

Space and Phase-Space saturation: A simple Bag-Model-inspired picture for a smooth transition to QGP

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(paper in preparation)

- Introduction.
- Brief remind on the bag model and compressible bags.
- The partition function of a gas of compressible hadrons.
- The isobaric partition function.
- The grand canonical partition function.
- Conclusions & Outlook.

The crossover transition

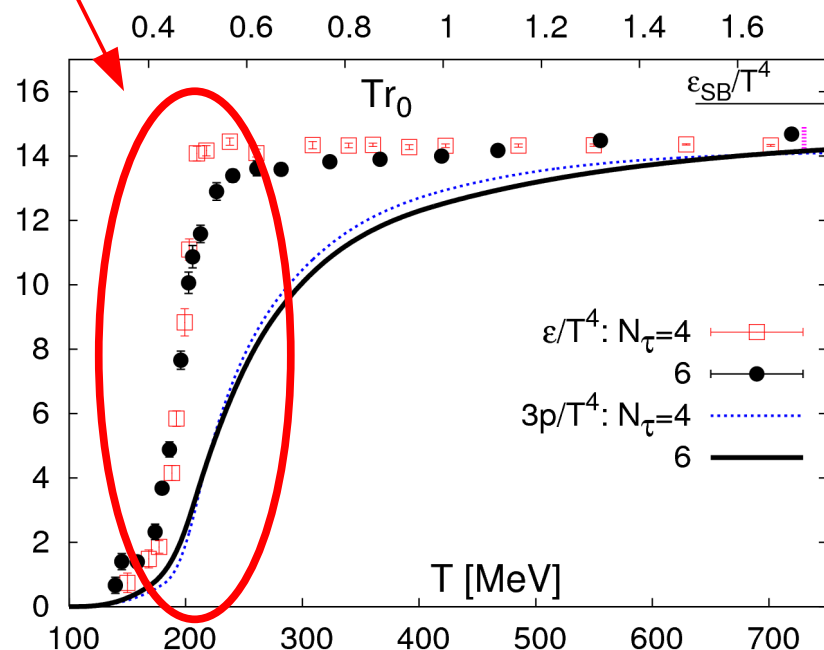
Crossover

Y. Aoki, G. Endrodi, Z. Fodor, S. D. Katz
and K. K. Szabo, Nature **443**, 675 (2006)

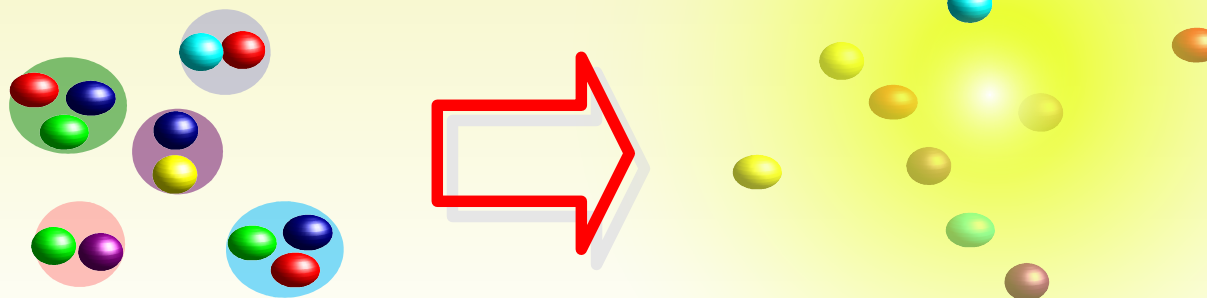
It is now evident that the confined phase of QCD can be described by hadron-gas models.

Around the transition a dual approach should be possible!

Try to understand this transition from the “hadronic side”

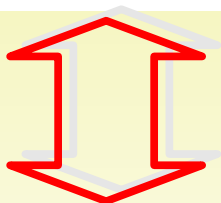
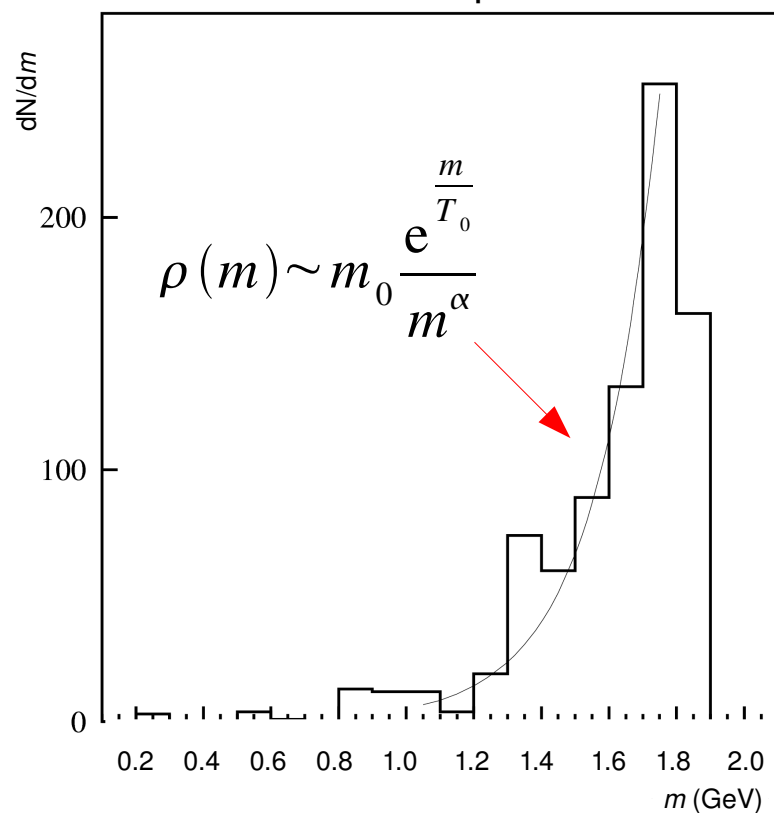


M. Cheng *et al.* Phys. Rev. D **77**, 014511 (2008)



Models of hadrons

The mass-spectrum



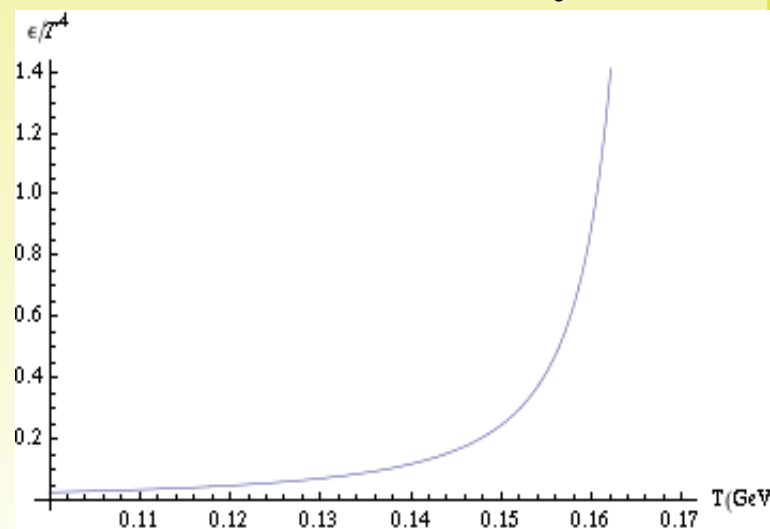
- The Statistical Bootstrap Model
(T_0 is the limiting temperature: the Hagedorn temperature)
- The Bag model
(T_0 is the transition temperature to the deconfined phase)

