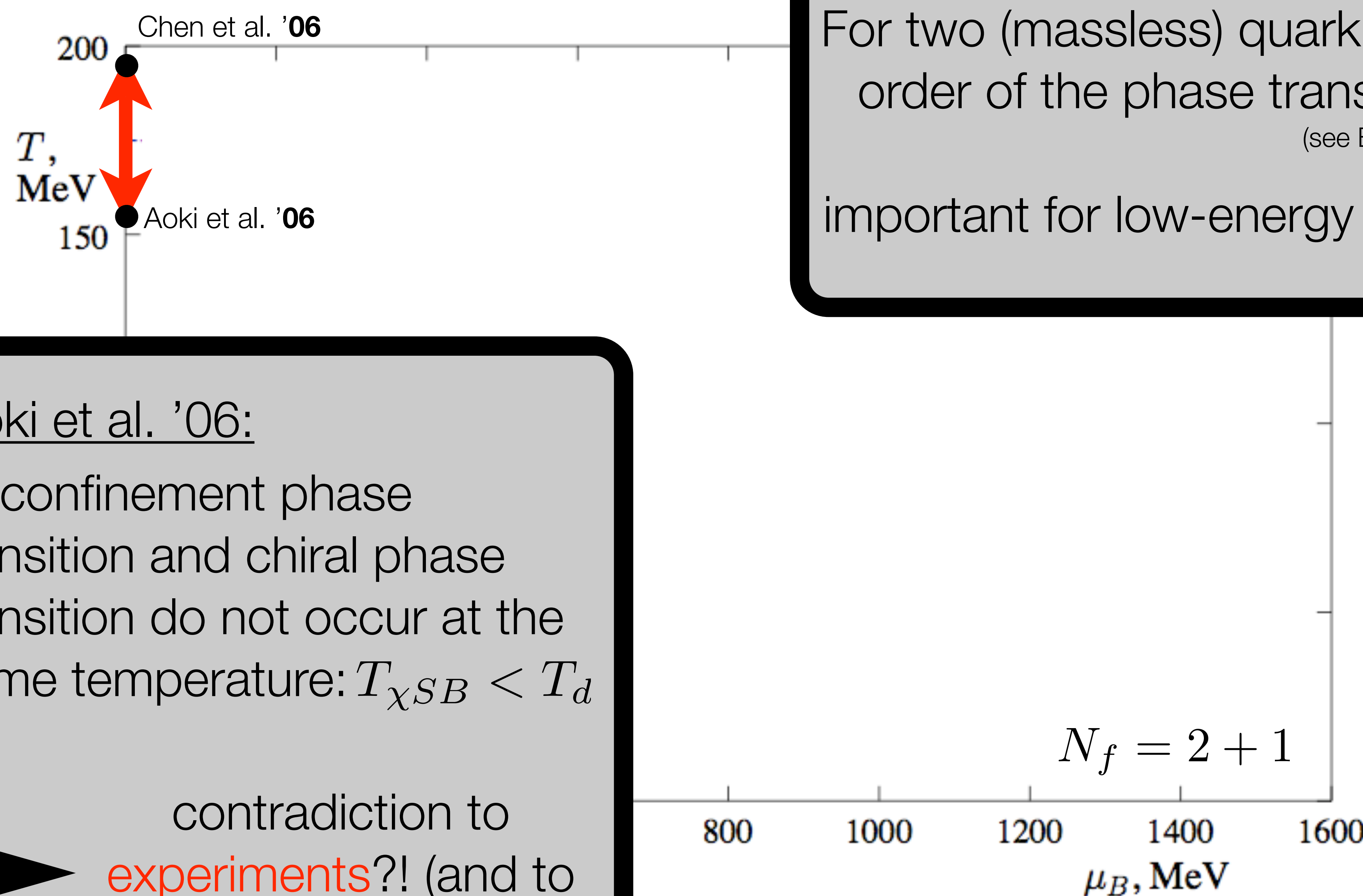
QCD phase diagram? Puzzles ...





- definition of the curvature: $\kappa = -T_c \left.\frac{dT_c(\mu_B)}{d\mu_B^2}\right|_{\mu_B=0}$

QCD phase diagram? Puzzles ...



For two (massless) quark flavors: order of the phase transition?

(see B .Klein's talk)

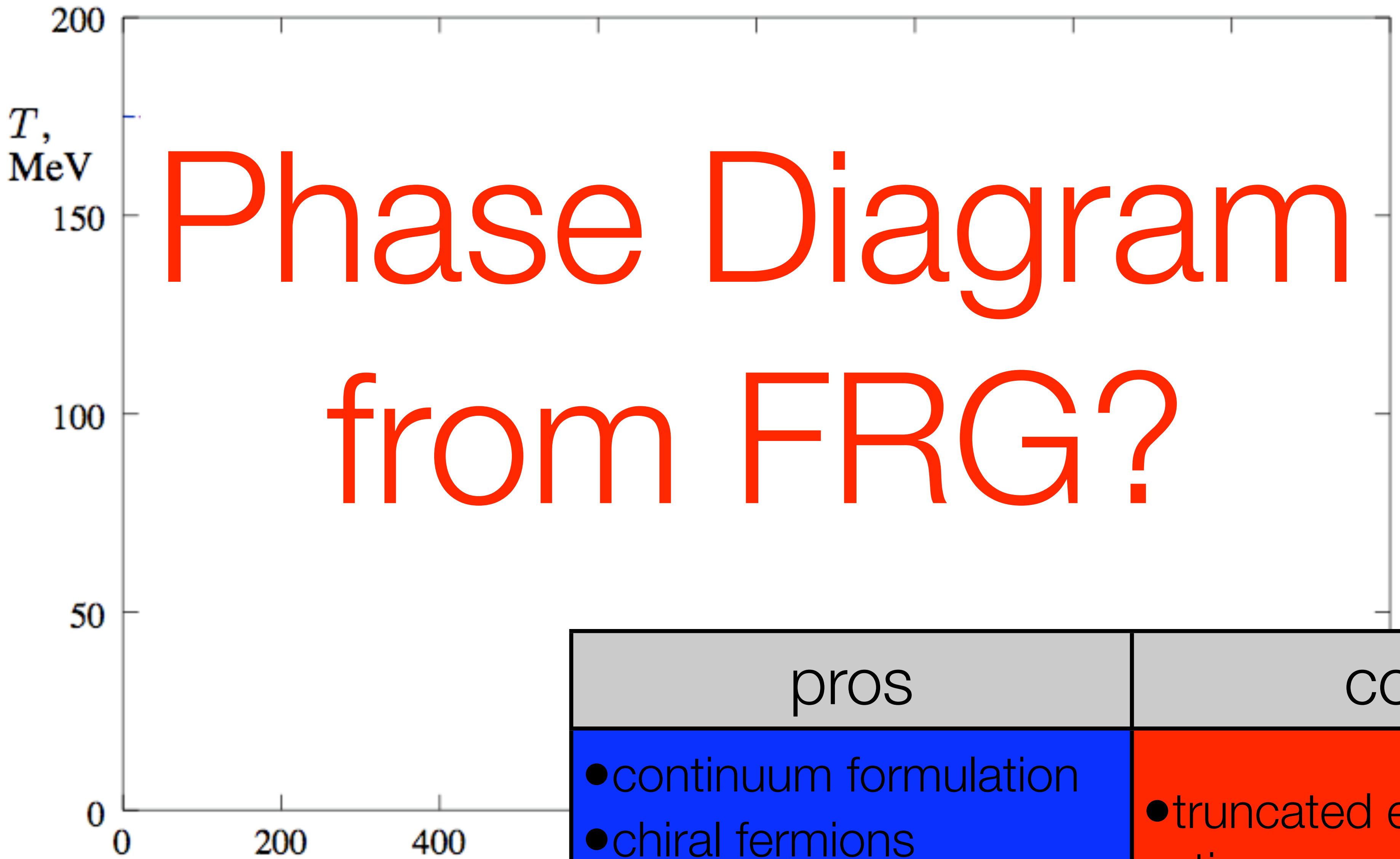
important for low-energy models

Aoki et al. '06:

Deconfinement phase transition and chiral phase transition do not occur at the same temperature: $T_{\chi SB} < T_d$

→ contradiction to experiments?! (and to Chen et al.)

QCD phase diagram from QCD RG flows?



Phase Diagram from FRG?

pros	cons
• continuum formulation • chiral fermions • no sign-problem	• truncated effective action

complementary to Lattice QCD

Outline

✓ Motivation

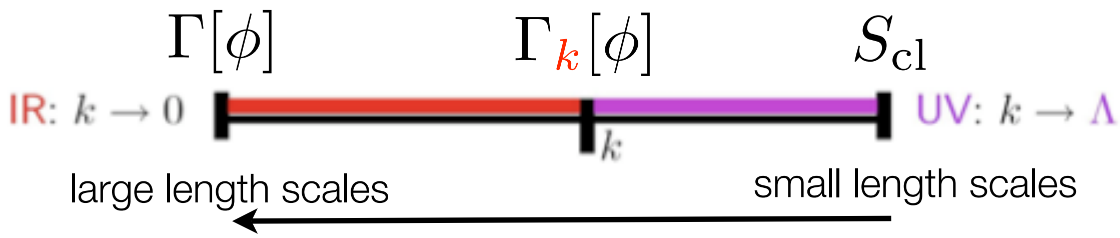
- Functional Renormalization Group

- Chiral Phase Boundary of QCD

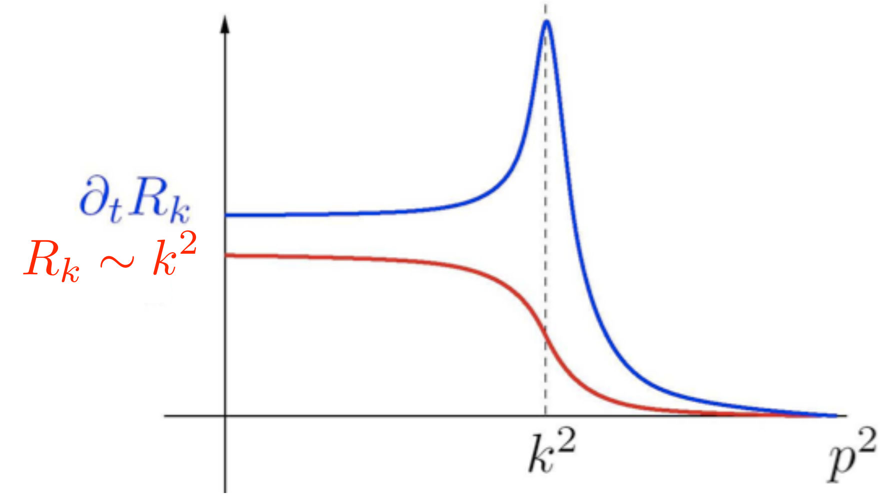
- Polyakov-Loop and (De-)Confinement Phase Transition

- Conclusions and Outlook

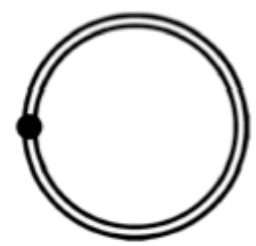
Functional Renormalization Group



start at classical action and include
quantum fluctuations successively (by lowering k)



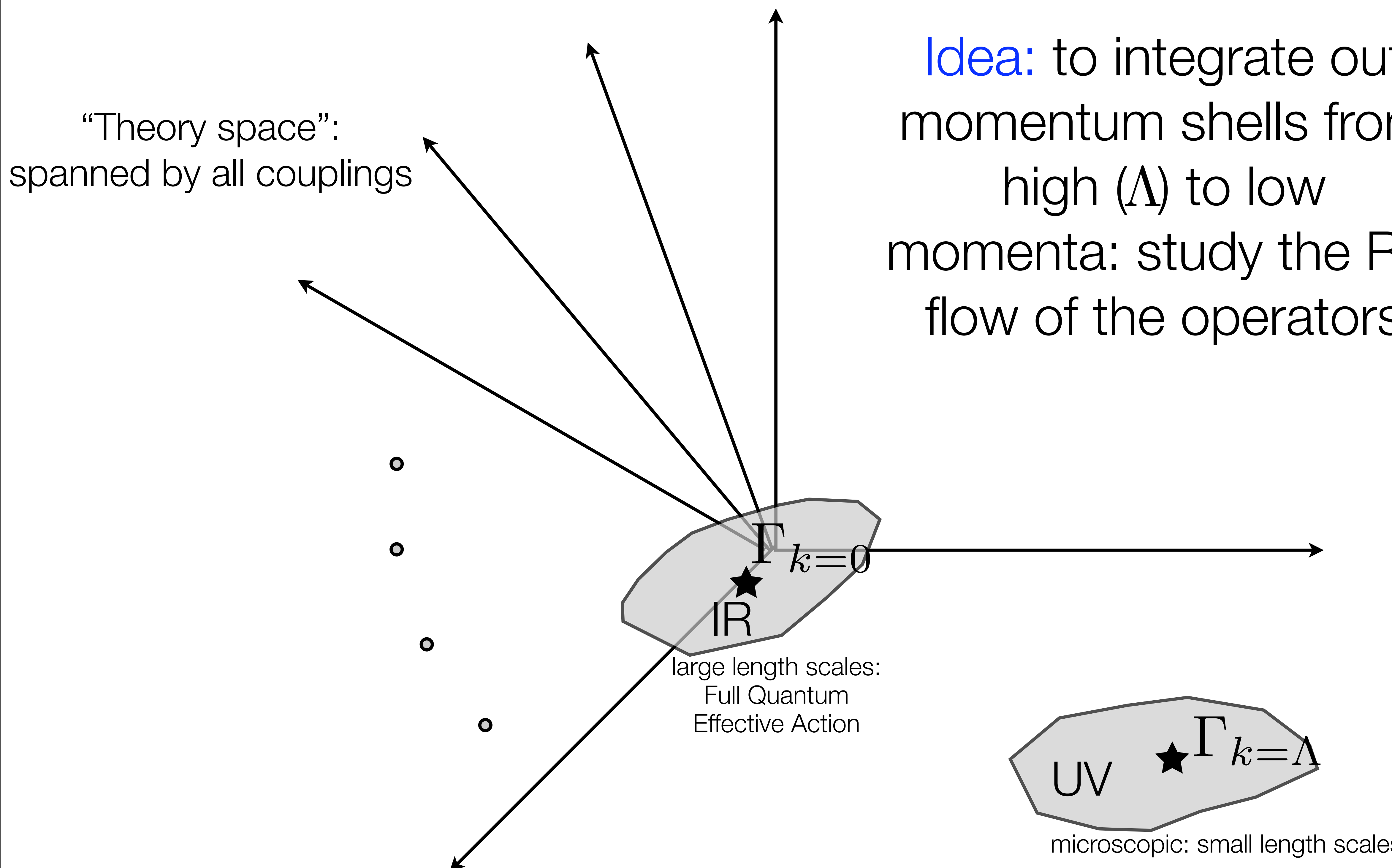
- RG flow equation for the effective action: (C. Wetterich ’93)

$$k\partial_k \Gamma_k \equiv \partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr} \frac{\partial_t R_k}{\Gamma_k^{(2)} + R_k} = \frac{1}{2}$$ 

