
Spectral functions near the QCD critical point in chiral models

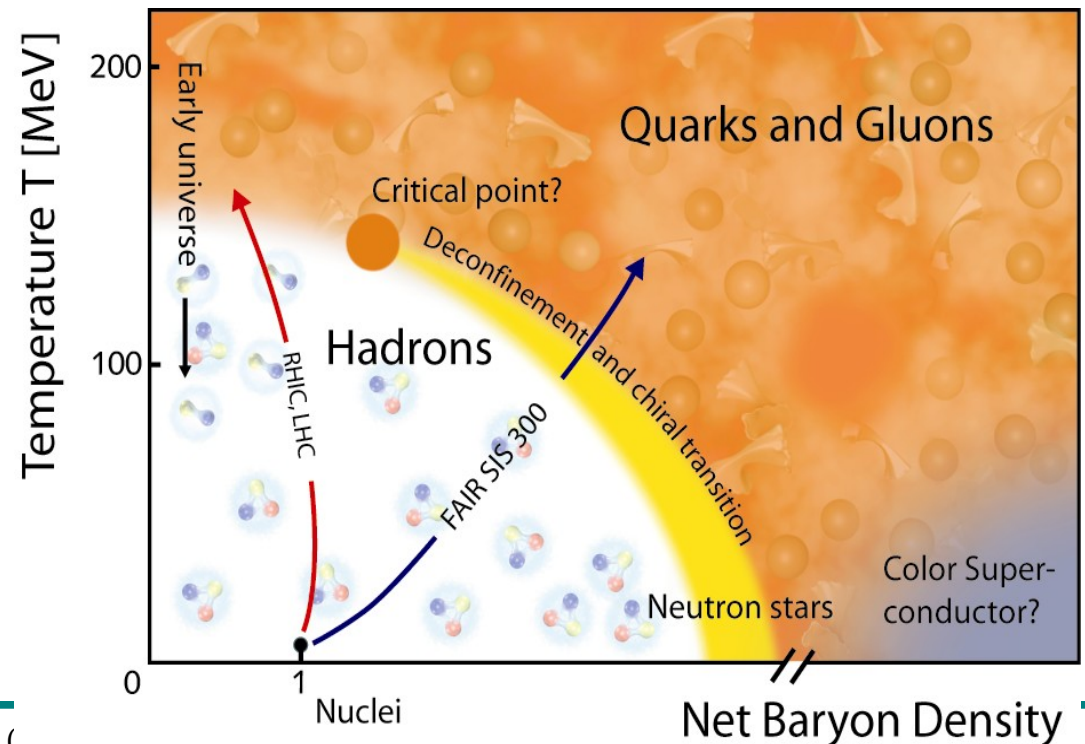
H. Fujii

HF, PRD67, 094018

HF-M.Ohtani, PRD70, 014016

Introduction

- LQCD + Models suggested the QCD-CP in T- μ
 - cross-over along T axis
 - 1st order along μ axis



Introduction

- **LQCD + Models suggested the QCD-CP in T - μ**
 - cross-over along T axis
 - 1st order along μ axis
- **Difficult to conclude in LQCD+Models today**
 - about the location
 - about the existence
- **Needs for exp'tal signatures / confirmation**
 - What is the characteristics of the QCD-CP?
 - fluctuations, focusing, particle ratios, ..., etc.

How to use chiral models

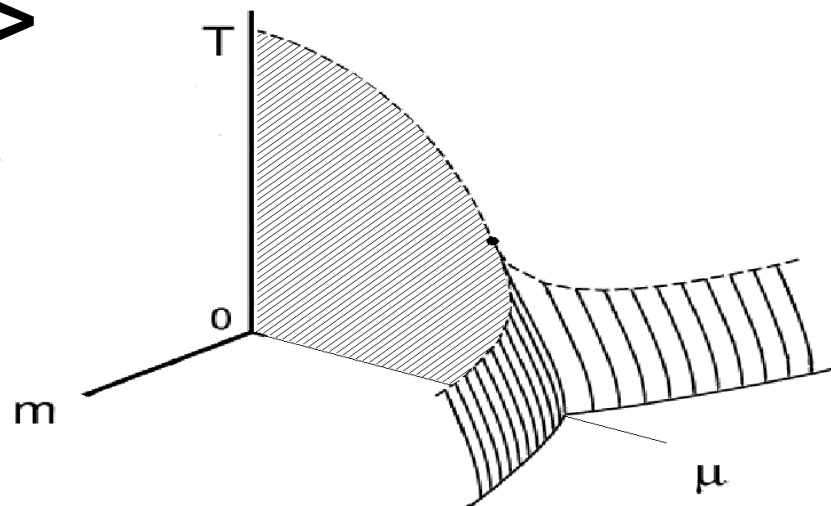
- Difficult to locate the QCD-CP in $T-\mu$
- But, generic features of the CP can be demonstrated in the (simplest) models
- What are the spectral properties of QCD-CP?

model

- **NJL** (and other chiral models)