Asymptotically exponential mass-spectrum

$$Z(T, V) = \sum_{N=0}^{\infty} \frac{(V m_0)^N}{N!} \left[\prod_{i=1}^N \int_0^{\infty} dm_i \frac{d^3 p_i}{(2\pi)^3} m_i^{-\alpha} \right] \times \exp \left(\sum_{i=1}^N \frac{m_i}{T_0} - \sum_{i=1}^N \frac{p_i^2 / 2 m_i + m_i}{T} \right)$$

(Boltzmann Statistics N.R. limit)

The partition function diverges at $T=T_0$



The bag model

In its simplest formulation, hadrons are described as bags of u.r. Ideal gas (the QGP).



The Bag model seems to be a good candidate as it incorporates asymptotic freedom and confinement

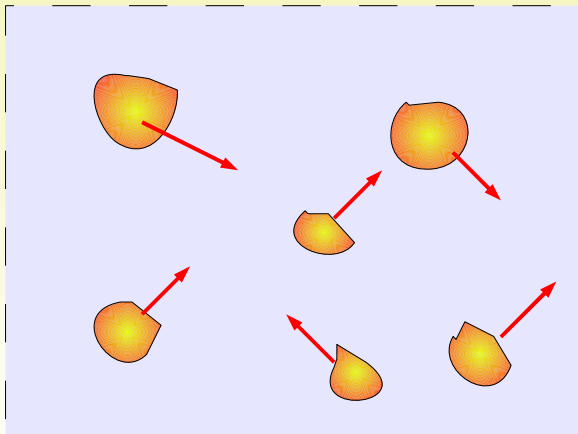


It exhibits a first order phase transition

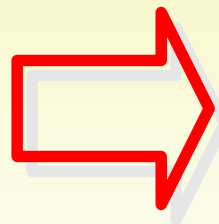


Is there any way to get rid of the excess of d.o.f. without invoking an actual phase transition?

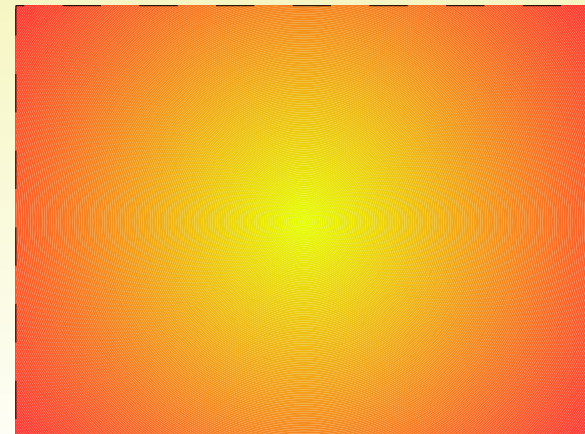
Hadron (bag) gas



Crossover



QGP



The bag model

$$m = U + B V_b$$

where

$$U = 3 p_r V_b$$

(large V_b)

$$m = 4 B V_b$$

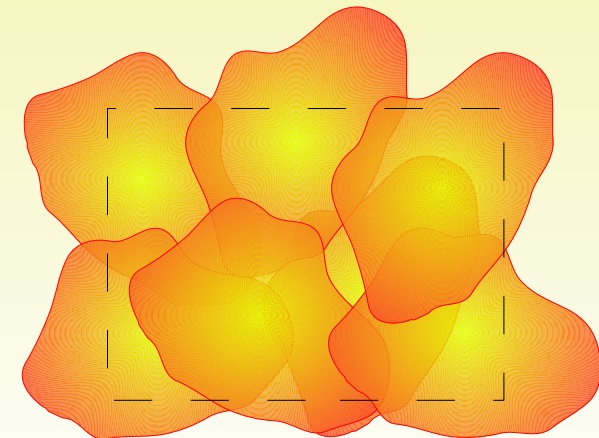
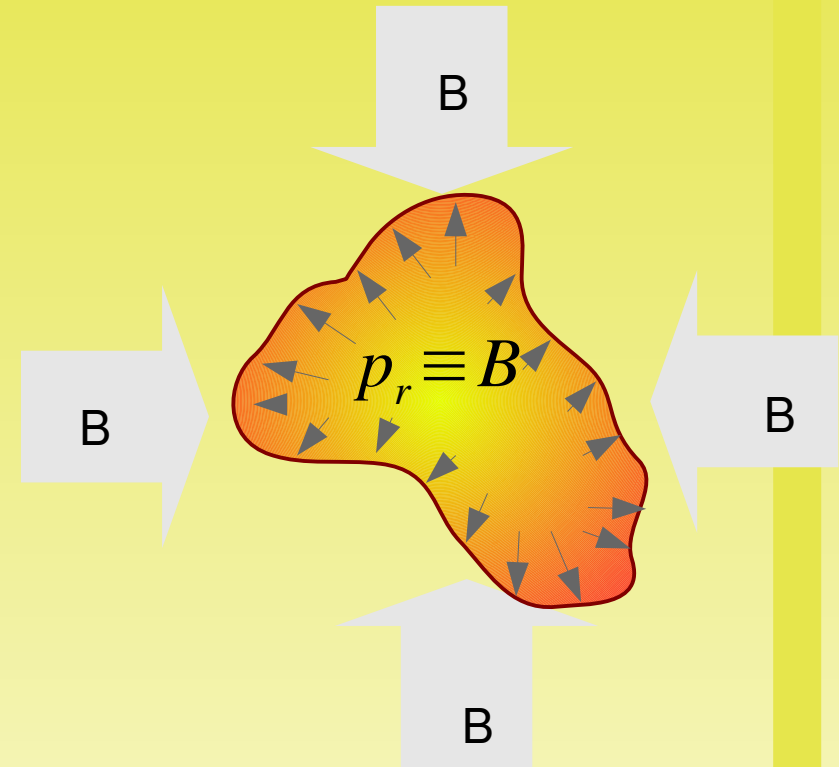
$$S - S_0 = \frac{4 U}{3 k p_r^{1/4}} = \frac{m}{T_0}$$

(depends on the d.o.f. of the inner field (or u.r. gas))

$$\rho(m) \propto e^{\frac{m}{T_0}}$$



$$T \rightarrow T_0 \quad m \rightarrow \infty \quad V_b \rightarrow \infty$$

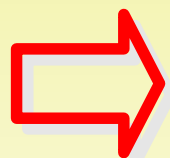


Reduced phase space

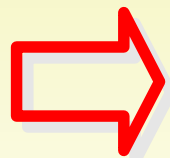
$$\left[\prod_{i=1}^N \frac{V}{(2\pi)^3} \int d^3 p \right] \rightarrow \left[\prod_{i=1}^N \frac{1}{(2\pi)^3} \int d^3 p \right] (V - \sum_{i=1}^N V_i)^N \theta(V - \sum_{i=1}^N V_i)$$

M.I. Gorenstein, V.K. Petrov and G. M. Zinovev, Phys. Lett. B **106**, 327 (1981).