- chiral symmetry is preserved: $R_k = R_k(\mathrm{i}\not{\partial})$

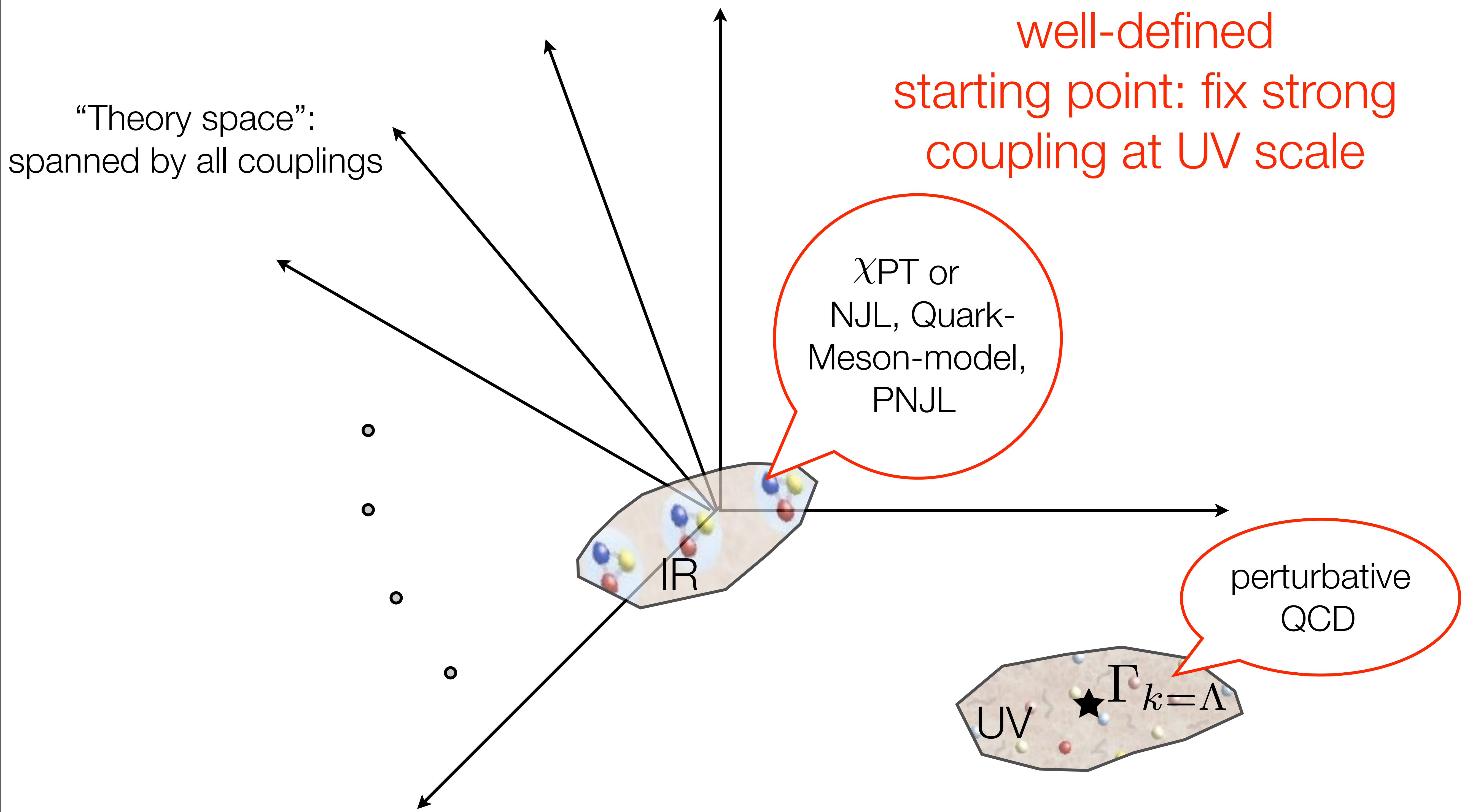
- gauge symmetry → modified Ward-Takahashi identities
  (Reuter & Wetterich ’94; Freire, Litim, Pawlowski ’00)

Functional Renormalization Group



Idea: to integrate out momentum shells from high (Λ) to low momenta: study the RG flow of the operators

Functional Renormalization Group - QCD

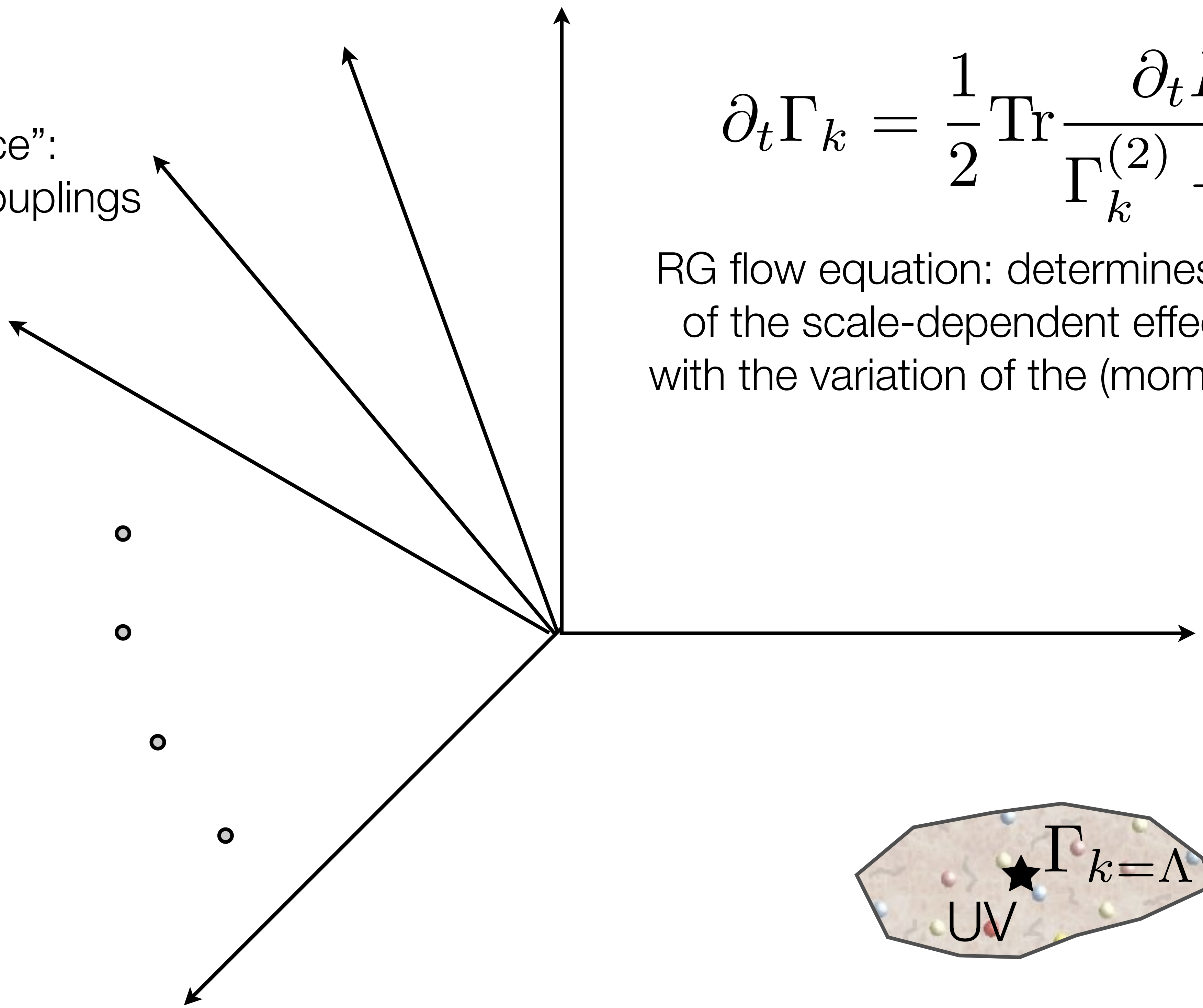


Functional Renormalization Group - QCD

“Theory space”:
spanned by all couplings

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr} \frac{\partial_t R_k}{\Gamma_k^{(2)} + R_k}$$

RG flow equation: determines the change of the scale-dependent effective action with the variation of the (momentum) scale



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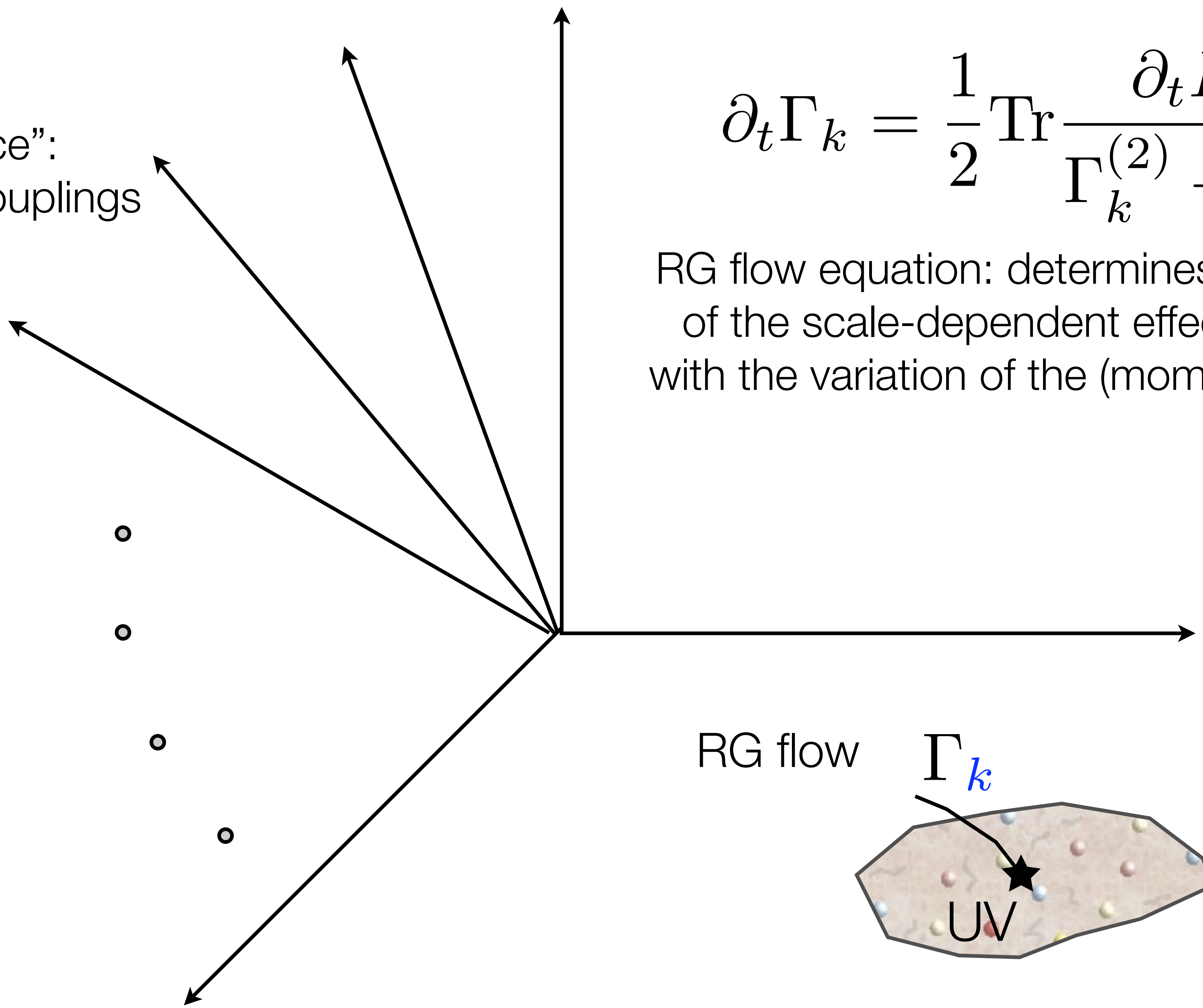

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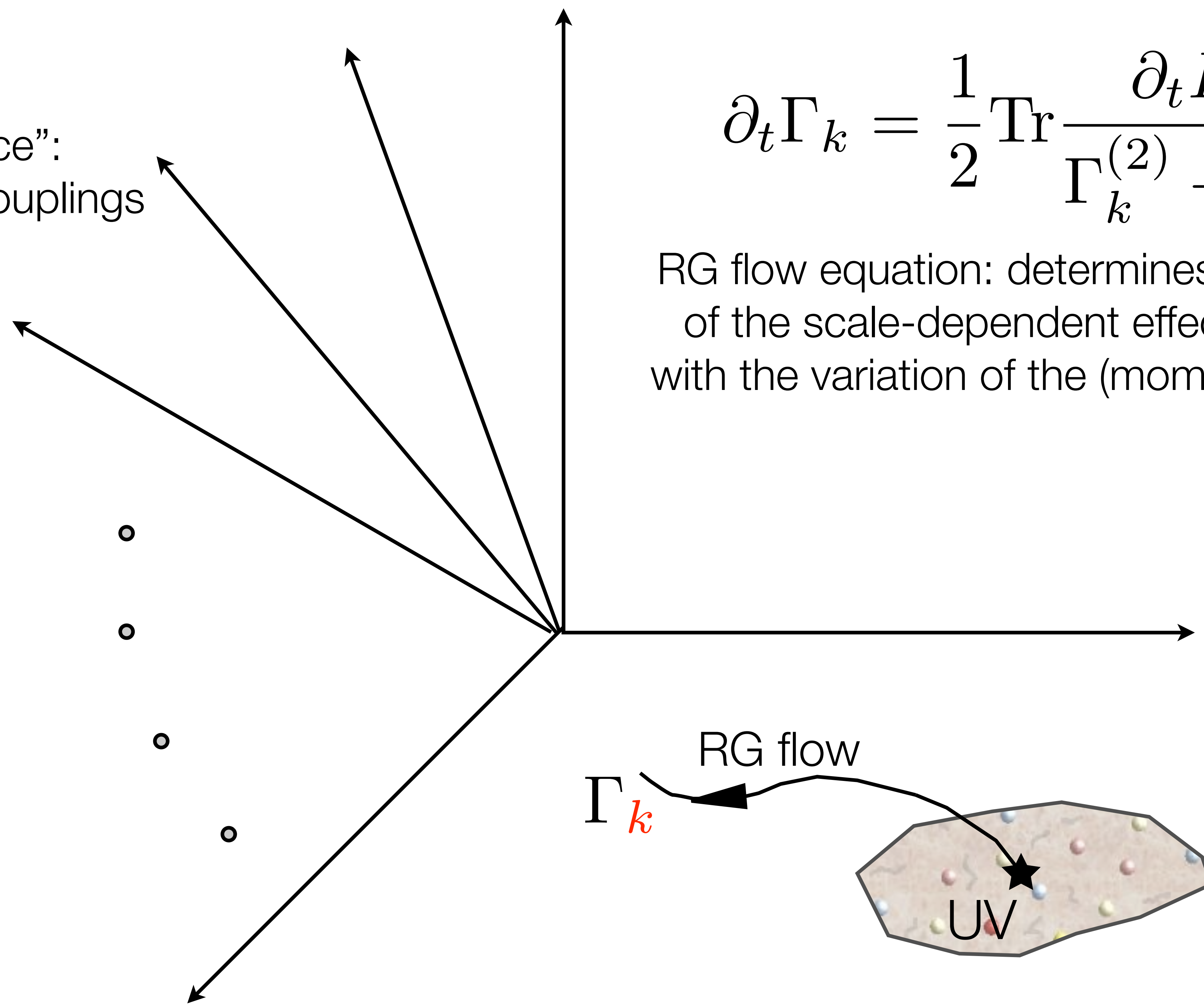

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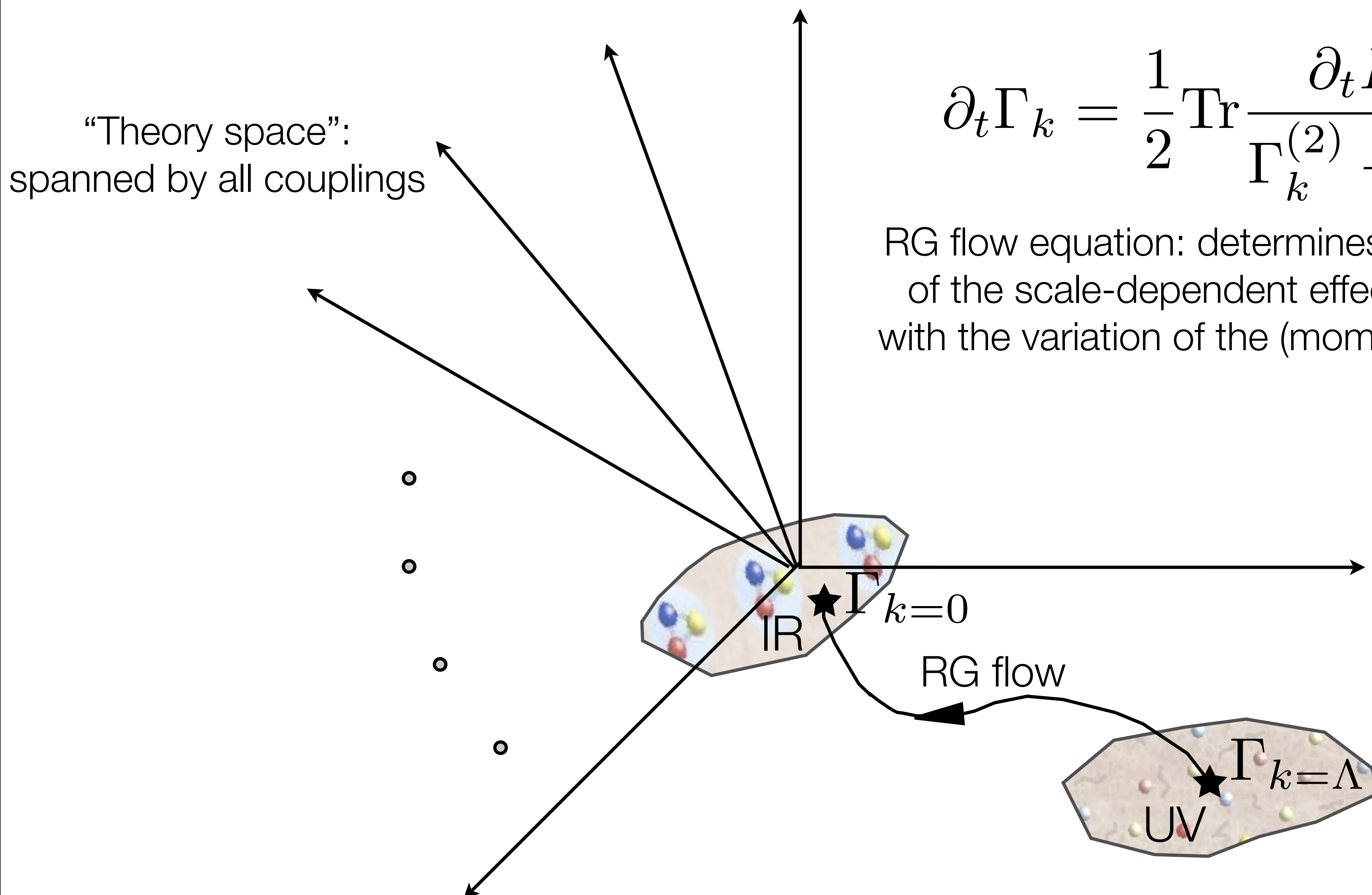