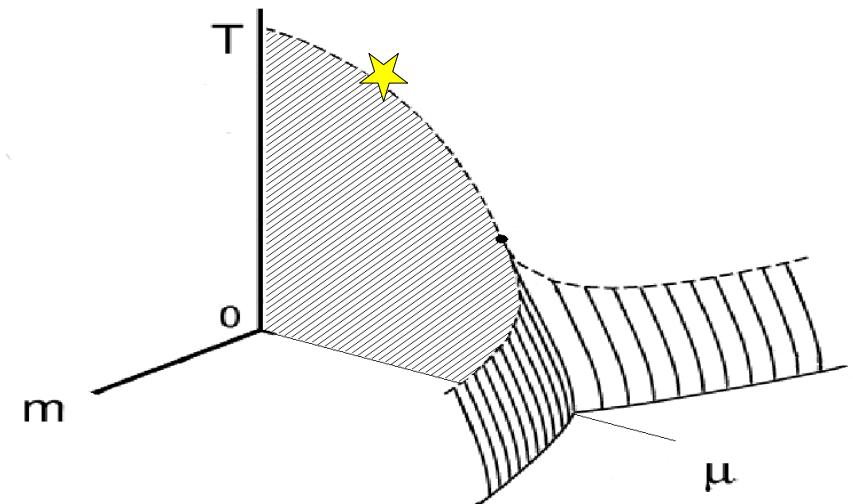
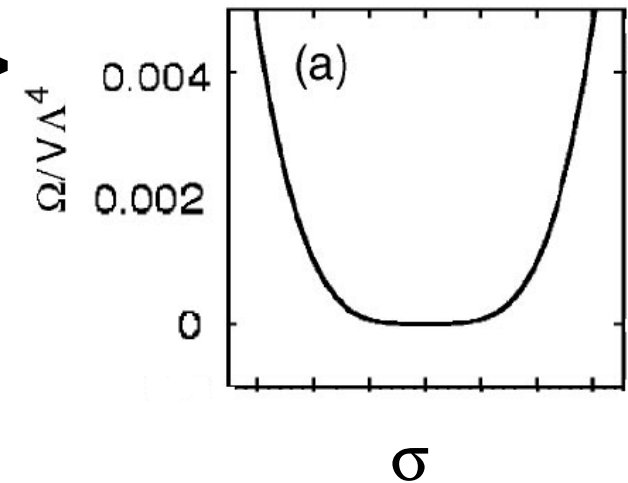
$$\mathcal{L} = \bar{q}(i\not{\partial} - m)q + g[(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau^a q)^2]$$

- simplest quark dynamics
- chiral symmetry breaking by $q^{\text{bar}}\text{-}q$ attraction
- no dynamic gluons, no confinement (nucleons)
- σ and π as flucst of $\langle q^{\text{bar}}q \rangle$



Chiral transition

- order parameter $\langle q^{\text{bar}} q \rangle \sim \langle \sigma \rangle$
 - $=0$ in Wigner, $\neq 0$ in NG phase
- flat potential at CP
- chiral susceptibility
 - $\chi_{mm} \sim \int d^3 r \frac{e^{-Mr}}{r} \sim \frac{1}{M^2} \rightarrow \infty$
 - long-range fluctuation
 - M : screening mass
- $\chi_{\mu\mu}$, C finite



CP with nonzero m

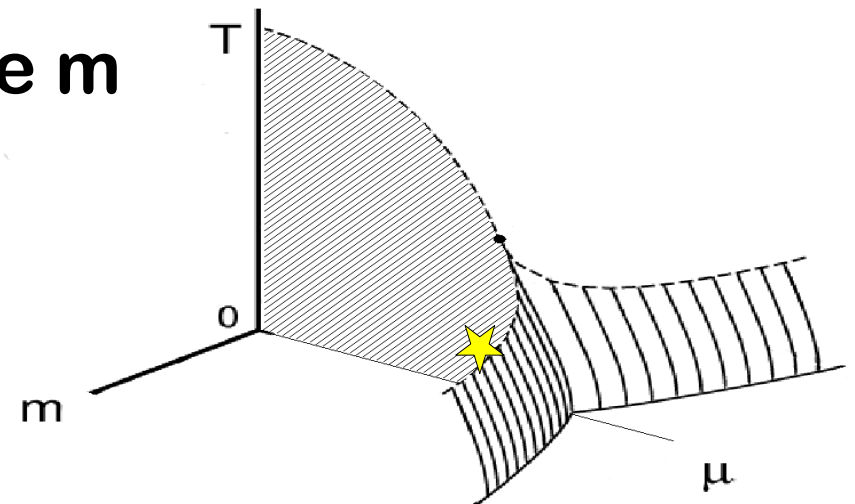
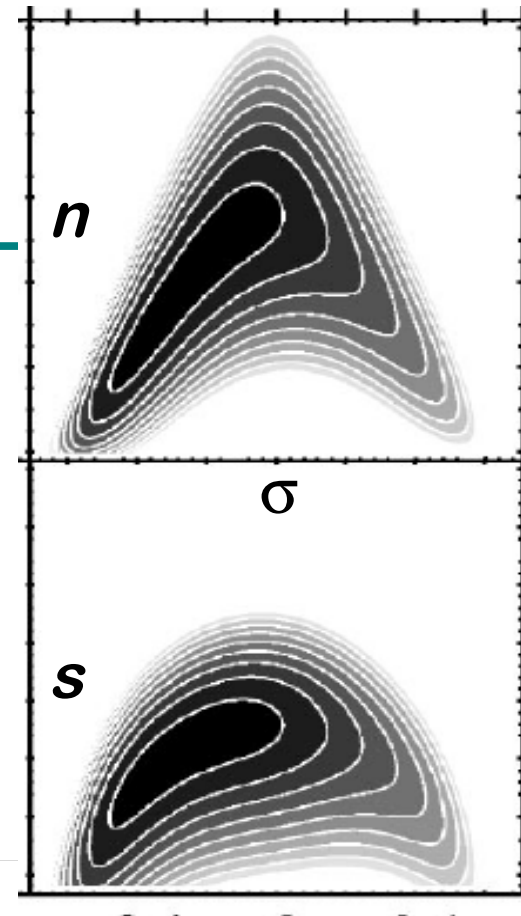
- Across co-existence surface
(Clapeyron-Clausius)

$$\frac{dT}{d\mu} = -\frac{\Delta n}{\Delta s}, \quad \frac{dT}{dm} = -\frac{\Delta \sigma}{\Delta s}$$