Assuming: $V_i = \frac{m_i}{4B}$

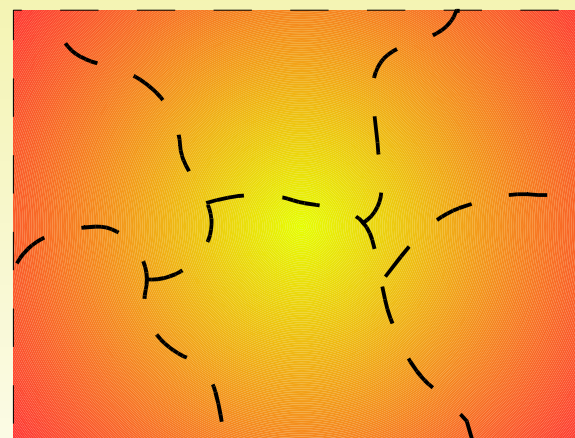


$Z(V, T)$ well defined at any T



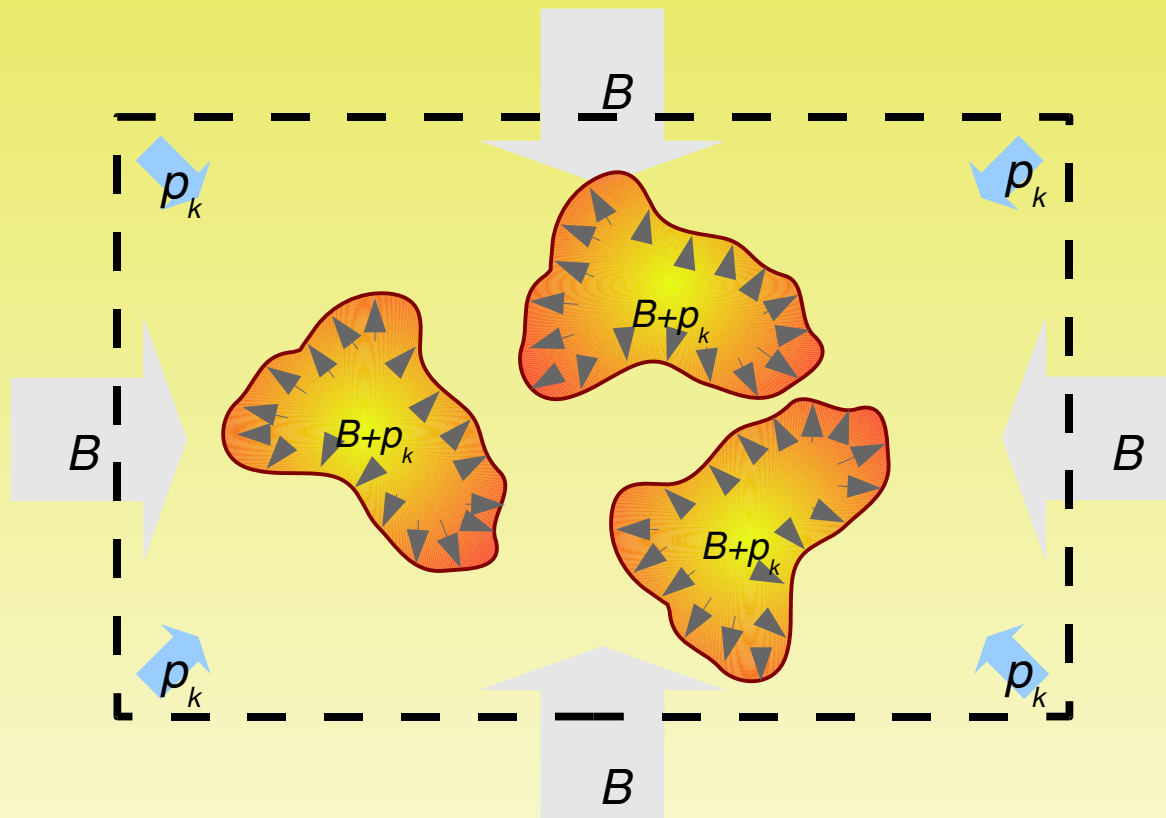
Does not yield the desired thermodynamic at high T

$$\epsilon_{max} = 4B$$



Compressible hadrons

We search for a possible solution by adopting the idea of compressible-bag models

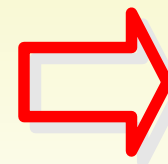


We study the simplest system:

- N.R.
- Boltzmann Statistics
- No Van Der Waals
- No Surface Energy
- Ideal gas Bag-like hadrons
- Continuous mass-spectrum

Stability Condition:

$$p_r \equiv B \rightarrow p_r(V, T) \equiv B + p_k(V, T)$$



Self Consistency Relation

Associated to the thermal motion of the hadrons themselves

The partition function

$$V_i \rightarrow \frac{U_i}{3 p_r}$$

$$m_i \rightarrow U_i \left(1 + \frac{B}{3 p_r} \right)$$

$$S_i = \frac{m_i}{T_0} \rightarrow \frac{4 U_i}{3 k p_r^{1/4}}$$

After integrating over particle's momenta...