Outline

✓ Motivation

✓ Functional Renormalization Group

- Chiral Phase Boundary of QCD
    - Quark-gluon dynamics and the chiral phase boundary: from two to many quark flavors

    - QCD with one quark flavor: from quarks and gluons to mesons
- Polyakov-Loop and (De-)Confinement Phase Transition
- Conclusions and Outlook

Aspects of the NJL model

- classical action of the NJL model:

$$S = \int_x \left\{ \bar{\psi}\mathrm{i}\partial\!\!\!/\psi + \bar{\lambda}_\sigma \left[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2 \right] \right\}$$

- spontaneous symmetry breaking if quark condensate is non-vanishing: $\langle \bar{\psi}\psi \rangle \neq 0$

Aspects of the NJL model

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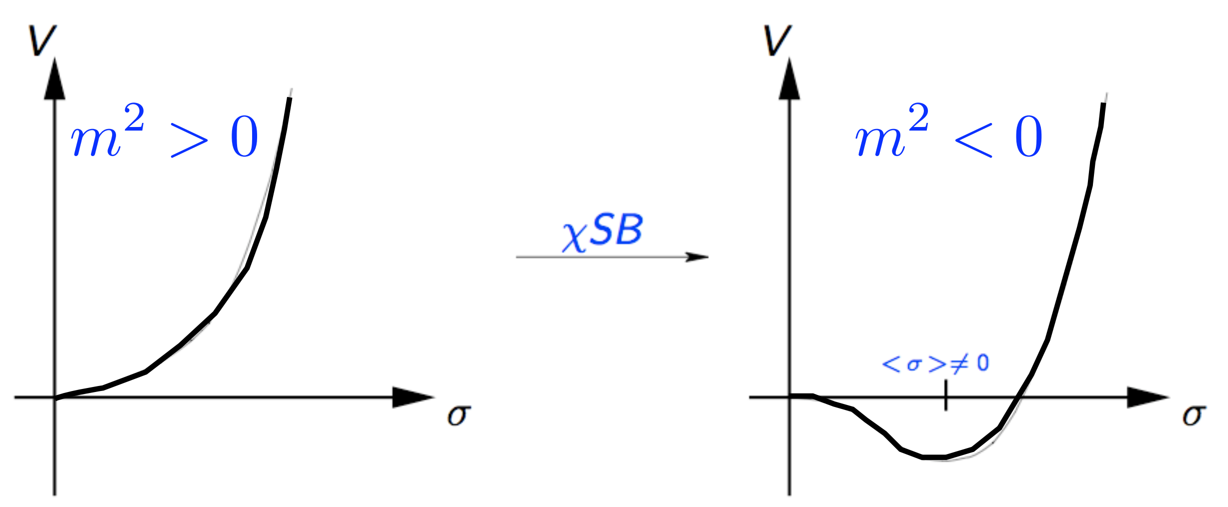
$$S = \int_x \{\bar{\psi} \mathrm{i} \partial\!\!\!/ \psi + \bar{\lambda}_\sigma [(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2]\}$$

- bosonization of the NJL model yields $(\sigma = -2\bar{\lambda}_\sigma \bar{\psi}\psi\,, \pi = -2\bar{\lambda}_\sigma \bar{\psi}\gamma_5\psi)$

$$S = \int_x \left\{ \bar{\psi} \mathrm{i} \partial\!\!\!/ \psi + \bar{\psi}(\sigma + \mathrm{i}\gamma_5 \pi)\psi - \frac{1}{\bar{\lambda}_\sigma}(\sigma^2 + \pi^2) \right\}$$ ("Quark-Meson model")

$\Rightarrow$ $\bar{\lambda}_\sigma$ is inverse proportional to the scalar mass parameter, $m^2 \propto \dfrac{1}{\bar{\lambda}_\sigma}$



Four-Fermion Interactions in QCD

- at the UV scale $(k = \Lambda \gg \Lambda_{\mathrm{QCD}})$:

$$\Gamma_{\Lambda} = \int_x \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{\psi}(\mathrm{i}\partial\!\!\!/ + \bar{g} A\!\!\!/)\psi \right\}$$

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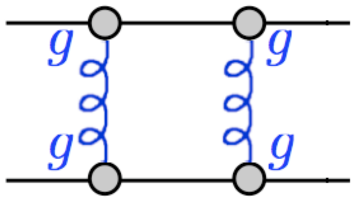
$$k = \Lambda - \delta k$$



$$\Gamma_{\Lambda - \delta k} = \int_x \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{\psi}(\mathrm{i}\partial\!\!\!/ + \bar{g} A\!\!\!/)\psi + \frac{\lambda_\sigma}{2k^2}[(\bar{\psi}^a \psi^b)^2 - (\bar{\psi}^a \gamma_5 \psi^b)^2] + \ldots \right\}$$

- quark-gluon dynamics generate four-fermion interactions

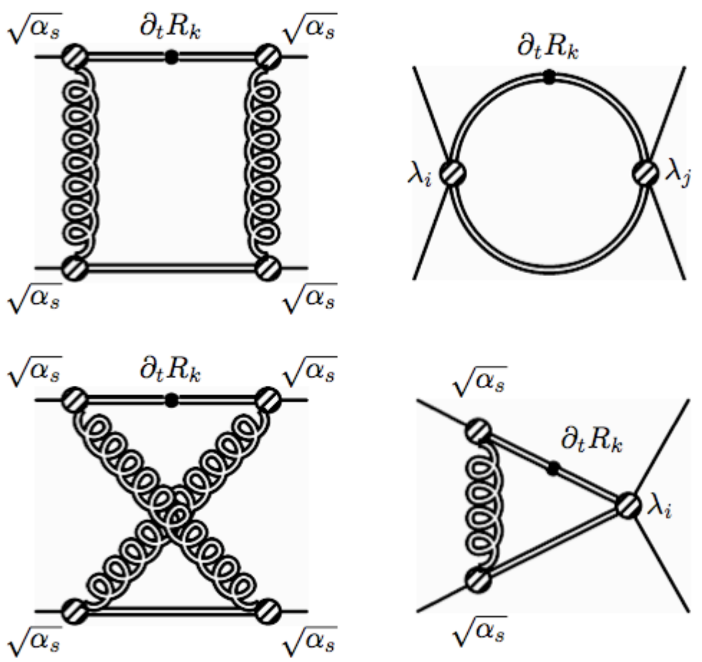
RG flow for the chiral QCD sector

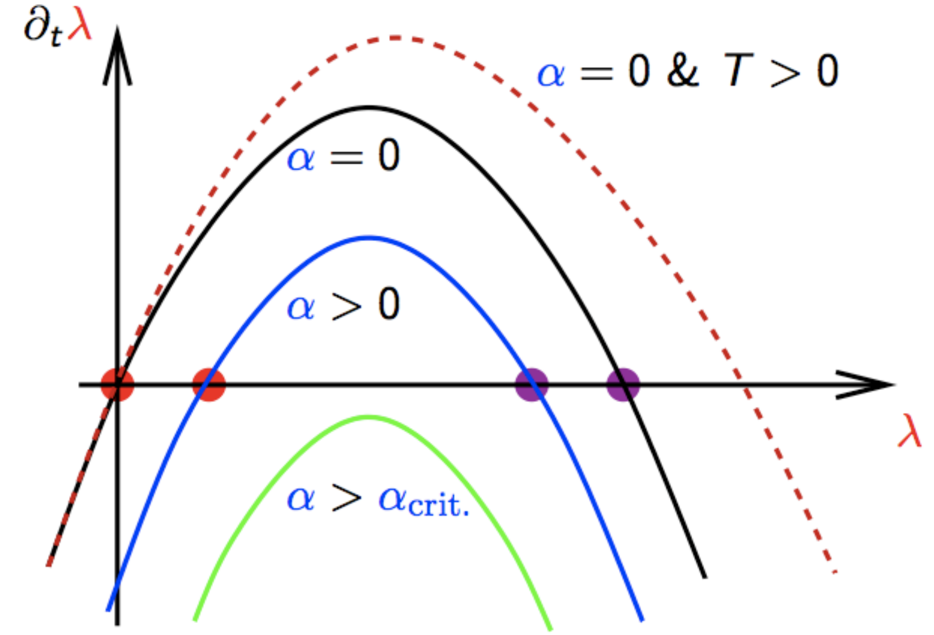
- effective action:

$$\Gamma_k = \int_x \Big\{ \frac{\bar{g}^2}{g^2} F^a_{\mu\nu} F^a_{\mu\nu} + w_2 (F^a_{\mu\nu} F^a_{\mu\nu})^2 + w_3 (F^a_{\mu\nu} F^a_{\mu\nu})^3 + \dots \Big\}$$
$$+ \int_x \Big\{ \bar{\psi}(\mathrm{i} Z_\psi \partial\!\!\!/ + Z_1 \bar{g} A\!\!\!/)\psi + \frac{1}{2}\Big[\frac{\lambda_-}{k^2}(\mathrm{V}-\mathrm{A}) + \frac{\lambda_+}{k^2}(\mathrm{V}+\mathrm{A})$$
$$+ \frac{\lambda_\sigma}{k^2}(\mathrm{S}-\mathrm{P}) + \frac{\lambda_{\mathrm{VA}}}{k^2}[2(\mathrm{V}-\mathrm{A})^{\mathrm{adj}} + (1/N_c)(\mathrm{V}-\mathrm{A})]\Big] \Big\}$$

- no Fierz-ambiguity
- four-fermion interactions $\left(\lim_{\Lambda \to \infty} \lambda_i = 0\right)$
- truncation checks: momentum dependencies, regulator dependencies, higher order interactions (H. Gies, J. Jaeckel, C. Wetterich '04)

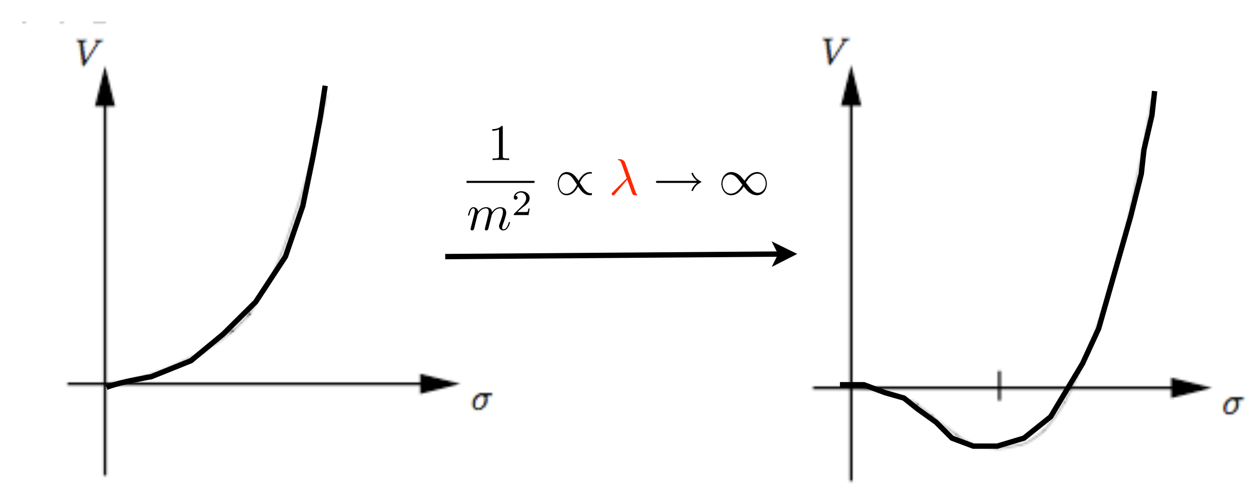
“Criticality” at zero and finite temperature





- flow of four-fermion couplings:

$$\partial_t \lambda = 2\lambda - \lambda A(\tfrac{T}{k})\lambda - b(\tfrac{T}{k})\lambda\alpha_s - c(\tfrac{T}{k}){\alpha_s}^2$$



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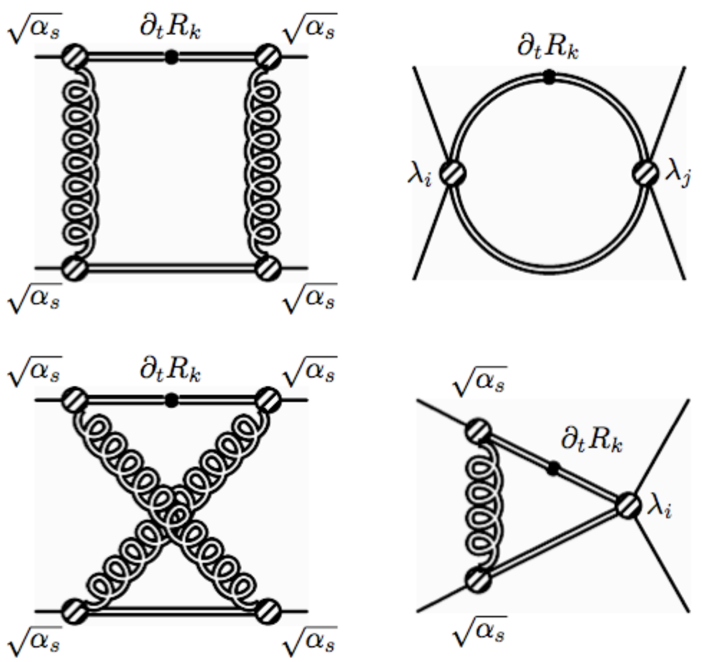
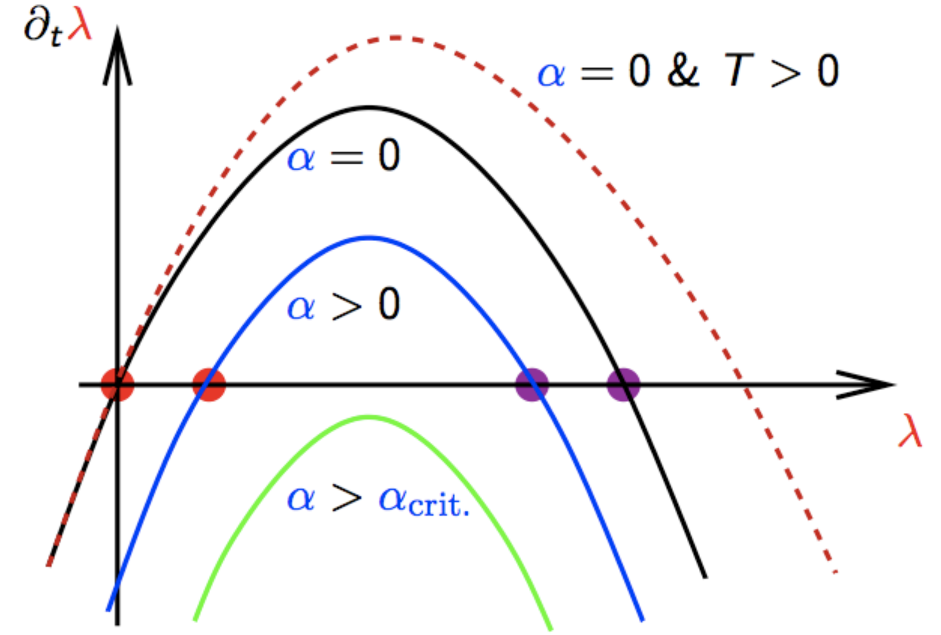