- order parameter = deviations from equilibrium σ , n , or s

– linear mixing due to finite m

- χ_{mm} , $\chi_{\mu\mu}$, and C diverge



χ' s & spectral change

- strong $q^{\text{bar}}-q$ attraction generates the CP
 - Which mode softens?

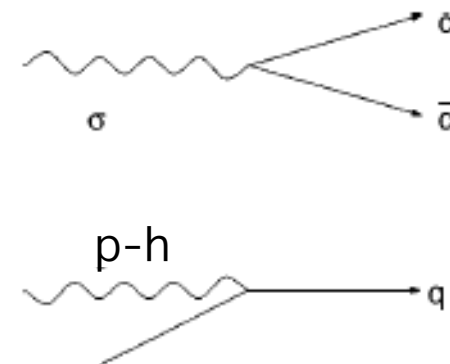
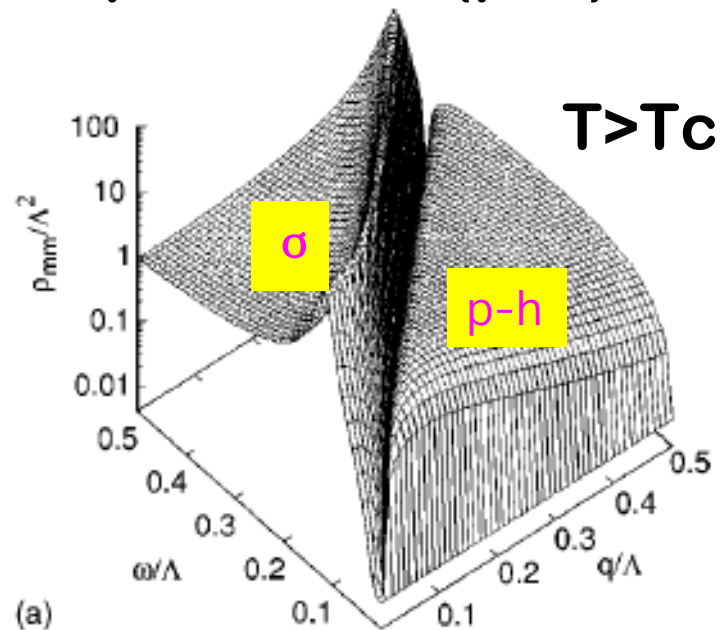
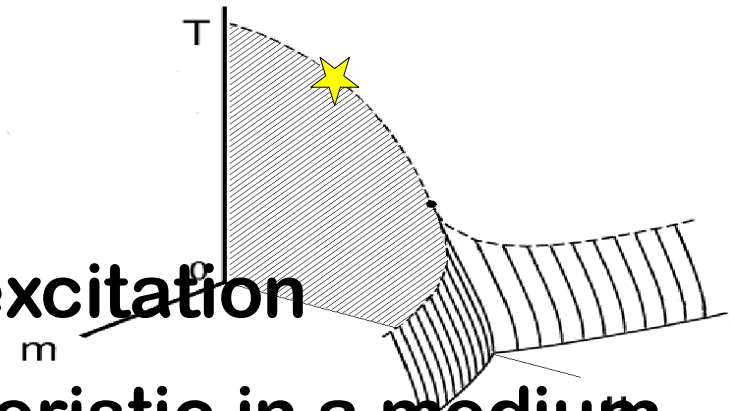
χ_{mm} & dropping mass