$$Z(T, V) = \sum_{N=0}^{\infty} \left(\frac{T}{2\pi} \right)^{3N/2} \frac{m_0^N}{N!} \left[\prod_{i=1}^N \int_0^{\infty} dU_i \left(U_i + B \frac{U_i}{3 p_r} \right)^{3/2-\alpha} \right] \\ \times \exp \left\{ \left[\frac{4}{3 k p_r^{1/4}} - \frac{1}{T} \left(1 + \frac{B}{3 p_r} \right) \right] \sum_{i=1}^N U_i \right\} \left(V - \sum_{i=1}^N \frac{U_i}{3 p_r} \right)^N \theta \left(V - \sum_{i=1}^N \frac{U_i}{3 p_r} \right)$$

$$U_i \rightarrow m_i = \frac{4}{3} U_i$$

Lower bound $m_{cut} = m_{\pi}$

$$Z(T, V) = \sum_{N=0}^{\infty} \left(\frac{T}{2\pi} \right)^{3N/2} \frac{m_0^N}{N!} \left[\prod_{i=1}^N \int_{m_{cut}}^{\infty} dm_i \left(\frac{3}{4} m_i + B \frac{m_i}{4 p_r} \right)^{3/2-\alpha} \right] \\ \times \exp \left\{ \left[\frac{1}{k p_r^{1/4}} - \frac{1}{T} \left(\frac{3}{4} + \frac{B}{4 p_r} \right) \right] \sum_{i=1}^N m_i \right\} \left(V - \sum_{i=1}^N \frac{m_i}{4 p_r} \right)^N \theta \left(V - \sum_{i=1}^N \frac{m_i}{4 p_r} \right)$$

Note: for $p_r = B$ one recover the previous partition function with the new phase space

The isobaric partition function

$$p_r(V, T) \equiv B + p_k(V, T)$$

Must be evaluated from the partition function itself

$s = p/T$ plays the role of an external pressure

$$\hat{Z}(T, s) \equiv \int_0^{\infty} dV Z(V, T) e^{-sV}$$

$$Z(V, T) \equiv e^{p_k V/T}$$

$$p_k = sT$$

The integral in dV can be solved and gives:

$$\hat{Z}(T, s) = \frac{1}{s} \sum_{N=0}^{\infty} \left[\frac{f(T, s)}{s} \right]^N = \frac{1}{s - f(T, s)}$$

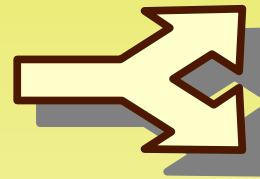
$$f(T, s) = m_0 \left(\frac{T}{2\pi} \right)^{3/2} \int_{m_{cut}}^{\infty} dm \left(\frac{3}{4} m + B \frac{m}{4(B + sT)} \right)^{3/2 - \alpha} \exp \left\{ \frac{m}{k(B + sT)^{1/4}} - \frac{m}{T} \right\}$$

The infinite volume limit

In the infinite volume limit the asymptotic behavior of $Z(V, T)$ is defined by the singularity of $\hat{Z}(V, T)$ with the largest real part.

We have two distinct cases depending on the parameter α on

$$\rho(m) = m_0 \frac{e^{\frac{m}{T_0}}}{m^\alpha}$$



1 $\alpha \leq 5/2$

2 $\alpha > 5/2$

Case

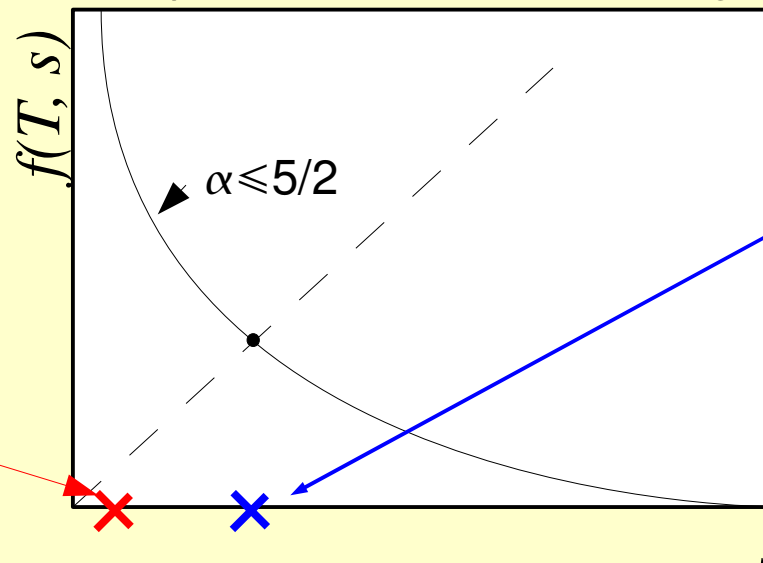


$$\hat{Z}(T, s) = \frac{1}{s} \sum_{N=0}^{\infty} \left[\frac{f(T, s)}{s} \right]^N = \frac{1}{s - f(T, s)}$$

The isobaric partition function has two singularities:

1) The divergence of the function f itself when the exponent of the integrand vanishes:

$$s_f(T) = \frac{T^3}{k^4} - \frac{B}{T}$$



2) The pole:

$$s_0(T) = f(T, s_0(T))$$

Is the solution for the pressure $p_k(\infty, T)$

This one is always the rightmost!

The infinite volume limit

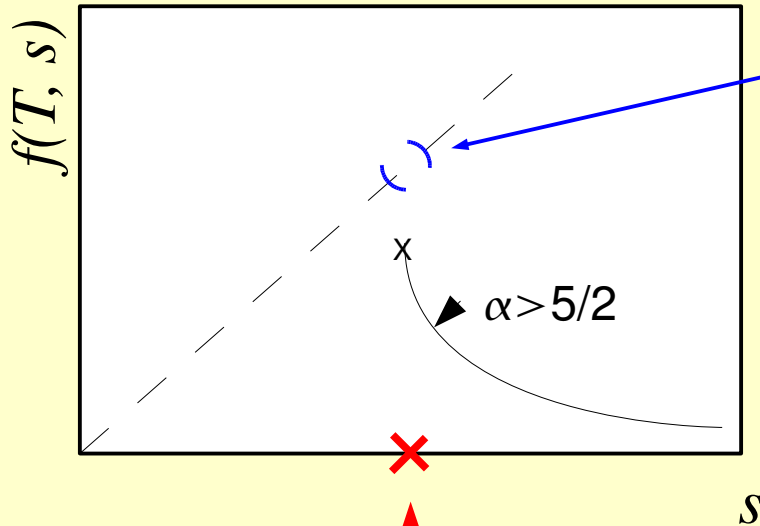
Case

2

 $(\alpha > 5/2)$

$$\hat{Z}(T, s) = \frac{1}{s} \sum_{N=0}^{\infty} \left[\frac{f(T, s)}{s} \right]^N = \frac{1}{s - f(T, s)}$$

$$f(T, s) = m_0 \left(\frac{T}{2\pi} \right)^{3/2} \int_{m_{cut}}^{\infty} dm \left(\frac{3}{4} m + B \frac{m}{4(B + sT)} \right)^{3/2 - \alpha} \exp \left\{ \frac{m}{k(B + sT)^{1/4}} - \frac{m}{T} \right\}$$



For sufficiently large T , the pole:

$$s_0(T) = f(T, s_0(T))$$

does not exist and there is **no solution** for the system's pressure!

$$s_f(T) = \frac{T^3}{k^4} - \frac{B}{T}$$

The function f is finite in $s=s_f$,
while diverges for $s < s_f$.