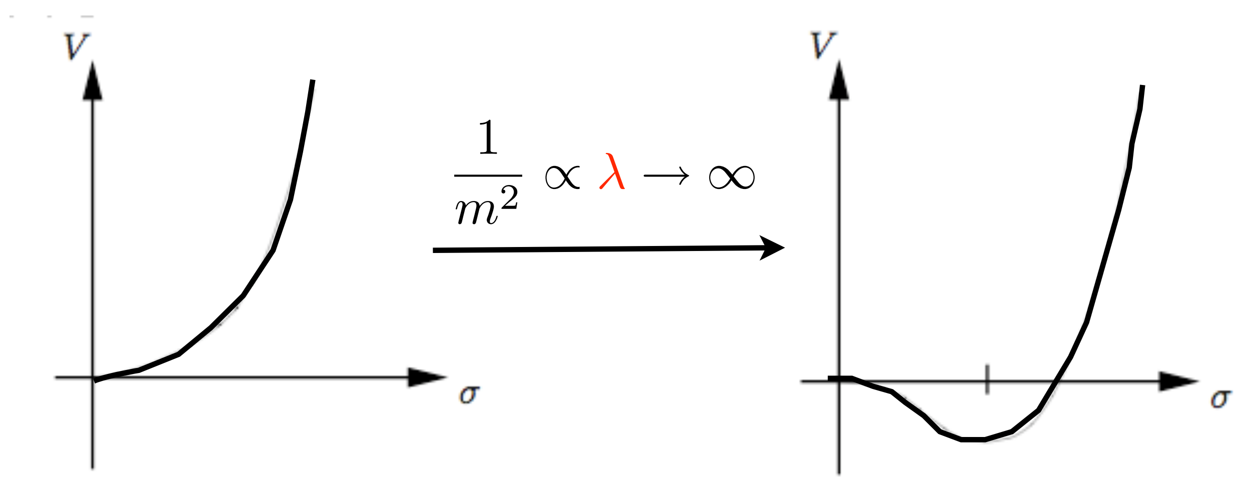


- critical gauge coupling α_{cr} :

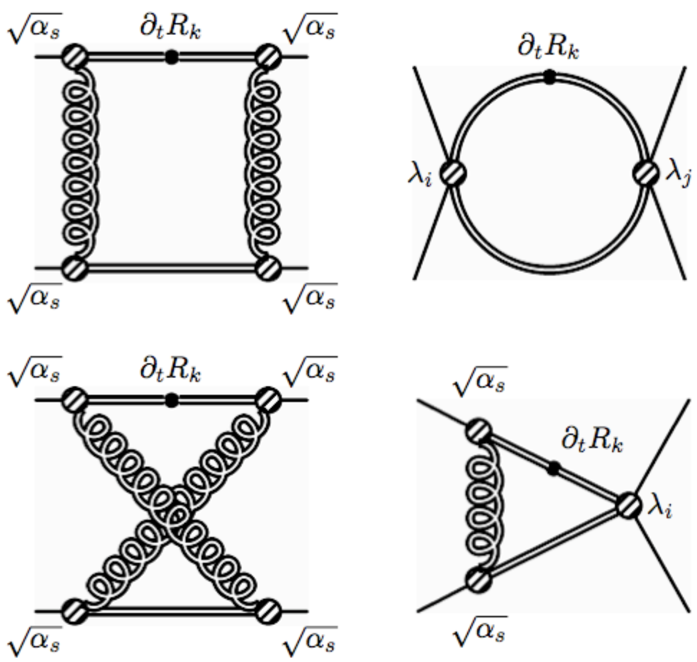
  if $\alpha_s > \alpha_{cr}$ ⟹ no fixed points ⟹ χSB

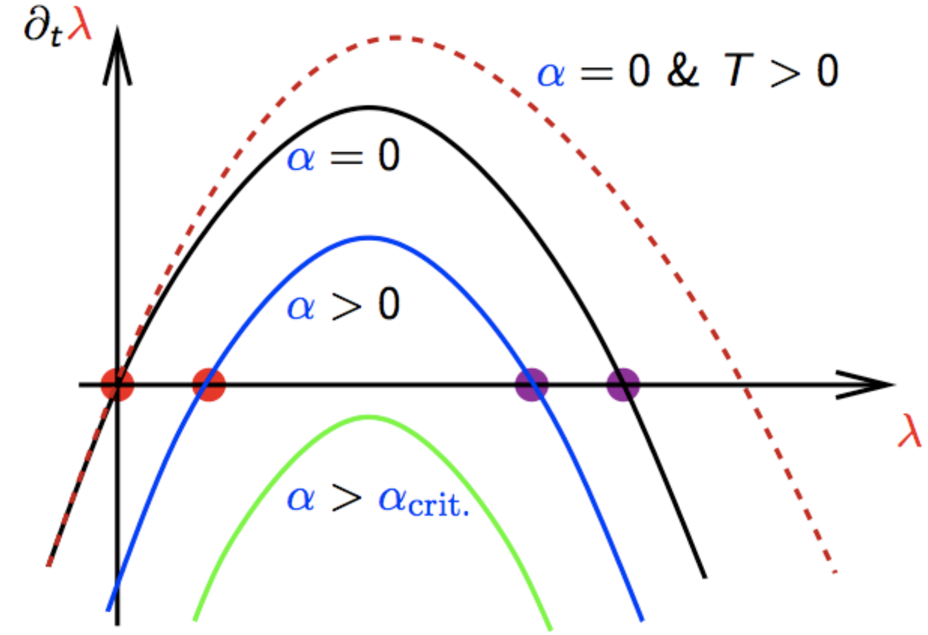
- at zero temperature: (H. Gies, J. Jaeckel ’05)

$$\alpha_{cr} \overset{N_c = N_f = 3}{\approx} 0.85$$



"Criticality" at zero and finite temperature





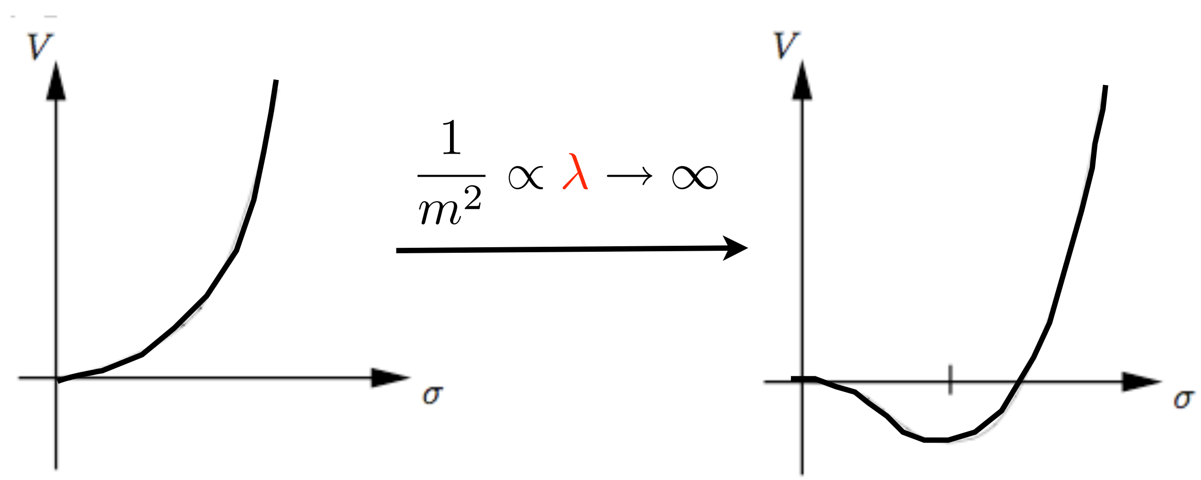
- critical gauge coupling α_{cr} :

  if $\alpha_s > \alpha_{cr}$ $\Longrightarrow$ no fixed points $\Longrightarrow$ χSB

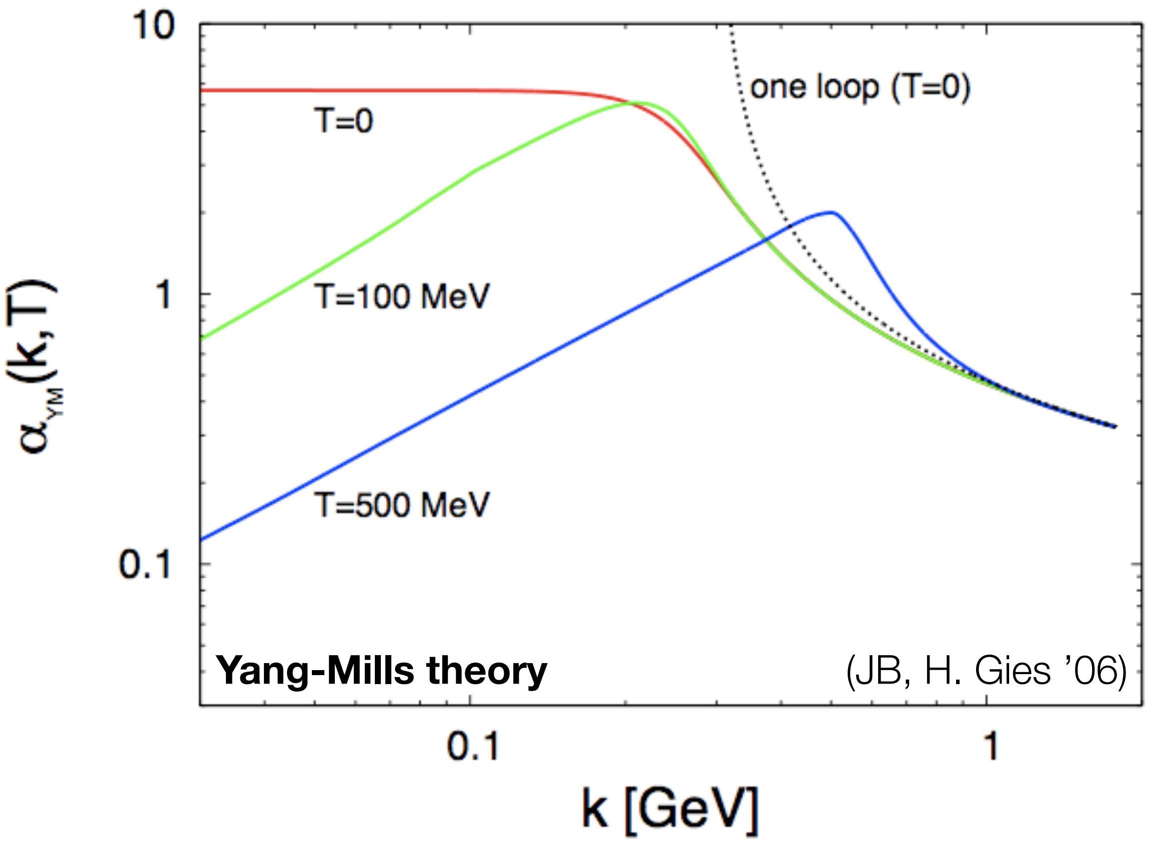
- at finite temperature: (JB, H. Gies '05)

$$\alpha_{cr}(T/k) > \alpha_{cr}(T=0)$$

quarks acquire a thermal mass



RG flow of gluodynamics



cf. vertex expansion in **Landau-gauge QCD**:

SDE: v. Smekal et al. '97, Fischer et al. '02;
RG: Pawlowski et al. '03, Fischer&Gies '04;
Gies '02; Gies&Braun '05/'
Lattice: e. g. Sternbeck et al. '05; ...

- $k_{max} \propto T$ decoupling of hard gluonic modes ⇨ "finite-size" effect:

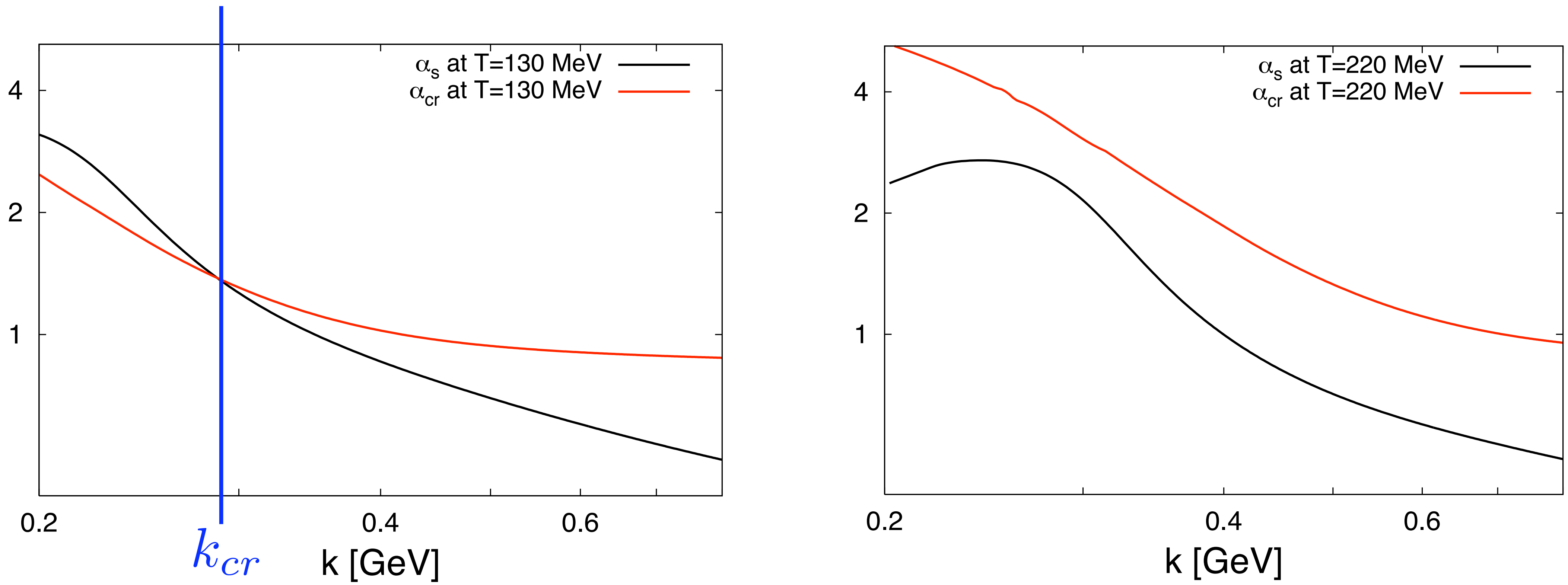
$$p_{g,0}^2 \equiv \omega_n^2 = 4n^2\pi^2T^2 \quad \rightarrow \quad \omega_0^2 = 0$$

- decrease for $T \gtrsim k$ due to existence of a non-trivial IR fixed point in 3d Yang-Mills theory: strong interactions at high temperatures (JB, H. Gies '06; Lattice: Cucchieri et al. '07)

$$\alpha_{4\mathrm{D}} \approx \alpha_{3\mathrm{D}}^* \frac{k}{T} + \mathcal{O}\left((k/T)^2\right) \quad \text{with} \quad \alpha_{3\mathrm{D}}^* \approx 2.7\,;\ \eta_{3d} \rightarrow 1$$

Chiral Phase Transition in QCD

- study: $\alpha_{cr}(T/k)$ vs. $\alpha_s(T/k)$
- intersection point of α_{cr} and α_s indicates onset of χSB



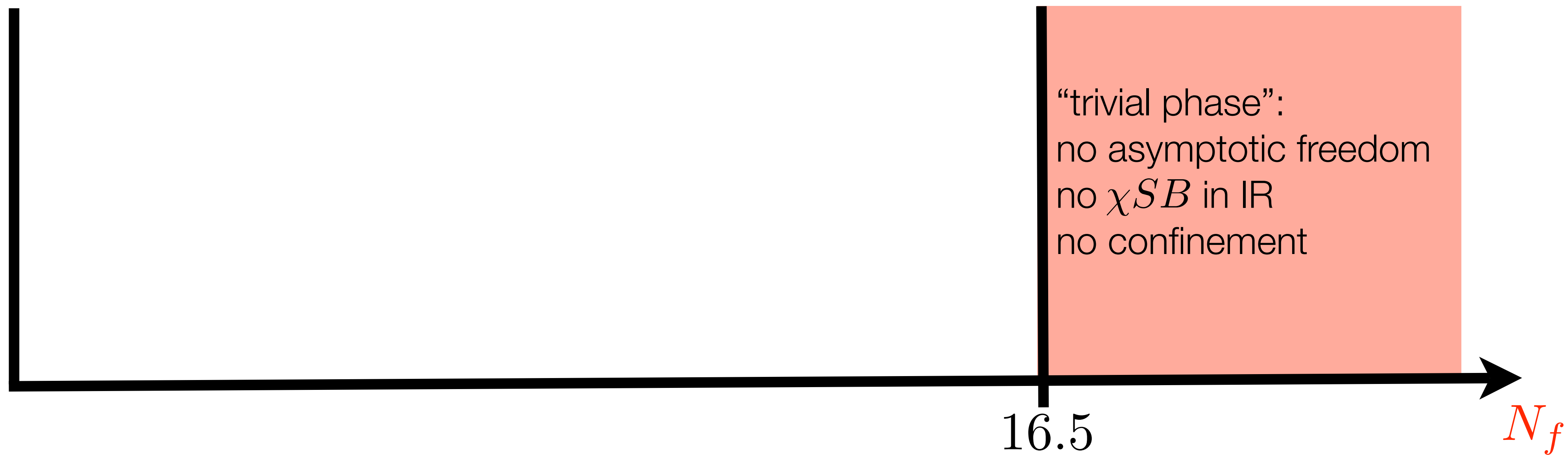
- single input parameter: $\alpha_s(m_\tau) = 0.322$

N_f	T_{cr}
2	$172\,\mathrm{MeV}$
3	$148\,\mathrm{MeV}$

(JB, H. Gies '06)

$N_f = 2+1$	T_{cr}
Lattice (Chen et al. '06)	$192\,\mathrm{MeV}$
Lattice (Aoki et al. '06)	$151\,\mathrm{MeV}$

What to expect for QCD with many flavors?