- Spectral fn near O(4) CP

- $\chi_{mm}(q) = \int \frac{d\omega}{2\pi} \frac{\rho_{mm}(\omega, q)}{\omega}$

- dynamic σ -mode as time-like excitation

- space-like (p-h) mode characteristic in a medium

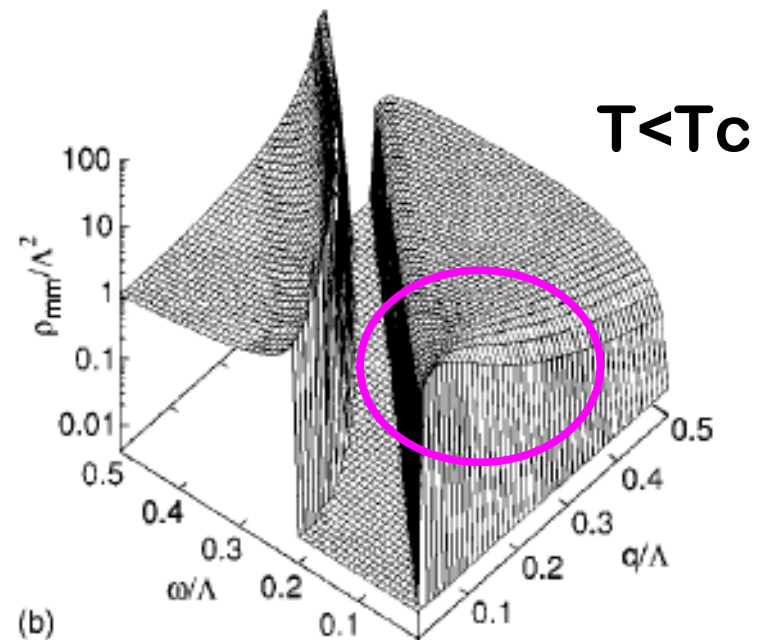
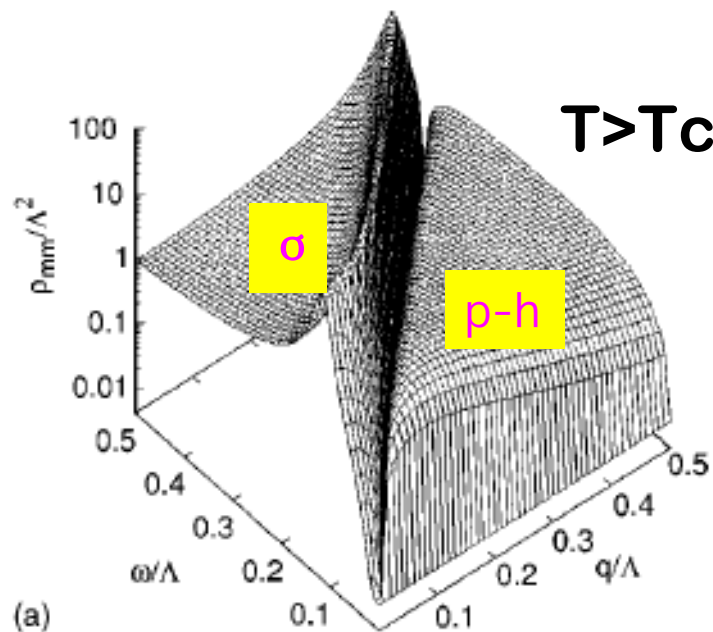
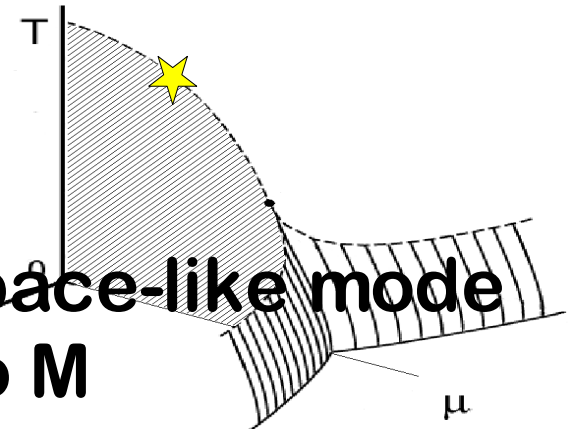


χ_{mm} & dropping mass

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- in approach from broken phase, space-like mode mixes in at $q=0$ because of nonzero M

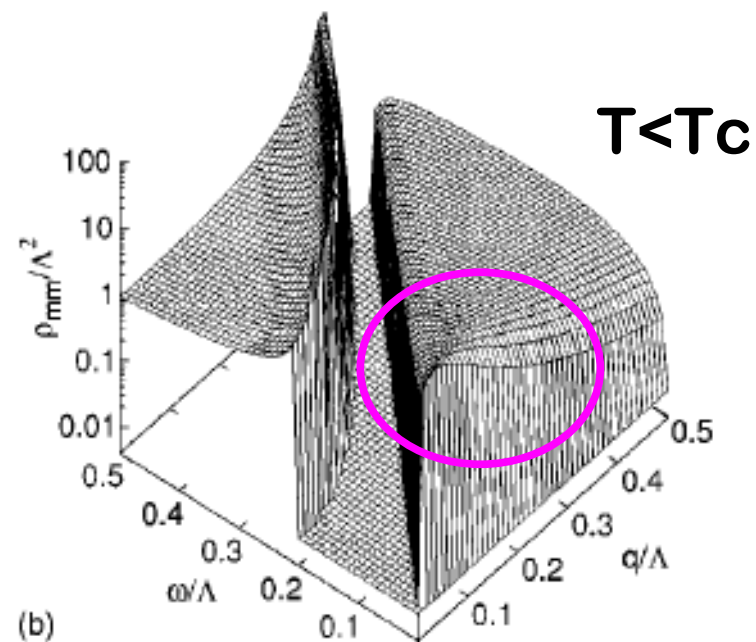
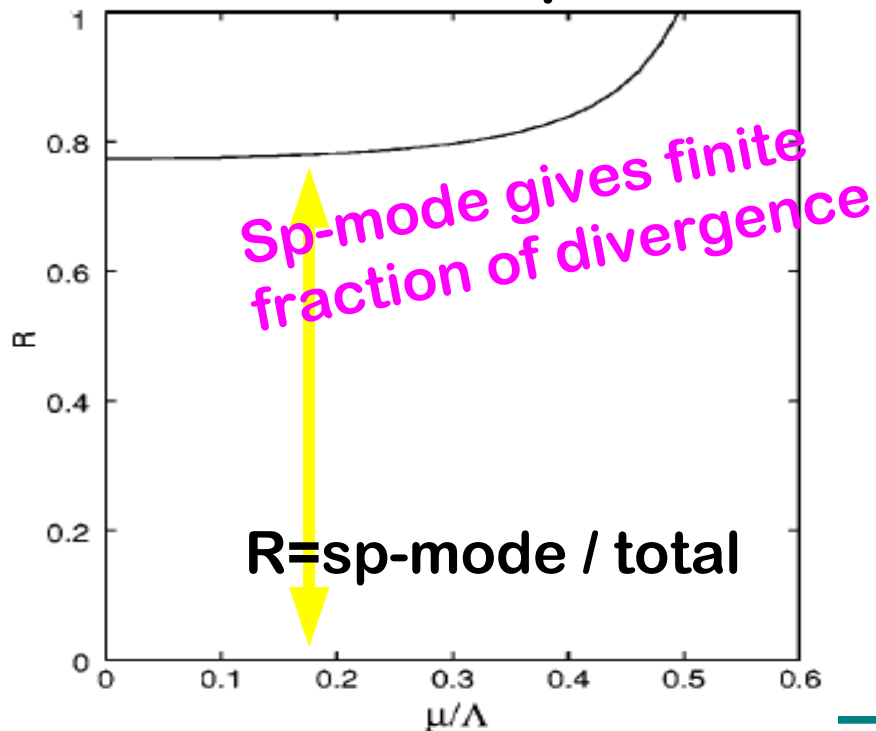
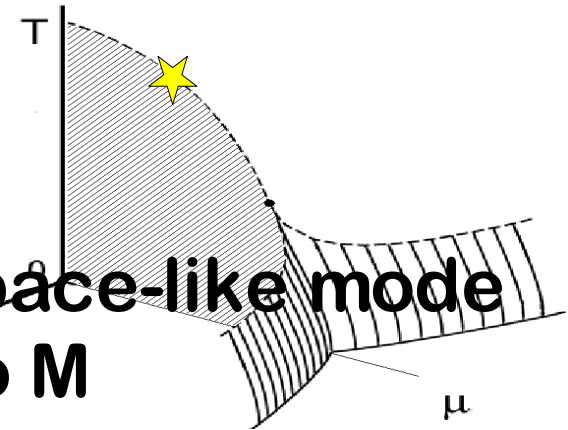