This situation has been discussed in:

M.I. Gorenstein, V.K. Petrov and G. M. Zinovev, Phys. Lett. B **106**, 327 (1981).

possible mechanism for a **phase transition**.

Here we focus on Case

1

Numerical analysis of the isobaric partition function

Standard values of the model parameters:

This will be the transition temperature

$$\left(\frac{90}{\pi^2 (16+21)} \right)^{1/4} \sim 0.7$$

$$T_0 \equiv k B^{1/4} = 0.17 \text{ GeV}$$

$$k = 0.68$$

$$\rightarrow B^{1/4} = 250 \text{ MeV}$$

Chosen on purpose to have the same high- T limit of LQCD

$$m_{\text{cut}} = m_{\pi} = 0.139 \text{ GeV}$$

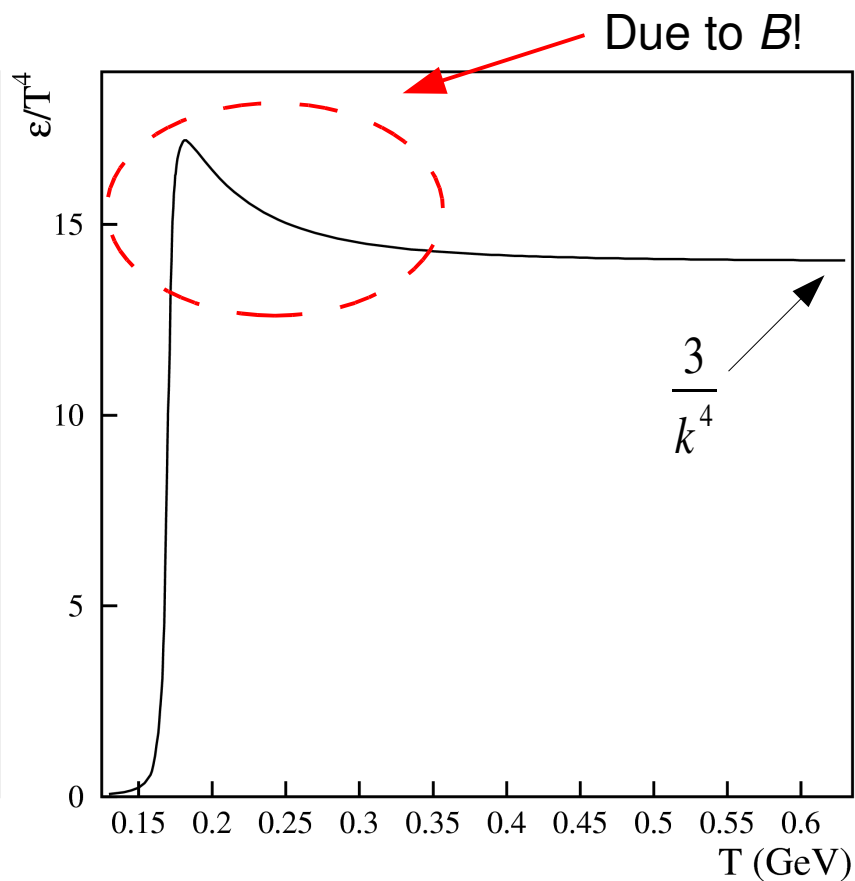
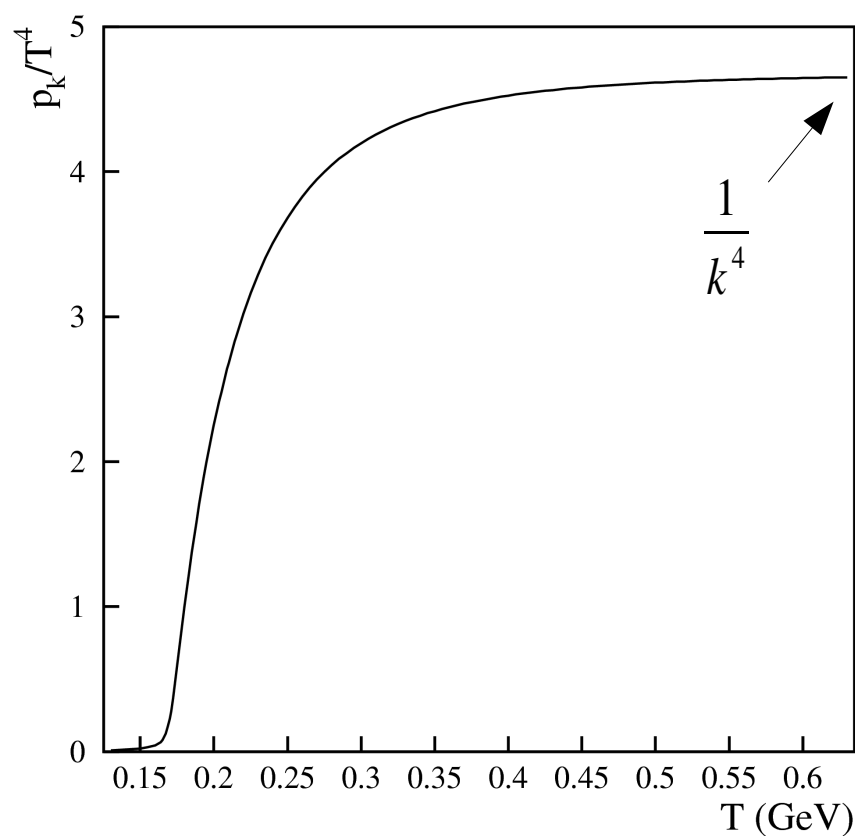
$$\alpha = 3/2$$