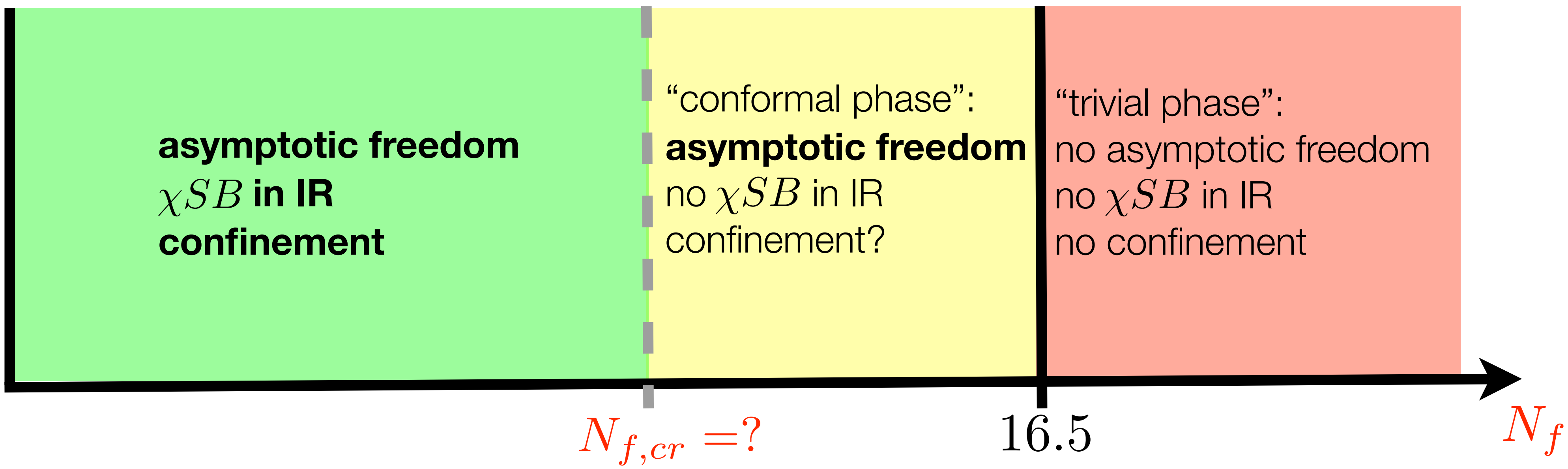


- one-loop β-function

$$\partial_t \alpha \equiv \beta(\alpha) = -\overbrace{\frac{1}{6\pi}(11 N_c - 2 N_f)}^{b_1}\,\alpha^2$$

- $b_1 < 0 \Longrightarrow N_f > \frac{11}{2} N_c \overset{N_c=3}{=} 16.5$ (QCD is NOT asymptotically free)

What to expect for QCD with many flavors?



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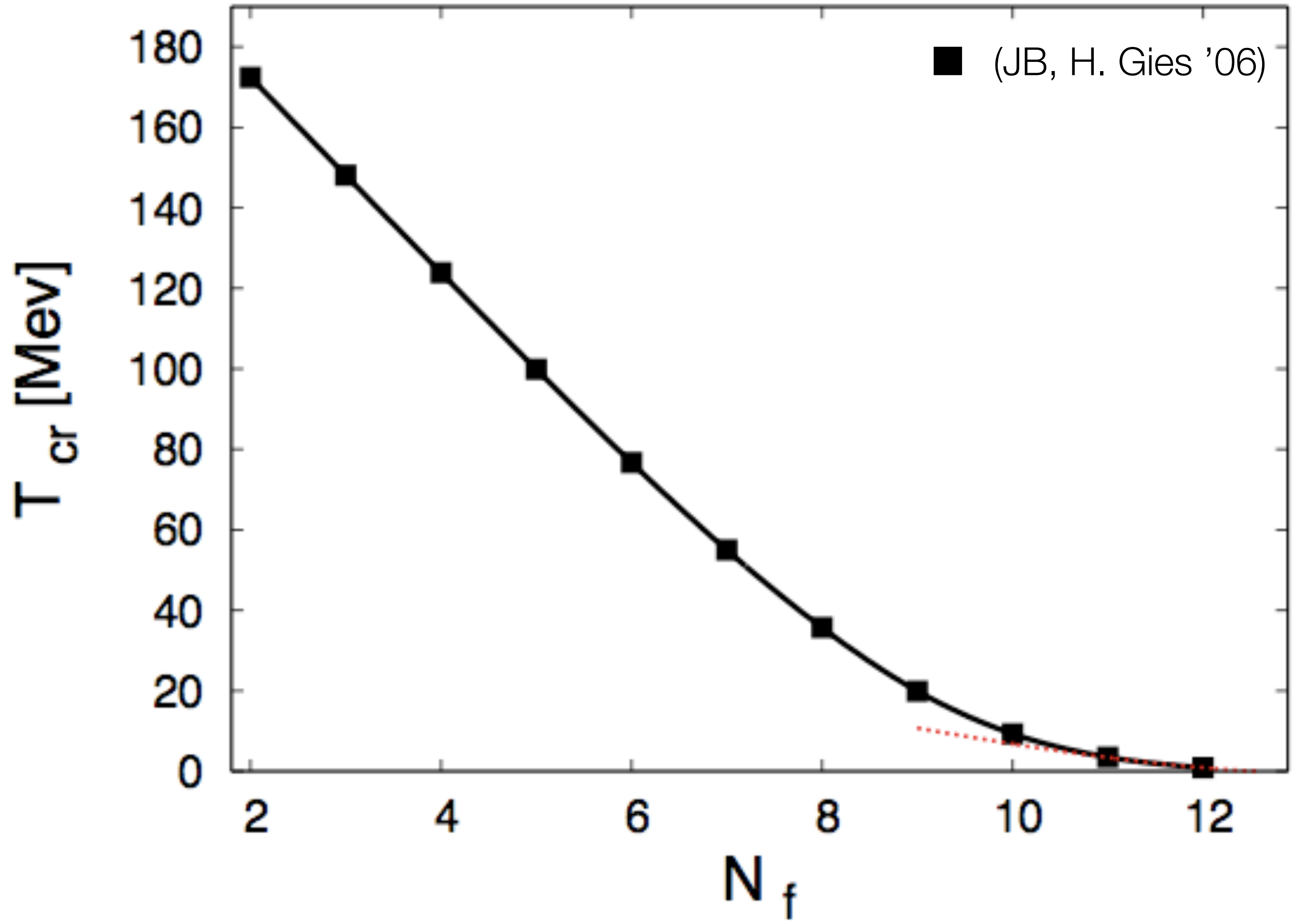
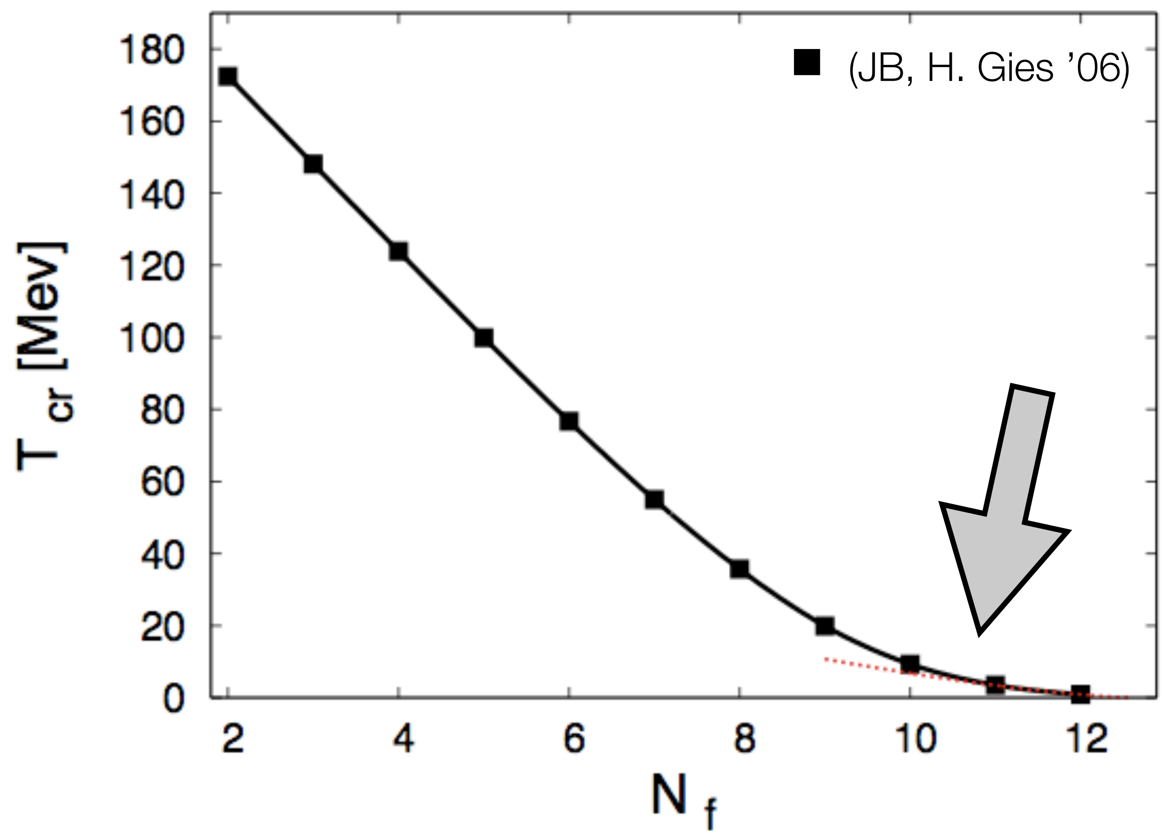
- $b_1 > 0$: QCD is asymptotically free

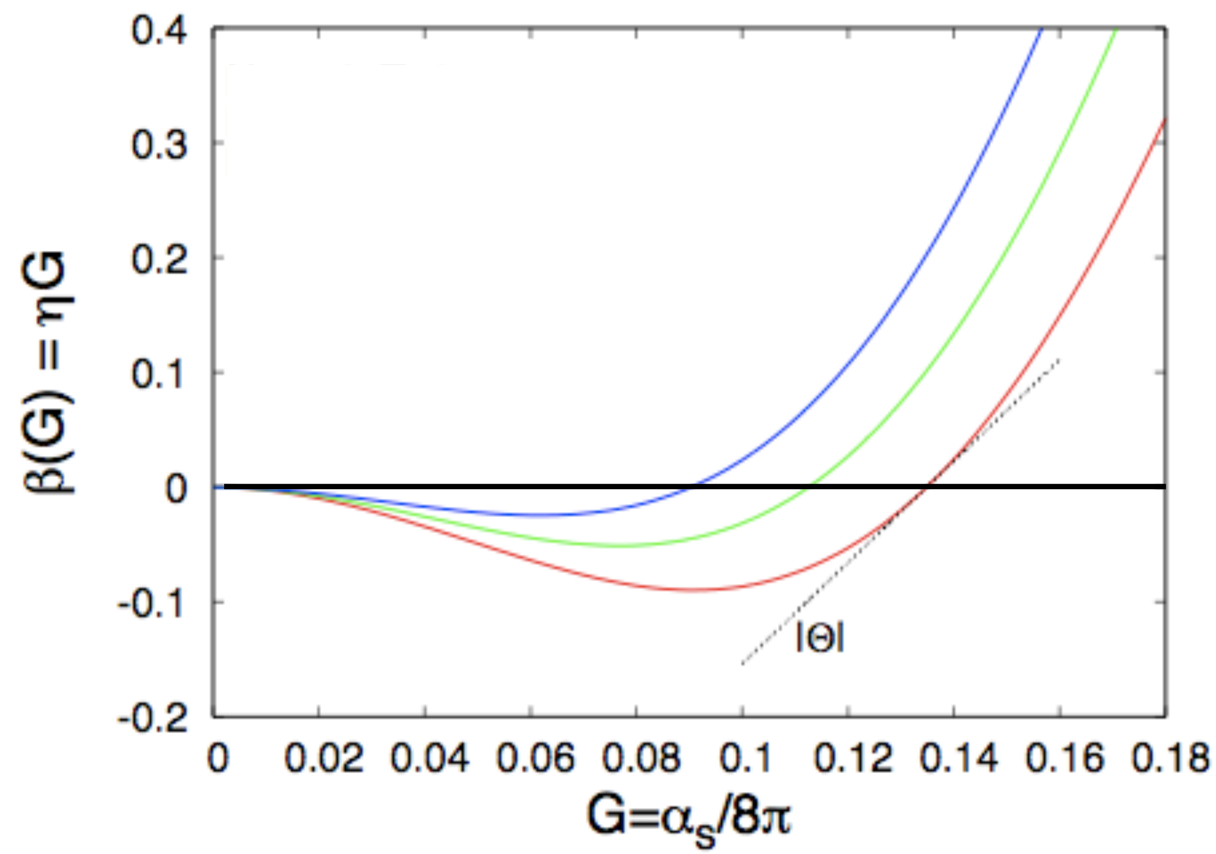
Many-flavor QCD



- small N_f : fermionic screening
- critical number of quark flavors: $N_{f,cr} = 12$ (cf. e. g. Appelquist '07 & '96)
- "conformal phase" for $N_{f,cr} < N_f < 16.5$: asymptotic freedom but no χSB

Many-flavor scaling regime





- fixed-point regime for large N_f: critical exponent $|\Theta|$

$$\partial_t g^2 \approx |\Theta|(g^2 - g_*^2)$$

- shape of the phase boundary for $N_f \approx N_{f,cr}$ (JB, H. Gies '06)

$$T_{cr} \propto |N_f - N_{f,cr}|^{\frac{1}{|\Theta|}} \quad \text{with} \quad |\Theta| \approx 0.71$$

(currently under investigation on the lattice, **Deuzeman, Lomarbdo, Pallante '08**)

Outline

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✓ Functional Renormalization Group

- Chiral Phase Boundary of QCD

  ✓ Quark-gluon dynamics and the chiral phase boundary: from two to many quark flavors

  - QCD with one quark flavor: from quarks and gluons to mesons 

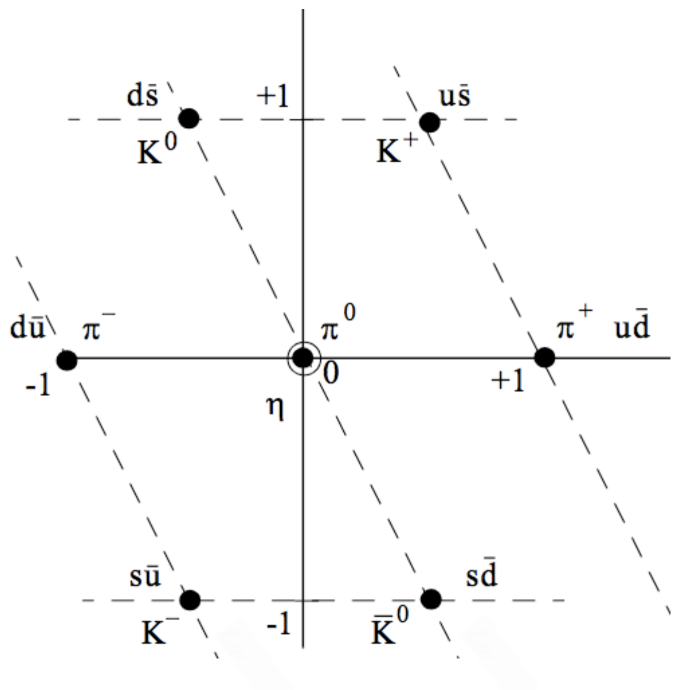
- Polyakov-Loop and (De-)Confinement Phase Transition

- Conclusions and Outlook

Challenge:
How to penetrate the phase boundary in order to get access to the low-energy observables?