χ_{mm} & dropping mass

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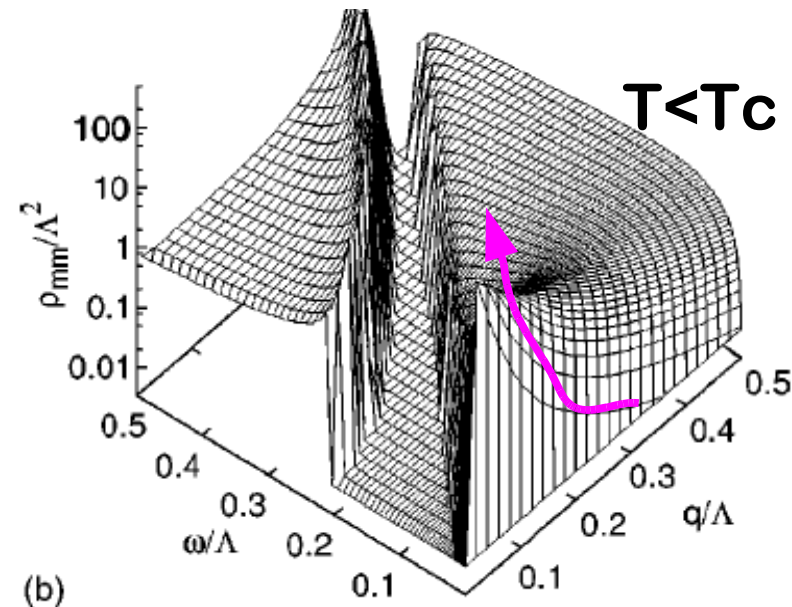
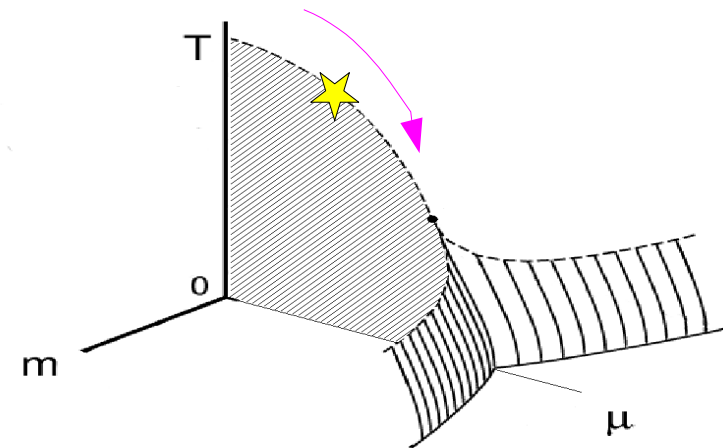
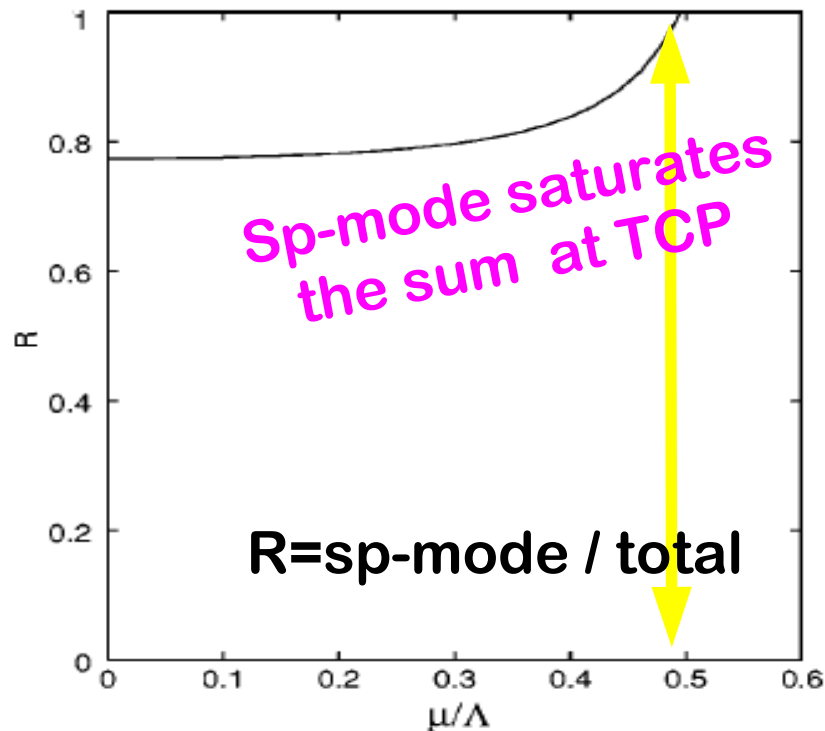