$$m_0 = 0.018 \text{ GeV}^{\alpha-1}$$

Fixed (for $\alpha=3/2$) by the actual mass spectrum

$$s_0(T) = f(T, s_0(T)) = \frac{p_k(\infty, T)}{T}$$

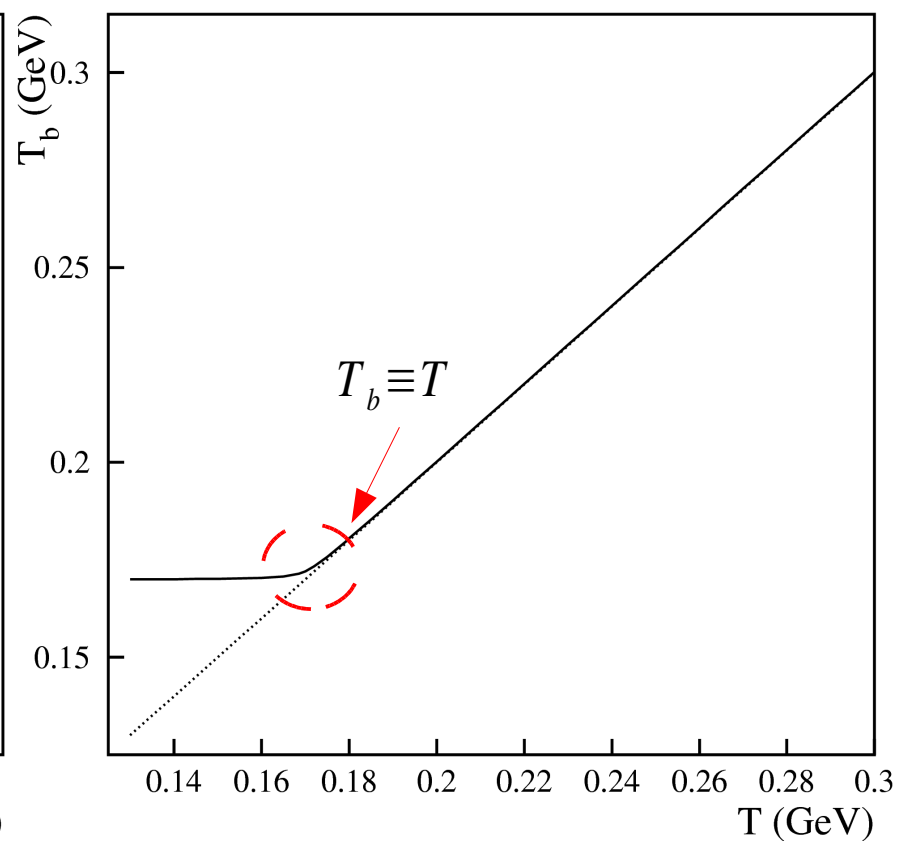
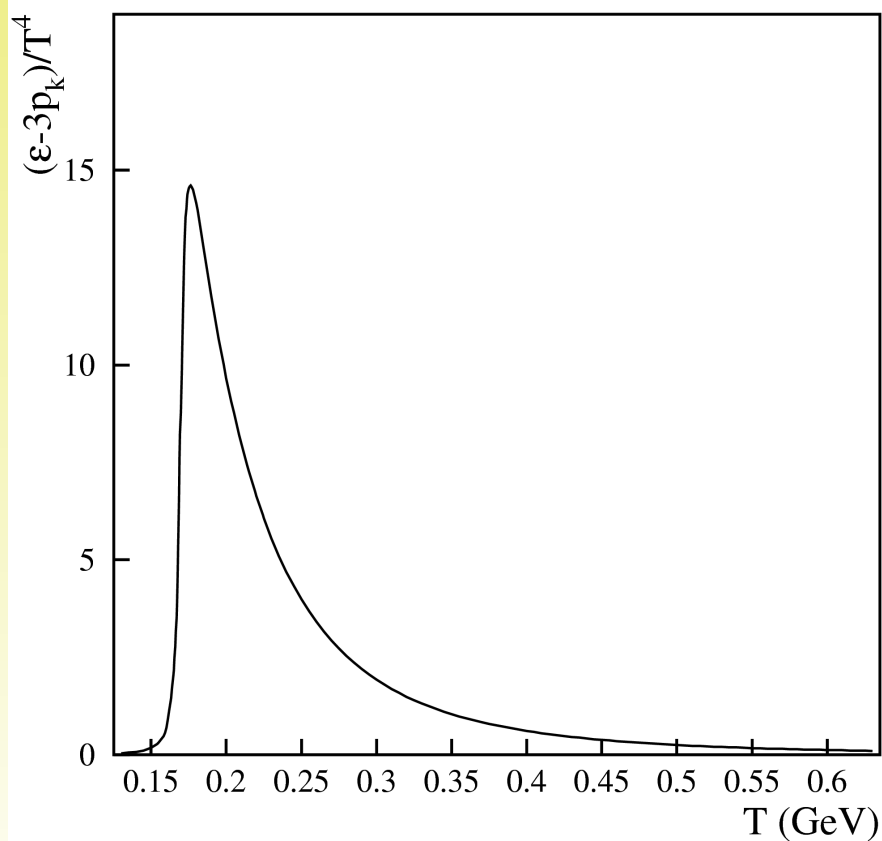
$$\epsilon = T \frac{\partial p_k(\infty, T)}{\partial T} - p_k(\infty, T)$$