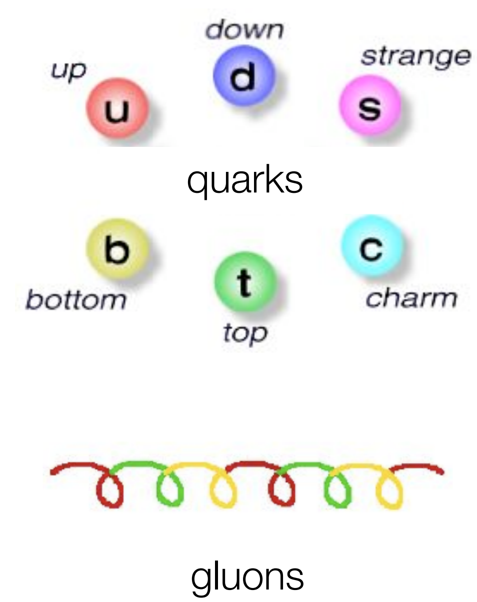
From microscopic to macroscopic DoFs

macroscopic DoF



RG flow

microscopic DoF



$k \to 0$

large length scales

k

$k \to \Lambda$

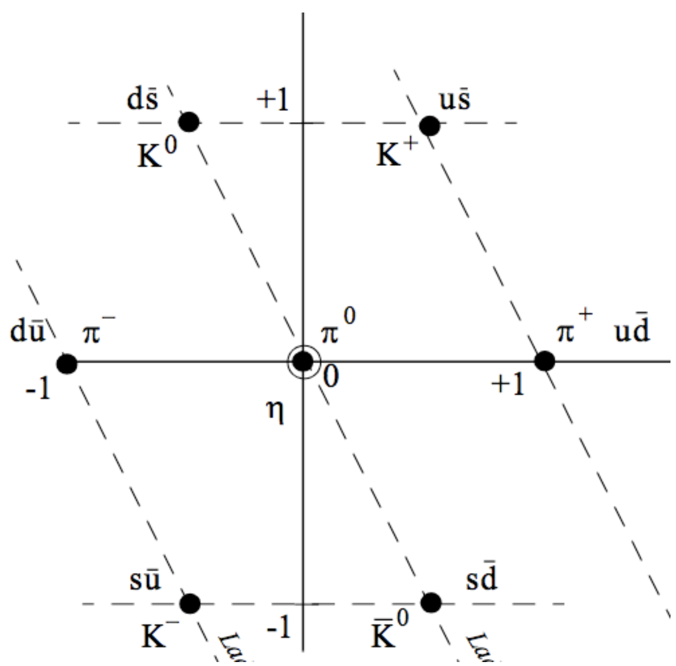
small length scales

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr} \frac{\partial_t R_k}{\Gamma_k^{(2)}[\phi] + R_k}$$

From microscopic to macroscopic DoFs: Do it by hand

macroscopic DoF

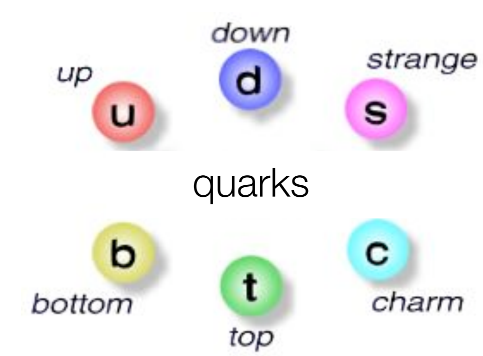
For example:
(constituent-quark-) meson-model



RG flow | RG flow

microscopic DoF

quark-gluon dynamics

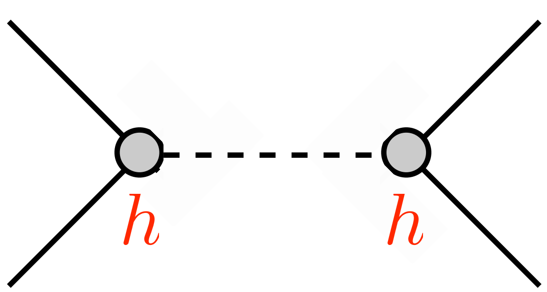


gluons

e. g.: $k \approx k_{cr}$
"set" by diverging
$\bar{\lambda}_\sigma(\mu, T)$

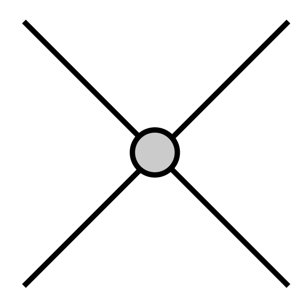
$h\bar{\psi}(\sigma + \mathrm{i}\gamma_5\pi)\psi + m^2(\sigma^2 + \pi^2)$

$\bar{\lambda}_\sigma[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2]$



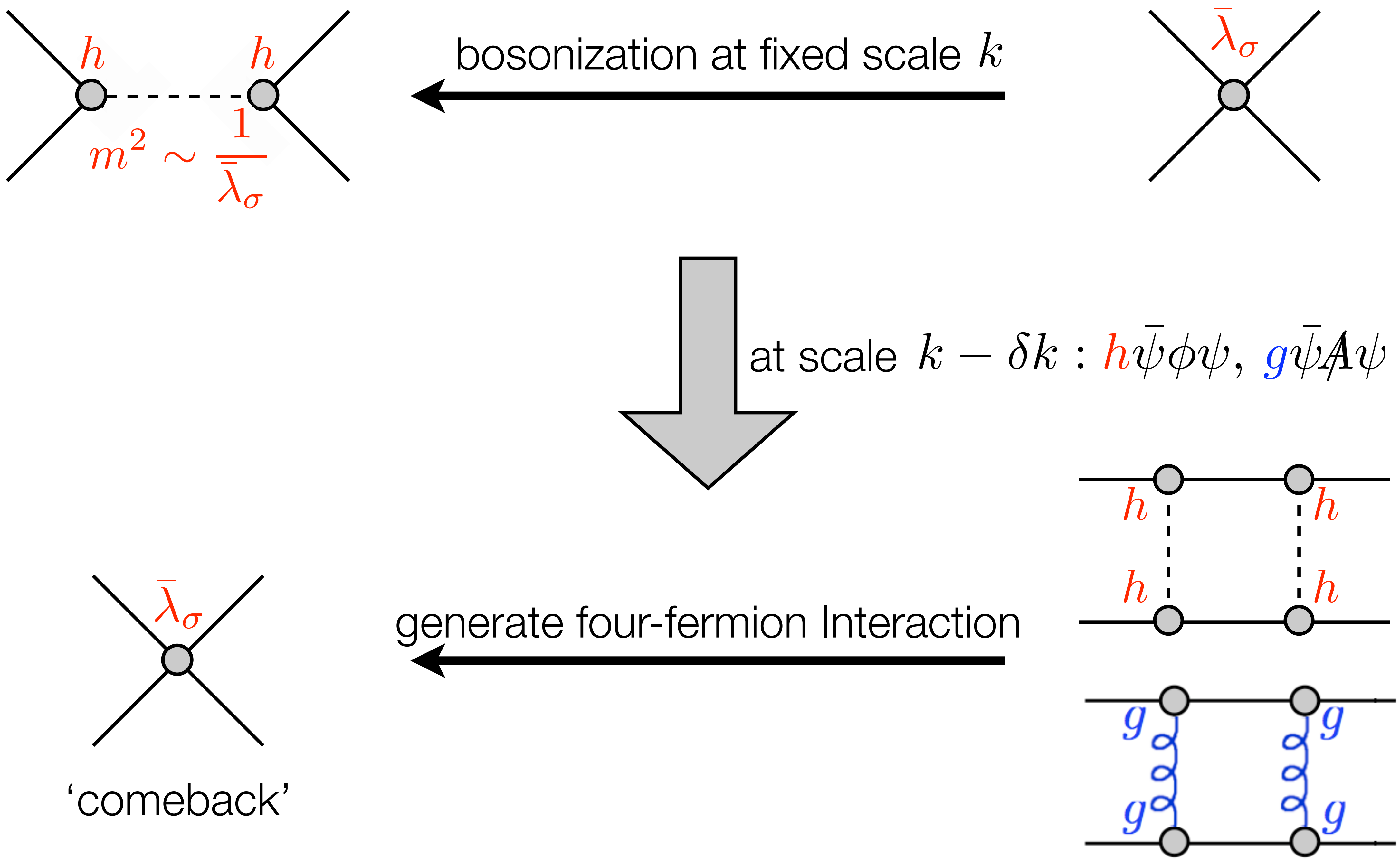
bosonization at fixed scale

Hubbard-Stratonovich transformation



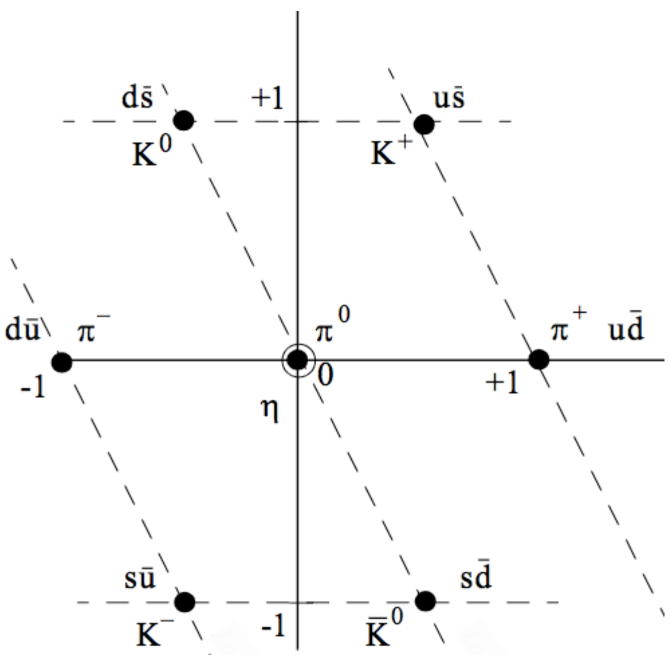
From microscopic to macroscopic DoFs

- problem:



From microscopic to macroscopic DoFs

macroscopic DoF

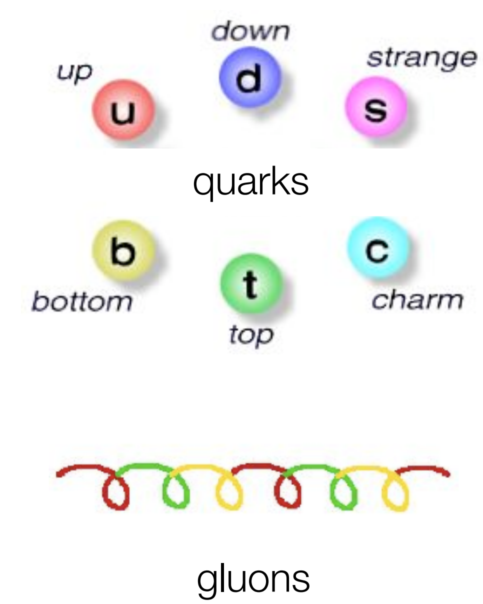


solution:

scale-dependent degrees of freedom

$$\partial_t \phi_k \sim \bar{\psi}_L \psi_R$$

microscopic DoF



$k \to 0$ k $k \to \Lambda$

$$\partial_t \Gamma_k[\phi_k] = \frac{1}{2} \mathrm{Tr} \frac{\partial_t R_k}{\Gamma_k^{(2)}[\phi_k] + R_k} - \int_x \frac{\delta \Gamma_k[\phi_k]}{\delta \phi_k} \partial_t \phi_k$$

(H. Gies, C. Wetterich '01, '02; J. M. Pawlowski '05)

QCD with one quark flavor

- allows (momentum-dependent) four-fermion interactions to arbitrary order to be included

- serves as a check for the approach incorporating "only" quark-gluon dynamics

- ansatz:

$$\Gamma_k = \int_x \Big\{ \bar\psi(\mathrm{i}\not{D} + \mathrm{i}\gamma_0\mu_q)\psi + \frac{\bar\lambda_\sigma}{2}\left[(\bar\psi\psi)^2 - (\bar\psi\gamma_5\psi)^2\right] + Z_\phi \partial_\mu\phi^*\partial_\mu\phi + U(\phi^2)$$
$$+\bar h\left[(\bar\psi_R\psi_L)\phi - (\bar\psi_L\psi_R)\phi^*\right]\Big\} + \Gamma_{gauge}$$

- initial conditions: $\bar\lambda_\sigma|_{\Lambda} = 0,\ \bar\lambda_\phi|_{\Lambda} = 0,\ \bar h|_{\Lambda} = 0,\ Z_\phi|_{\Lambda} = 0,\ \alpha_s(M_Z) = 0.118$

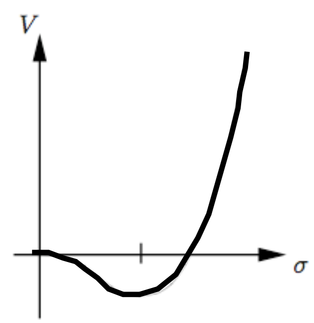
$$\Gamma_k = \int_x \left\{ \frac{1}{4} F^a_{\mu\nu}F^a_{\mu\nu} + \bar\psi(\mathrm{i}\not{D} + \mathrm{i}\gamma_0\mu_q)\psi \right\}$$

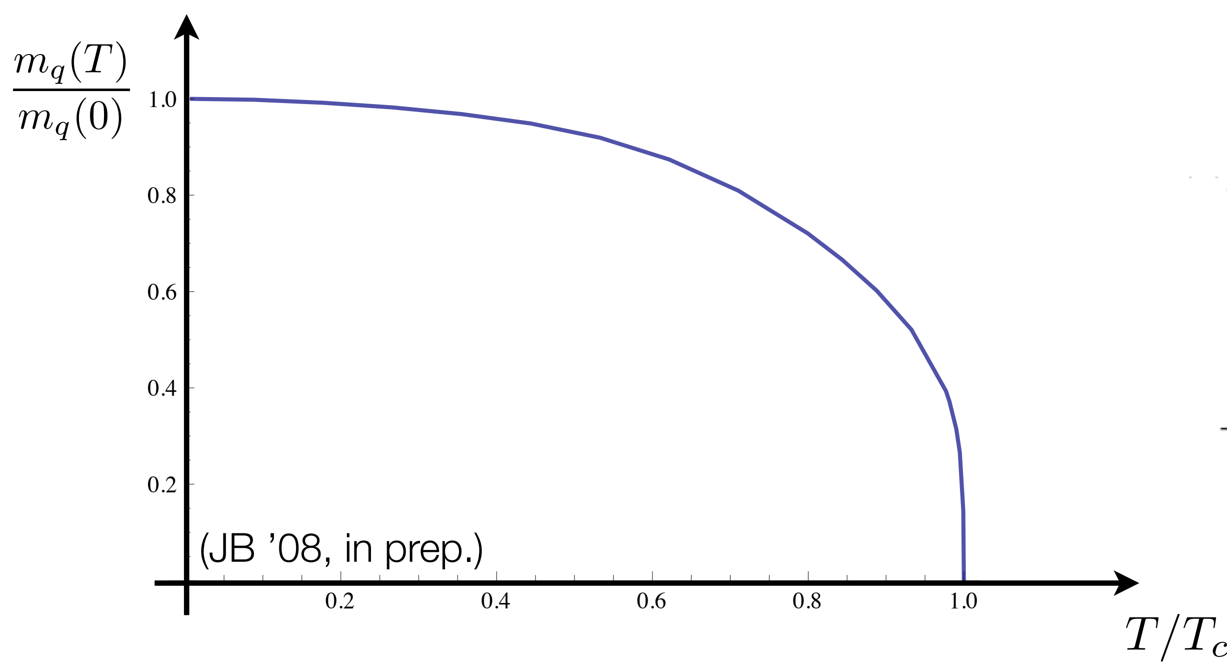
QCD with one quark flavor

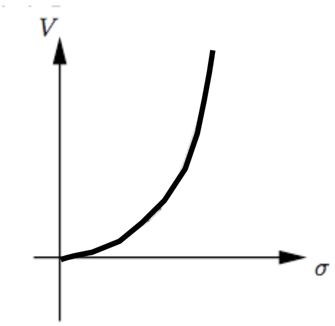
- ansatz:

$$\Gamma_k = \int_x \Big\{ \bar\psi(\mathrm{i}\not{D} + \mathrm{i}\gamma_0\mu_q)\psi + \frac{\bar\lambda_\sigma}{2}\left[(\bar\psi\psi)^2 - (\bar\psi\gamma_5\psi)^2\right] + Z_\phi \partial_\mu\phi^*\partial_\mu\phi + U(\phi^2) + \bar h\left[(\bar\psi_R\psi_L)\phi - (\bar\psi_L\psi_R)\phi^*\right] \Big\} + \Gamma_{gauge}$$