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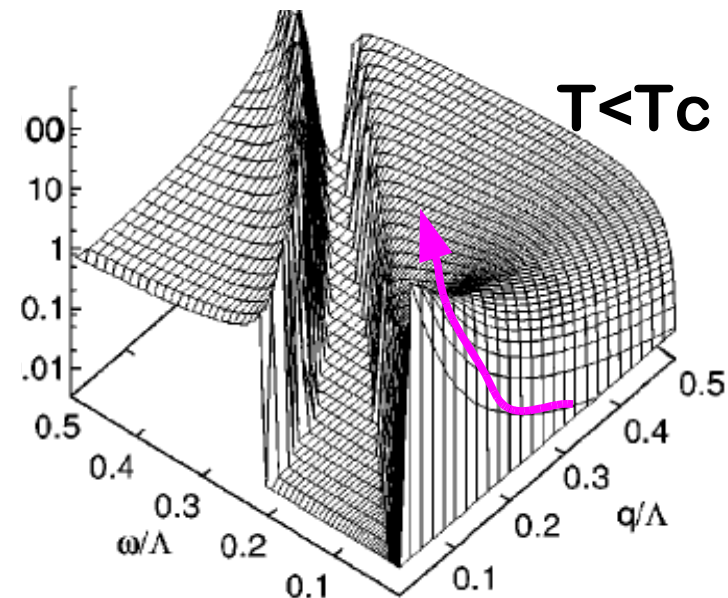
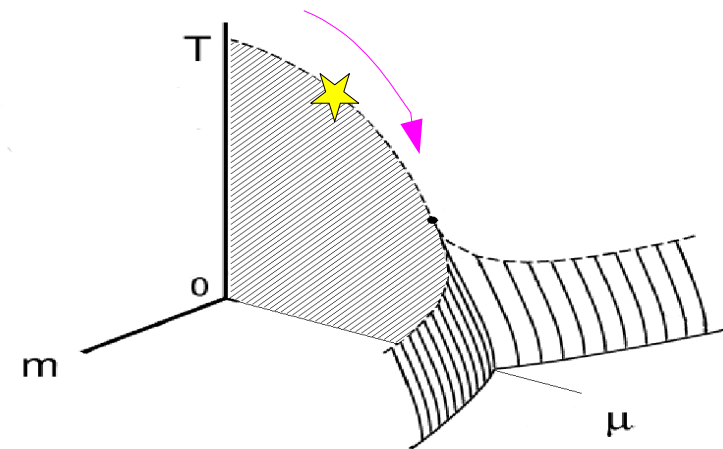
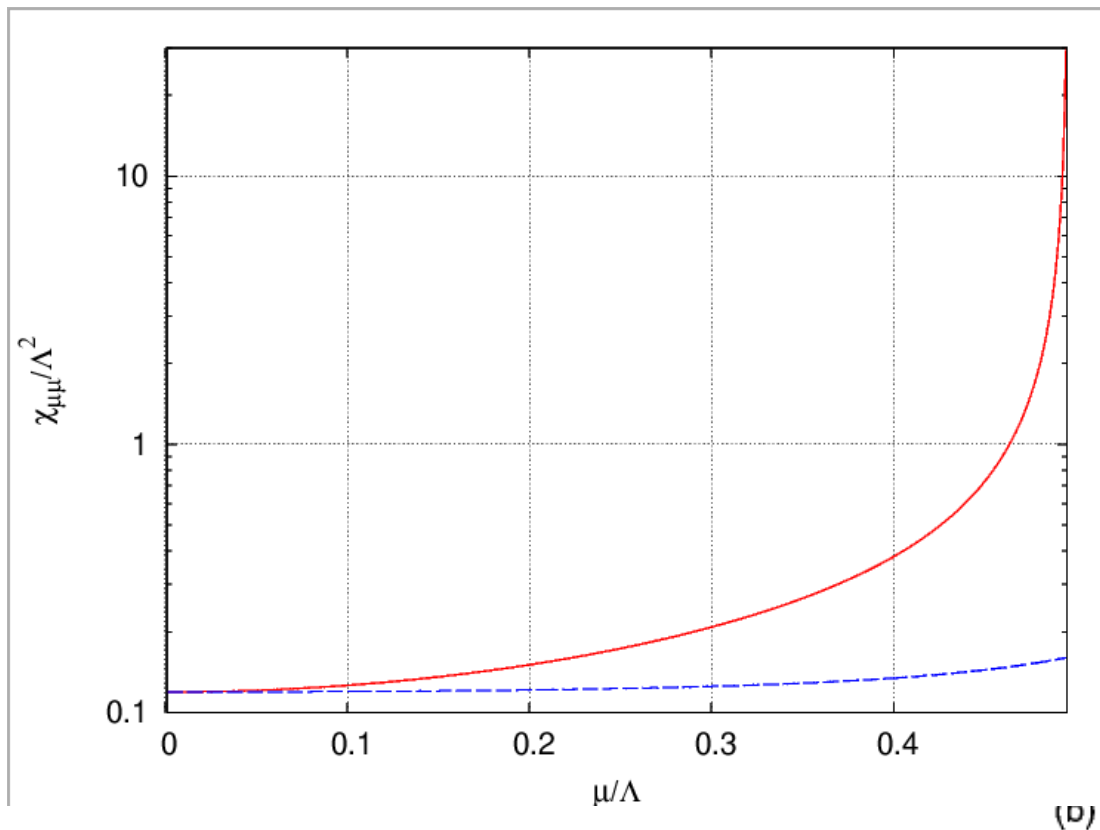
χ_{mm} & dropping mass