$$\frac{(\epsilon - 3p_k)}{T^4}$$

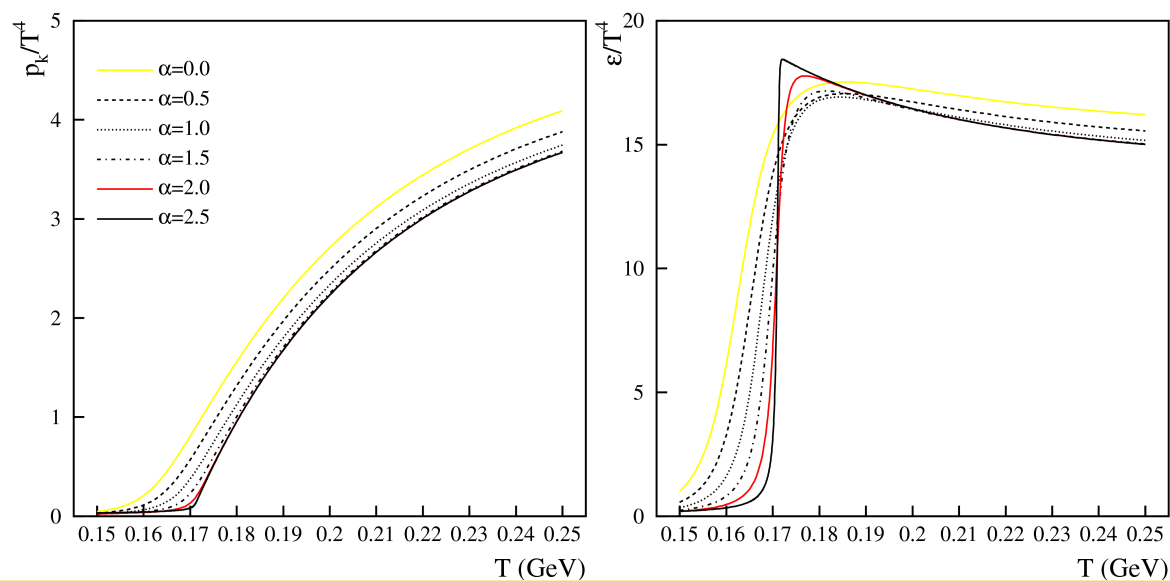
Effective hadrons
inner temperature

$$T_b \equiv k(p_k + B)^{1/4}$$



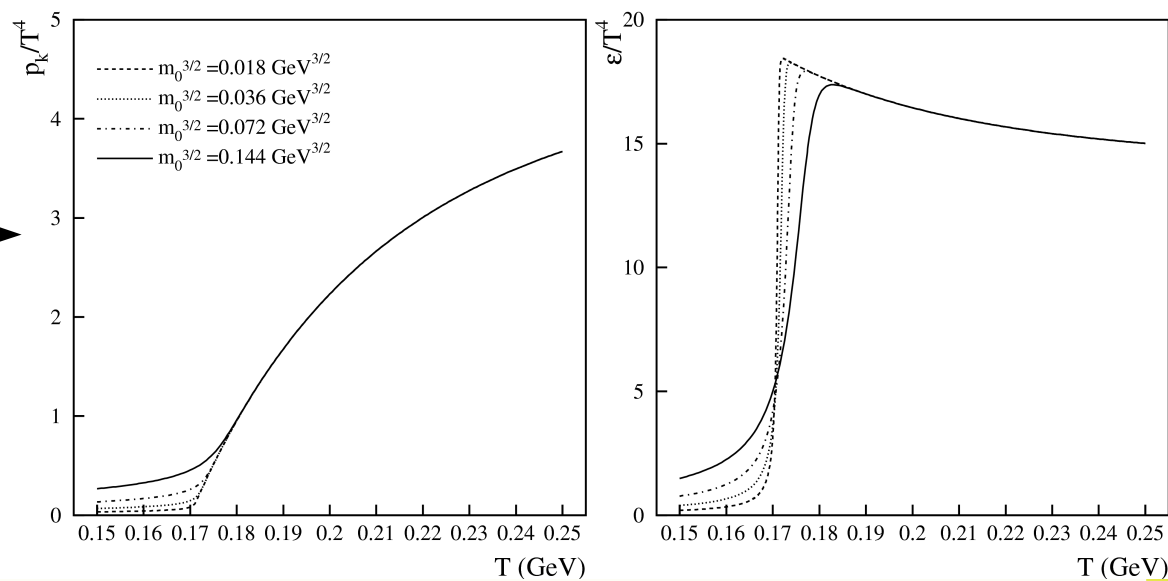
Numerical analysis of the isobaric partition function

Lorenzo Ferroni LBNL



Same as before with various choices of α

Same as before with various choices of m_0 ($\alpha=5/2$)



The grand canonical partition function

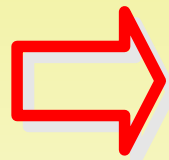
Lorenzo Ferroni LBNL

$$Z(T, V) = \sum_{N=0}^{\infty} Z_N(T, V)$$

Now the pressure of N hadrons

$$Z_N(T, V) = \left(\frac{T}{2\pi} \right)^{3N/2} \frac{m_0^N}{N!} \left[\prod_{i=1}^N \int_{m_{cut}}^{\infty} dm_i \left(\frac{3}{4} m_i + B \frac{m_i}{4 p_{r,N}} \right)^{3/2 - \alpha} \right] \\ \times \exp \left\{ \left[\frac{1}{k p_{r,N}^{1/4}} - \frac{1}{T} \left(\frac{3}{4} + \frac{B}{4 p_{r,N}} \right) \right] \sum_{i=1}^N m_i \right\} \left(V - \sum_{i=1}^N \frac{m_i}{4 p_{r,N}} \right)^N \theta \left(V - \sum_{i=1}^N \frac{m_i}{4 p_{r,N}} \right)$$

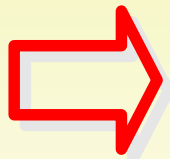
We maximize the integrand with respect $p_{r,N}$



$\langle T_b \rangle$

$$\frac{T}{k \langle p_{r,N} \rangle^{1/4}} \langle p_{r,N} \rangle = B + \frac{NT}{V - \sum_{i=1}^N \frac{m_i}{4 \langle p_{r,N} \rangle}}$$

$$V - \sum_{i=1}^N V_i$$



For $T/\langle T_b \rangle = 1$ This automatically gives the **stability condition**:

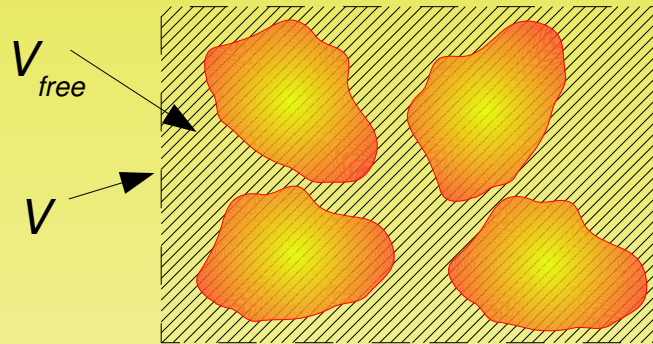
$$\langle p_{r,N} \rangle \equiv B + \langle p_{k,N} \rangle$$