- initial conditions: $\bar\lambda_\sigma|_\Lambda = 0,\ \bar\lambda_\phi|_\Lambda = 0,\ \bar h|_\Lambda = 0,\ Z_\phi|_\Lambda = 0,\ \alpha_s(M_Z) = 0.117$







- with global $U_A(1)$ symmetry: 2nd order phase transition
- anomalously broken $U_A(1)$: crossover

QCD with one quark flavor: phase boundary

$$\frac{T_c(\mu_q)}{T_c(0)} = 1 - t_2 \left(\frac{\mu_q}{\pi T_c(0)}\right)^2 + \ldots$$

- large N_c expansion: $t_2 \sim \dfrac{N_f}{N_c}$ (D. Toublan '05)
- results from different approaches:

Method	$N_f = 1$	$N_f = 2$	$N_f = 3$	
FRG: QCD flow	0.97*	---	---	(JB '08, in prep.)
Lattice: imag. μ	---	0.500(54)	0.667(6)	(de Forcrand et al. '02, '06)
Lattice: Taylor+Rew.	---	---	1.13(45)	(Karsch et al. '03)

*with global $U_A(1)$ symmetry

- only one single input parameter: $\alpha_s(M_Z)$

QCD with one quark flavor: phase boundary

- results from different approaches:

Method	$N_f = 1$	$N_f = 2$	$N_f = 3$	
FRG: QCD flow	0.97*	---	---	(JB '08, in prep.)
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Lattice: Taylor+Rew.	---	---	1.13(45)	(Karsch et al. '03)

*with global $U_A(1)$ symmetry

- anomalously broken $U_A(1)$:

$$\Gamma_{\mathcal{I}} = \int d^4x \, m_{\mathcal{I}}(\bar{\psi}_R \psi_L - \bar{\psi}_L \psi_R)$$

(masslike) $\Rightarrow$ curvature t_2 becomes smaller

- estimate for lower bound for the curvature: $t_2 \gtrsim 0.40$

Outline

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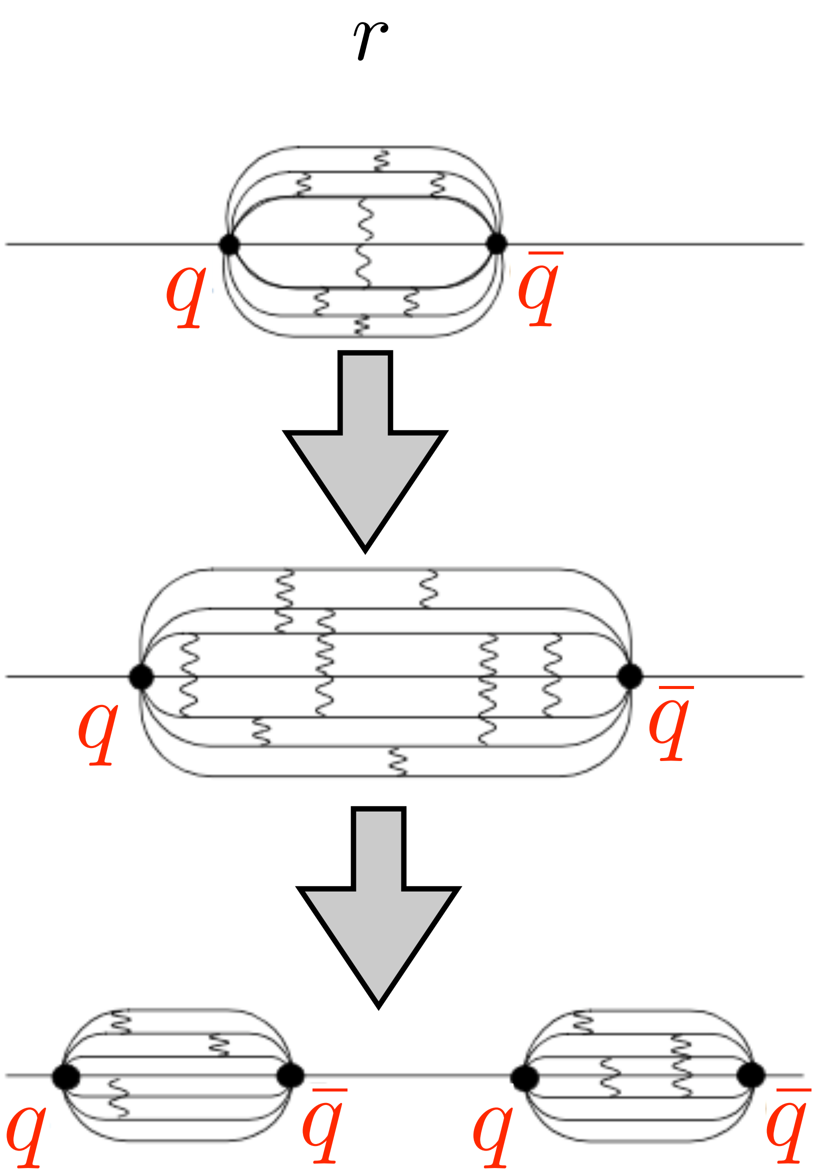
Confinement at zero temperature

potential of a quark-antiquark pair: $\mathcal{F}_{q\bar{q}}(r) \propto \sigma r$

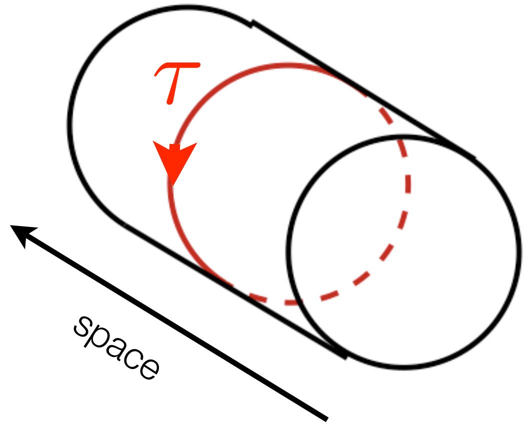


confinement at finite temperature

- infinitely heavy quark moving in Euclidean time direction:

$$\frac{\partial \Psi_q}{\partial \tau} = \mathrm{i}\bar{g} A_0 \Psi_q \quad \Longrightarrow \quad \Psi_q(\vec{x},\tau) = \left[\mathrm{P}\exp\left(i\bar{g}\int_0^\tau dt A_0 \right)\right] \Psi_q(\vec{x},0)$$

infinitely heavy quark propagating in (Euclidean) time direction



- Polyakov-Loop: $\tau = \beta = 1/T$ (Polyakov '78, Susskind '79)

$$\mathcal{P}(\vec{x}) = \frac{1}{N_c}\mathrm{P}\exp\left(i\bar{g}\int_0^\beta dt A_0(t,\vec{x}) \right)$$

confinement at finite temperature

- expectation value of Polyakov-loop is related to the quark free energy $\mathcal{F}_q$:

$$\langle \mathrm{Tr}_F \mathcal{P}(\vec{x}) \rangle \sim \int \mathcal{D}A \, \mathrm{Tr}_F \mathcal{P}(\vec{x}) \, e^{-S} \sim \mathrm{e}^{-\beta \mathcal{F}_q}$$

⇨ deconfinement:

$$\mathcal{F}_q \ \text{finite} \quad \Longleftrightarrow \quad \langle \mathrm{Tr}_F \mathcal{P}(\vec{x}) \rangle \neq 0$$

⇨ confinement:

$$\mathcal{F}_q \to \infty \quad \Longleftrightarrow \quad \langle \mathrm{Tr}_F \mathcal{P}(\vec{x}) \rangle = 0$$

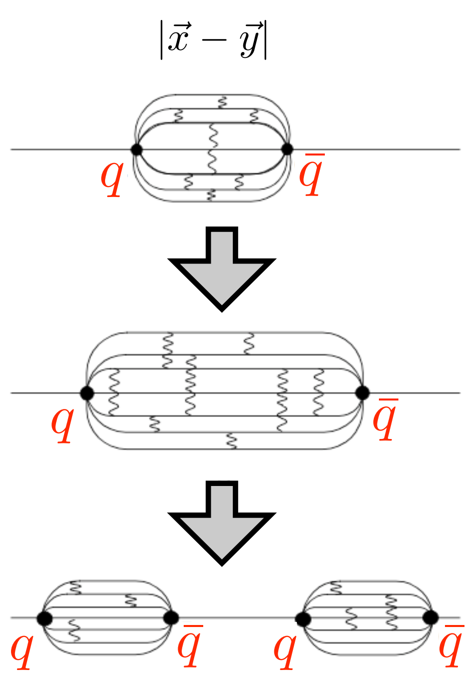
confinement at finite temperature

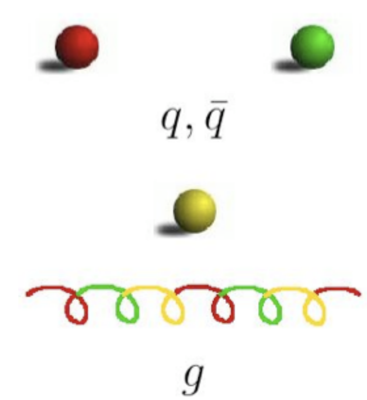
- quark-antiquark correlator:

$$\lim_{|\vec{x}-\vec{y}|\to\infty} \mathrm{e}^{-\beta\mathcal{F}_{q\bar{q}}} \sim \lim_{|\vec{x}-\vec{y}|\to\infty} \langle \mathrm{Tr}\,\mathcal{P}(\vec{x}) \cdot \mathrm{Tr}\,\mathcal{P}^\dagger(\vec{y})\rangle \leq |\mathrm{e}^{-\beta\mathcal{F}_q}|^2$$

$T < T_c$ $\qquad$ $T \geq T_c$

$$\mathcal{F}_{q\bar{q}} \sim \sigma|\vec{x}-\vec{y}|$$





deconfinement

$$\mathcal{F}_q \to \infty \quad \Longleftrightarrow \quad \langle \mathrm{Tr}_F \mathcal{P}(\vec{x})\rangle = 0$$

$$\mathcal{F}_q \text{ finite} \quad \Longleftrightarrow \quad \langle \mathrm{Tr}_F \mathcal{P}(\vec{x})\rangle \neq 0$$

confinement criterion at vanishing temperature

(JB, H. Gies, J. M. Pawlowski '07)

- (RG) Polyakov-loop potential in Landau-background-field-gauge

$$V(\beta\langle A_0\rangle) = \frac{1}{\Omega T^4}\left(\frac{1}{2}\mathrm{Tr}\ln\Gamma^{(2)}_{\mathrm{A}}[\beta\langle A_0\rangle] - \mathrm{Tr}\ln\Gamma^{(2)}_{\mathrm{gh}}[\beta\langle A_0\rangle]\right) + \mathcal{O}(\partial_t\Gamma^{(2)})$$

- (very) high-temperature: potential is dominated by modes $k \sim p \sim T$

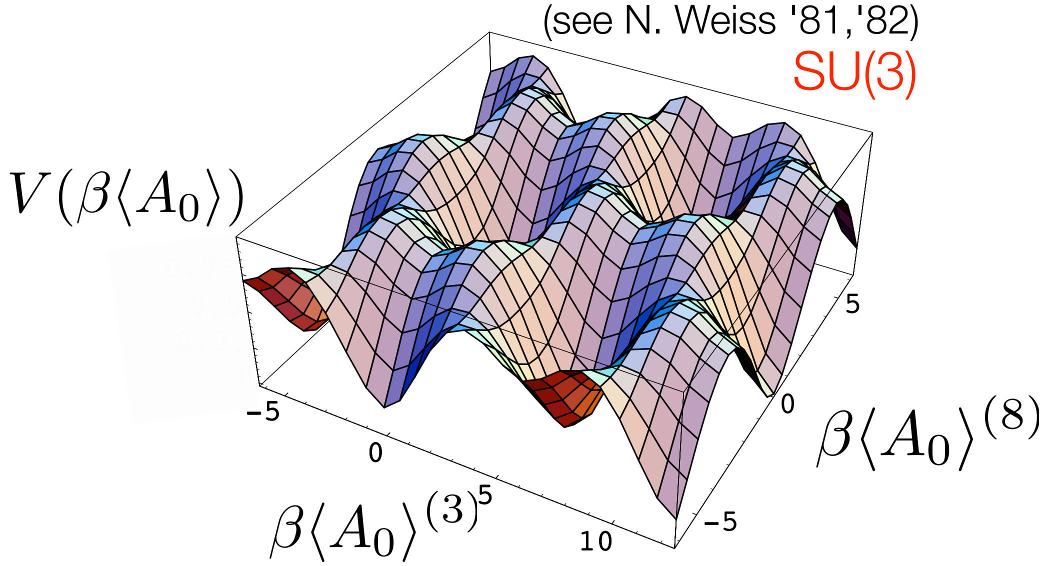
$$(\Gamma^{(2)}_{A}) \sim D^2[\langle A_0\rangle]\,, \qquad (\Gamma^{(2)}_{\mathrm{gh}}) \sim D^2[\langle A_0\rangle]$$

perturbative Polyakov-Loop potential

(JB, H. Gies, J. M. Pawlowski '07)

- perturbative Polyakov-loop potential in background-field gauge, $A_\mu = \delta_{\mu 0}\langle A_0\rangle$

(see N. Weiss '81,'82)



minimum at $\beta\langle A_0\rangle = 0$:
deconfinement (broken Z_3-symmetry)

$$\mathcal{F}_q \text{ finite} \quad \Longleftrightarrow \quad \langle \mathrm{Tr}_F \mathcal{P}(\vec{x})\rangle \neq 0$$

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- low-temperature: $k \sim p \sim T \lesssim \Lambda_{\mathrm{QCD}}$

$$(\Gamma^{(2)}_{A}) \sim (D^2[\langle A_0\rangle])^{1+\kappa_A}\,, \qquad (\Gamma^{(2)}_{\mathrm{gh}}) \sim (D^2[\langle A_0\rangle])^{1+\kappa_{gh}}$$

- Polyakov-loop potential in the IR (d=4)

$$V_{\mathrm{IR}}(\beta\langle A_0\rangle) = (2 + 3\kappa_A - 2\kappa_{gh})V_{\mathrm{UV}}(\beta\langle A_0\rangle)$$