- Toward the TCP
 - space-like mode enhances



χ_{mm} & dropping mass

- Toward the TCP
 - quark number $\chi_{\mu\mu}$ diverges



CP with nonzero m

- mass of sigma?
 - Scavenius et al. posed a [Q]:

PRC64, 045202

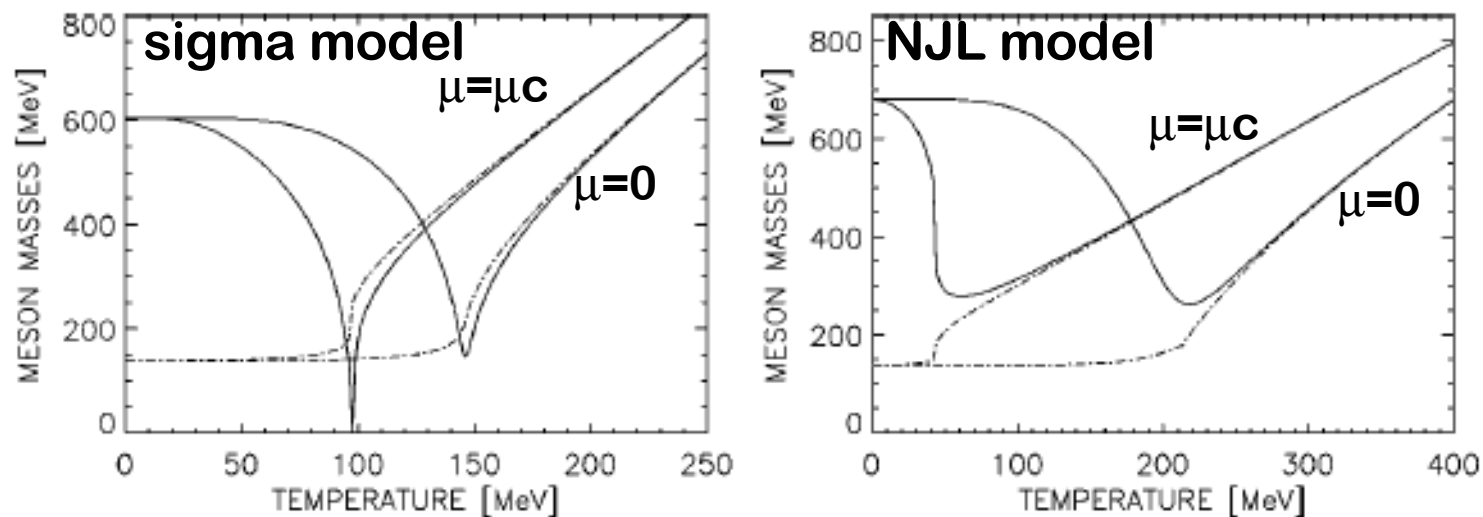
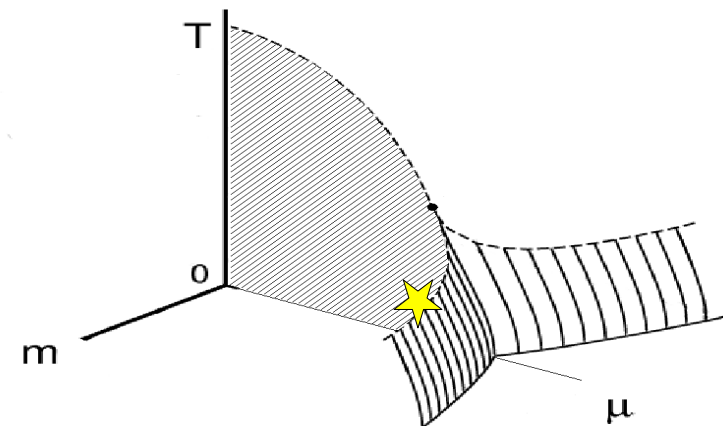


FIG. 6. The sigma mass (solid line) and pion mass (dashed line) in the sigma model (left) and NJL model (right) as functions of temperature for $\mu=0$ (right pair) and for $\mu=\mu_c$ (left pair).