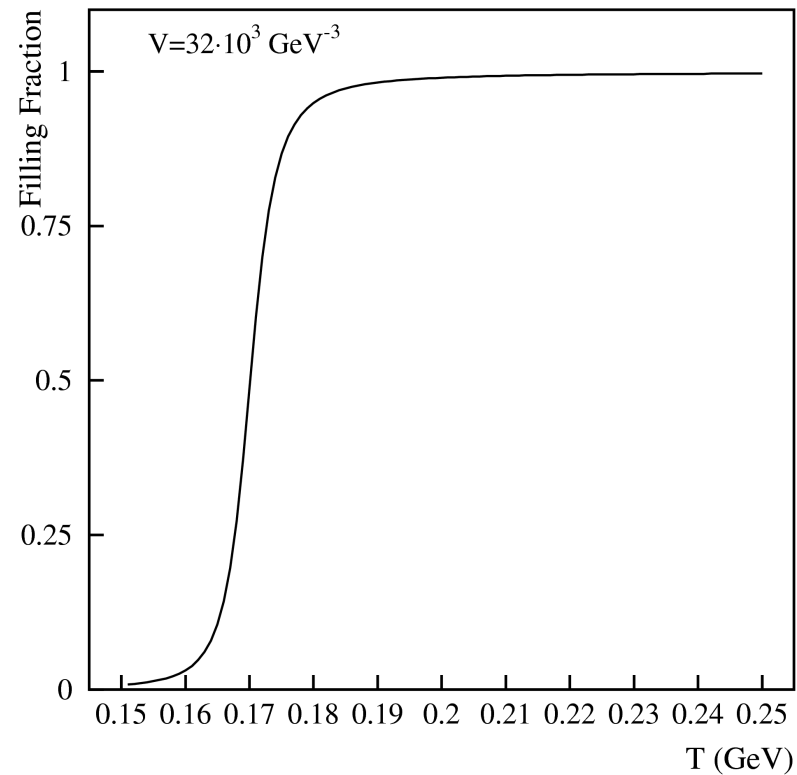
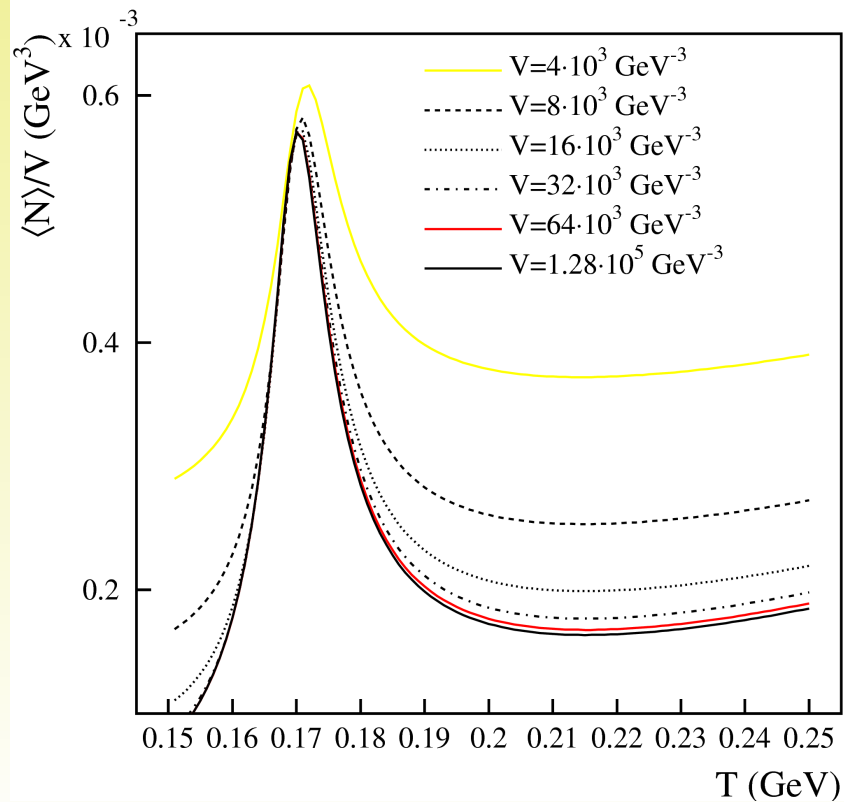
$$\langle p_{k,N} \rangle \equiv \frac{NT}{V - \sum_{i=1}^N \frac{m_i}{4 \langle p_{r,N} \rangle}} = \frac{NT}{V - \sum_{i=1}^N V_i}$$

The **self-consistency relation**.

Numerical analysis of the grand canonical partition function

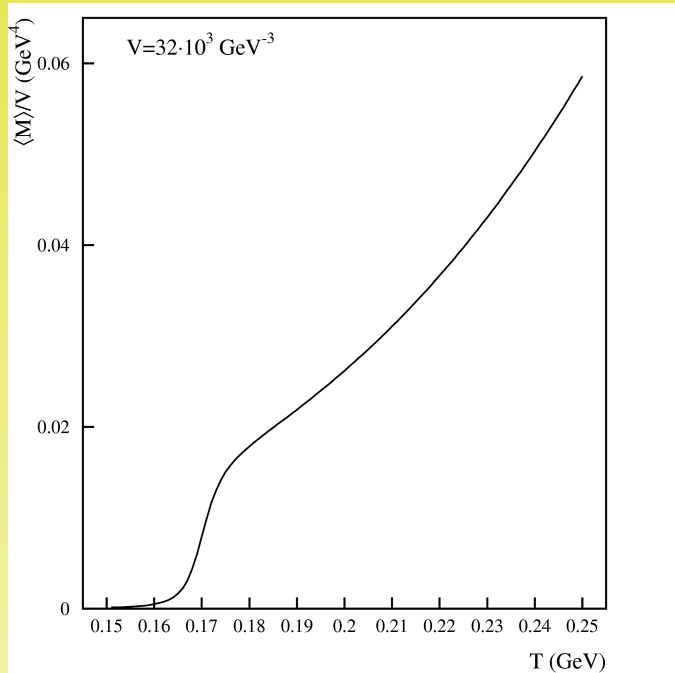


$$\text{Filling Fraction} = 1 - V_{\text{free}}/V$$



Conclusions & Outlook

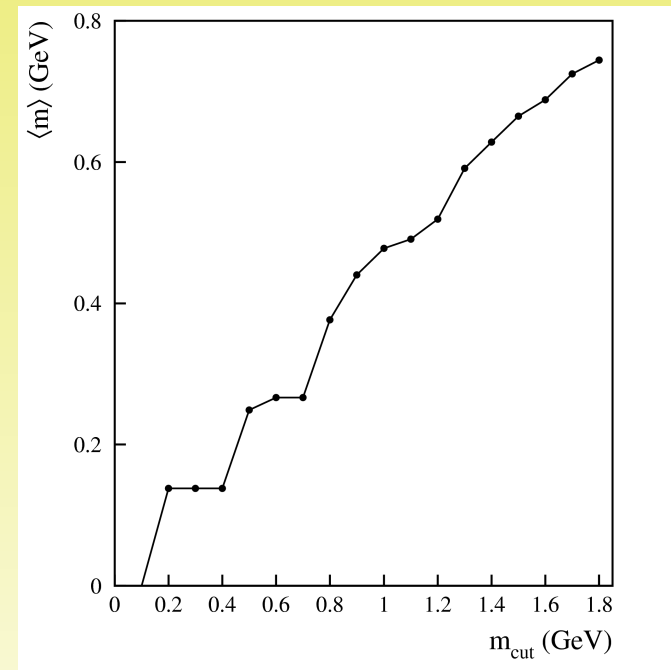
- We have adopted the simple idea of hadrons as compressible bags to describe the crossover transition at zero chemical potential.
- We have enforced the stability condition $p_r = B + p_k$
- We showed that for the most important configurations this stability condition can be obtained by minimizing the Free Energy.
- The results of the model are in a fair qualitative agreement with the Lattice QCD data.
- There are still issues to understand and to discuss.
- In the future, a more realistic approach would be desirable.
- Generalization to finite Baryochemical potential.



$$T = 160 \text{ MeV} \rightarrow \langle m \rangle = 2.8 \text{ GeV}$$

$$T = 170 \text{ MeV} \rightarrow \langle m \rangle = 14 \text{ GeV}$$

Actual hadron gas at 170 MeV:
average mass as a function of the
upper cut-off



Hagedorn spectrum at 160 MeV:
average mass as a function of the
upper cut-off