- **what if ...**

$$\boxed{3\kappa_A - 2\kappa_{gh} < -2} \quad \Rightarrow \quad \boxed{\kappa_{gh} > \frac{d-3}{4}}$$

perturbative Polyakov-Loop potential

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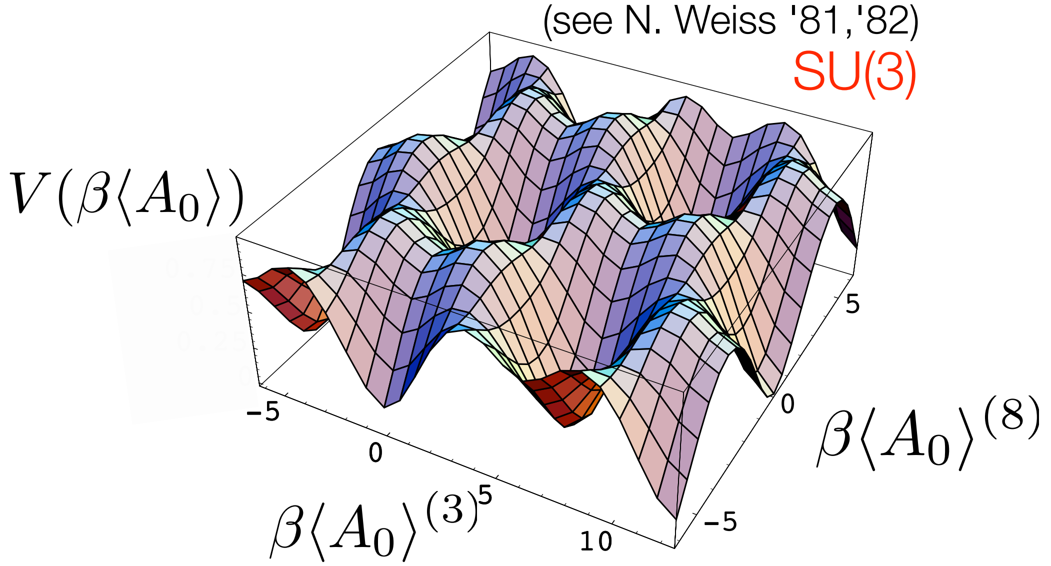
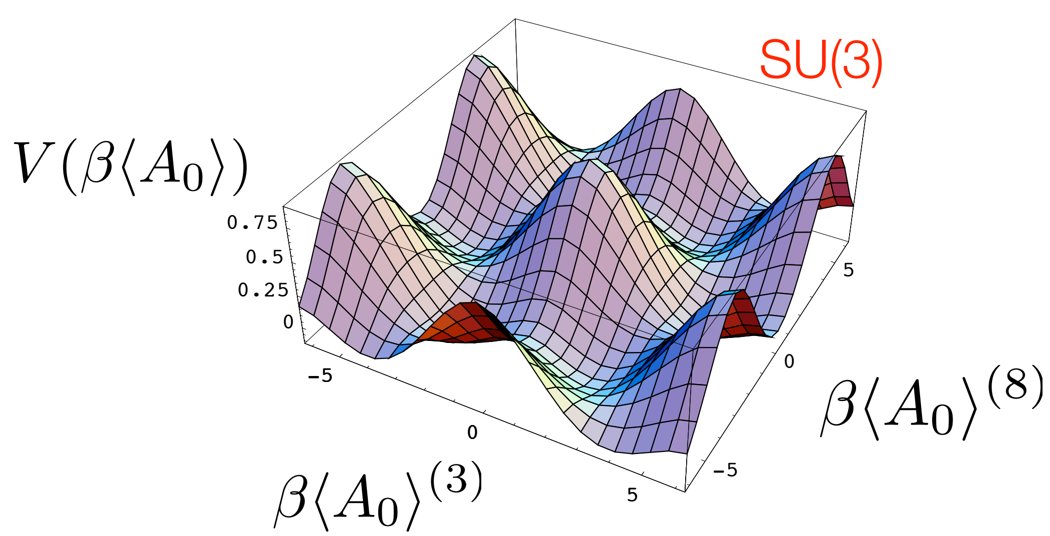


minimum at $\beta\langle A_0\rangle = 0$:
deconfinement (broken Z_3 -symmetry)

$$\mathcal{F}_q \ \text{finite} \quad \Longleftrightarrow \quad \langle \mathrm{Tr}_F \mathcal{P}(\vec{x})\rangle \neq 0$$

- for $T < T_c$



minimum at $\beta\langle A_0\rangle = (2/3)2\pi$:
deconfinement (broken Z_3 -symmetry)

$$\mathcal{F}_q \to \infty \quad \Longleftrightarrow \quad \langle \mathrm{Tr}_F \mathcal{P}(\vec{x})\rangle = 0$$

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- **quark confinement criterion (Landau gauge)**:

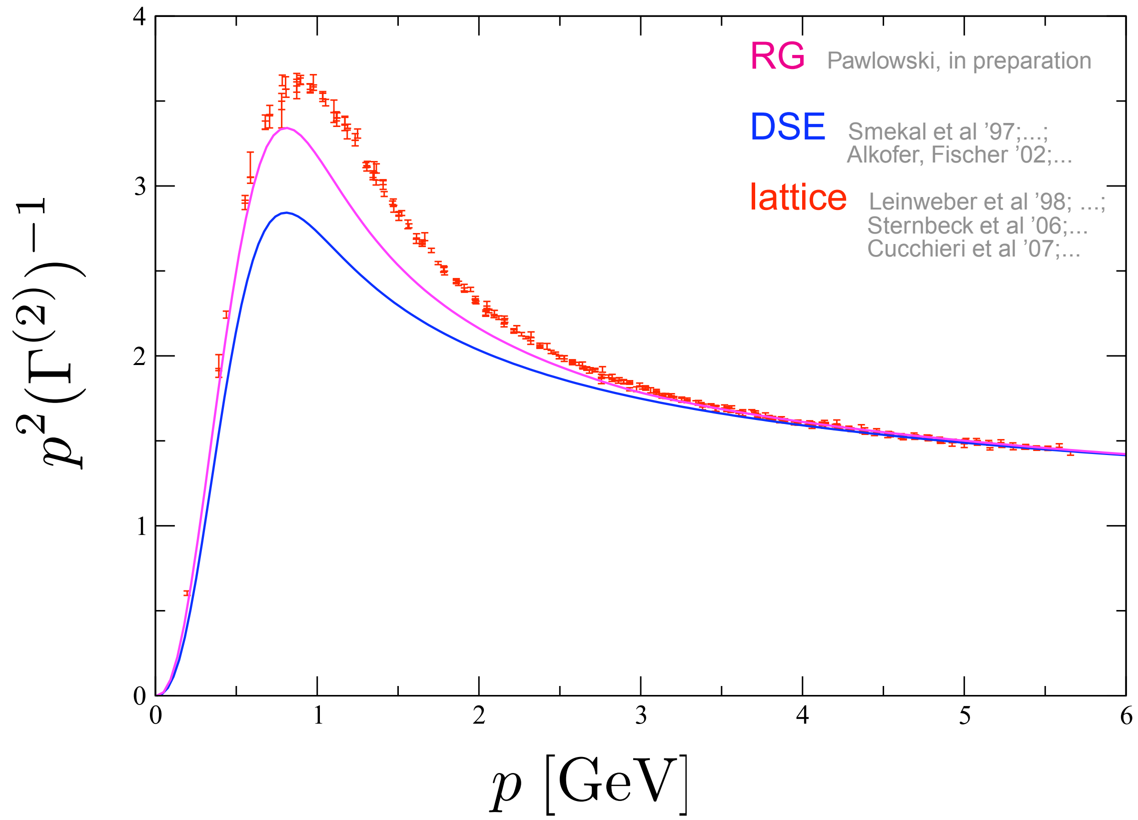
$$\boxed{\langle \mathrm{Tr}_F \mathcal{P}(\vec{x})\rangle = 0\ :\ 3\kappa_A - 2\kappa_{gh} < -2} \quad \Rightarrow \quad \boxed{\langle \mathrm{Tr}_F \mathcal{P}(\vec{x})\rangle = 0\ :\ \kappa_{gh} > \frac{d-3}{4}}$$

- quark confinement induced by IR gluon suppression

- confer: Kugo-Ojima criterion: $\kappa_{gh} > 0$ (Kugo, Ojima '79) Gribov-Zwanziger condition: $\kappa_{gh} > \frac{1}{2}$ (Gribov '78; Zwanziger '94,'03)

Landau-gauge propagators & color confinement

$$(\Gamma_A^{(2)})^{-1} \xrightarrow{\mathrm{IR}} \frac{1}{(p^2)^{1+\kappa}}$$



- results for κ_{gh}

Method	κ_{gh}
DSE/SQ	0.595
FRG	$0.539 \leq \kappa \leq 0.595$
Lattice	...

(Lerche, v. Smekal '02; Zwanziger '02)

(Pawlowski, Litim, Nedelko, v. Smekal '03; Fischer, Gies '04)

(Sternbeck et al.'05; Olivera, Silva '06; Cucchieri, Mendes '06;Cucchieri, Mendes '07; Sternbeck et al.'07)

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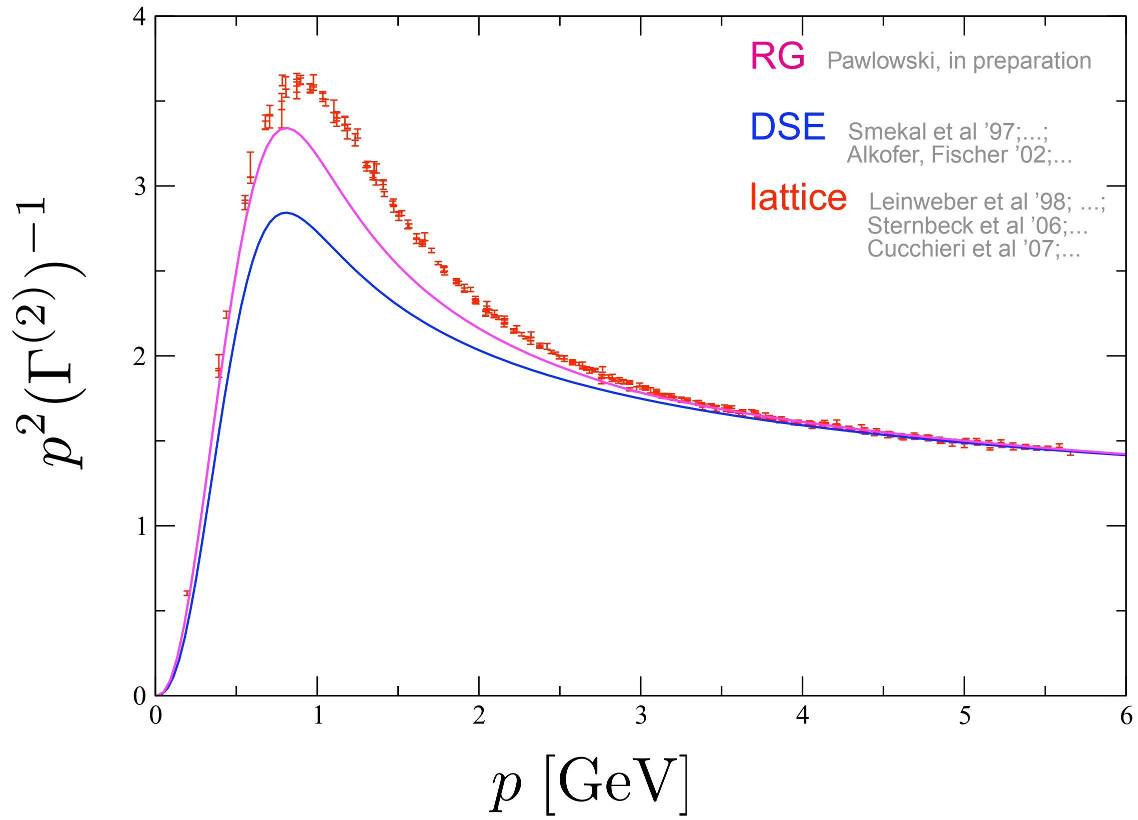

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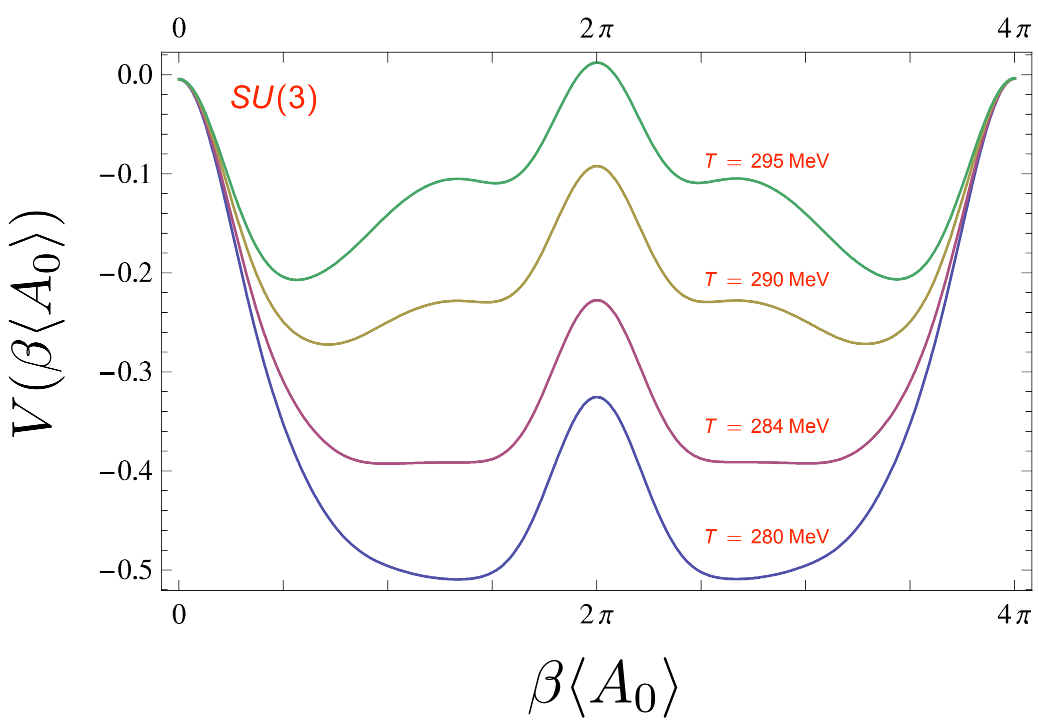
DSE/SQ	0.595	(Lerche, v. Smekal '02; Zwanziger '02)
FRG	$0.539 \leq \kappa \leq 0.595$	(Pawlowski, Litim, Nedelko, v. Smekal '03; Fischer, Gies '04)
Lattice	from naive extrapolation $\kappa_A = -1\,,\ \kappa_C = 0$	(Sternbeck et al.'05; Olivera, Silva '06; Cucchieri, Mendes '06;Cucchieri, Mendes '07; Sternbeck et al.'07)

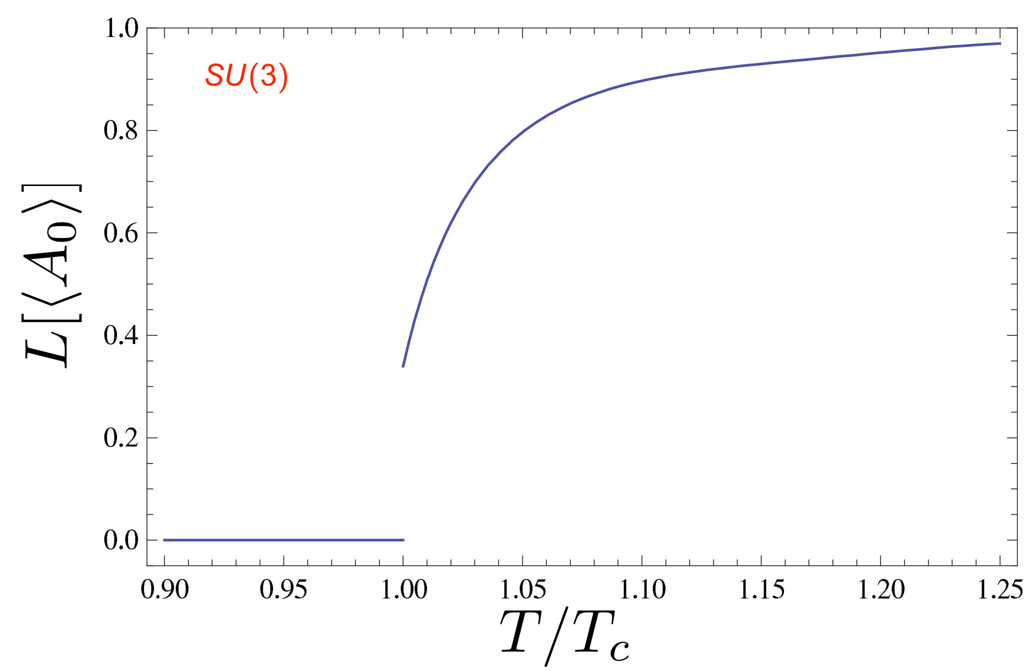
satisfies quark confinement criterion

Polyakov-Loop Potential in Landau-gauge

(JB, H. Gies, J. M. Pawlowski '07)

- order parameter $L[\langle A_0\rangle] = \frac{1}{N_c}\mathrm{Tr}_F \exp\left(i\int_0^\beta dt\langle A_0\rangle\right)$





- first order phase transition for SU(3) (and second order for SU(2))

- SU(3): $T_c = 284\,\mathrm{MeV} (= 0.646\sqrt{\sigma})$ Lattice QCD: $T_c = 0.646\sqrt{\sigma}$ (Kaczmarek et al.)

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Conclusions

- FRG allows to bridge the gap between regimes with different DoF
- good agreement with Lattice QCD studies for chiral as well as deconfinement phase transition
- critical number of quark flavors for SU(3): $N_{f,cr} = 12$
- shape of the phase boundary near $N_{f,cr}$ is determined by the underlying IR fixed point scenario (testable prediction!)
- promising results for finite chemical potential
- criterion for quark confinement

Outlook

- quantitative study of the effect of anomalously broken $U_A(1)$ on the phase boundary (together with J. Pawlowski)

- order of the phase transition?
  (insights from finite-volume scaling (?), together with B. Klein)

- study finite chemical potential for $N_f = 2$ and $N_f = 3$

- deconfinement and chiral phase transition at the same temperature?

- ...