CP with nonzero m

- mass of sigma?
 - Scavenius et al. posed a [Q]:
 - comparing different things
 - screening vs pole masses

PRC64, 045202

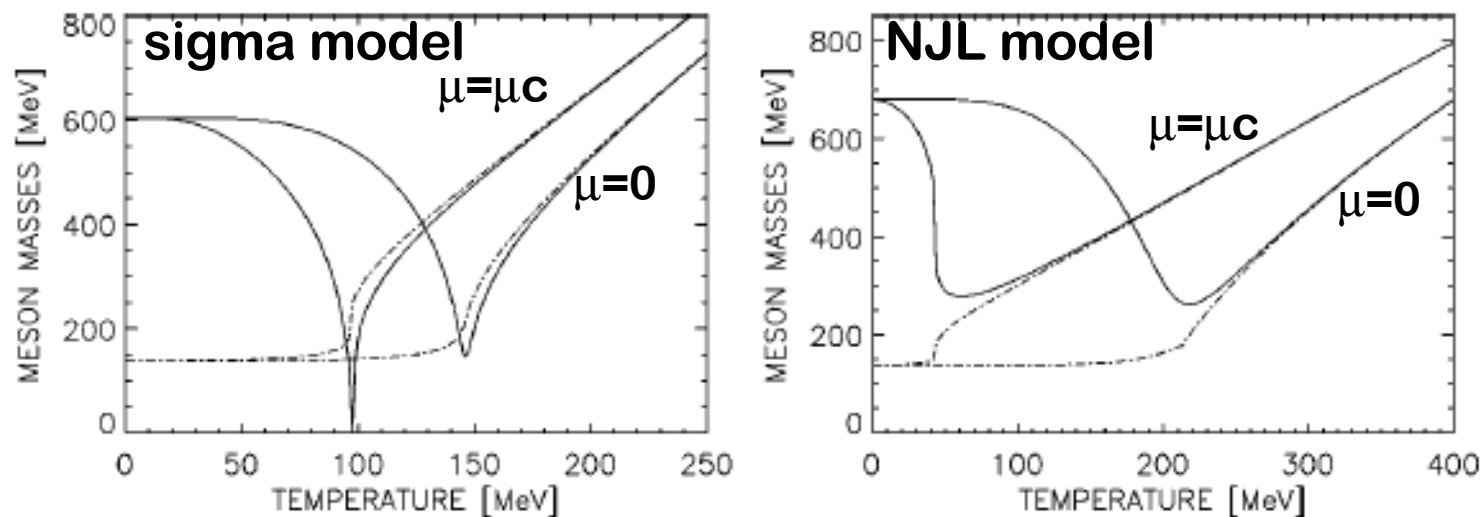
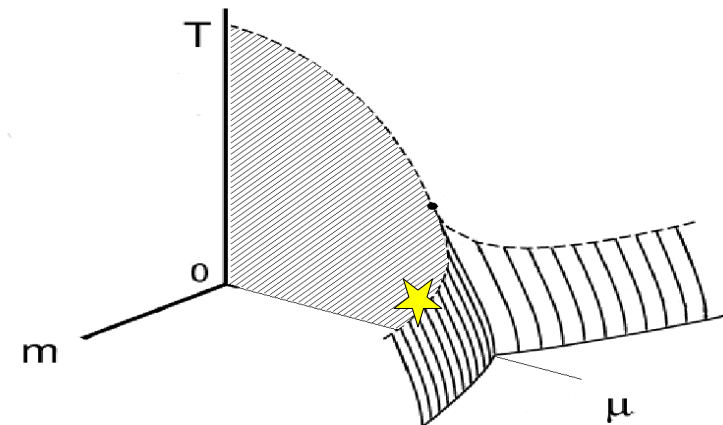
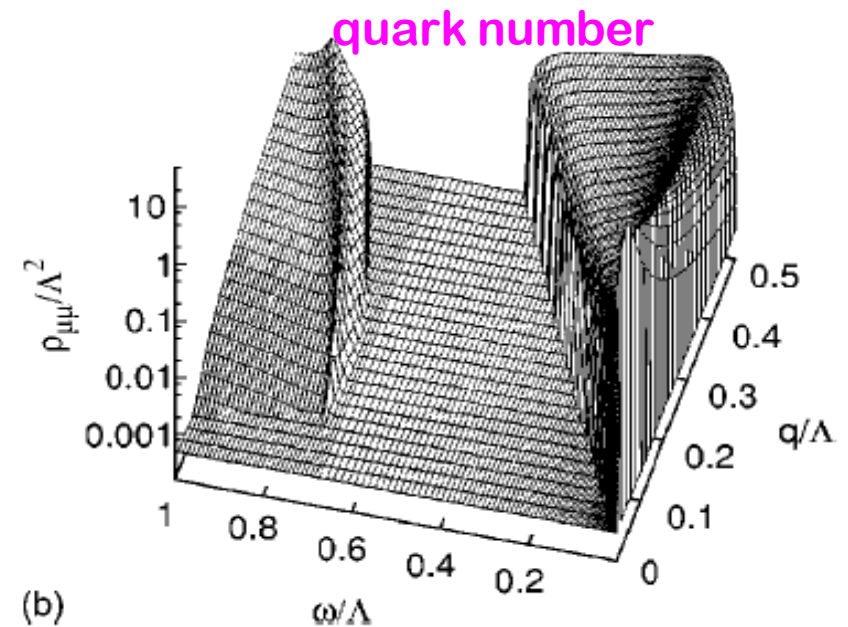
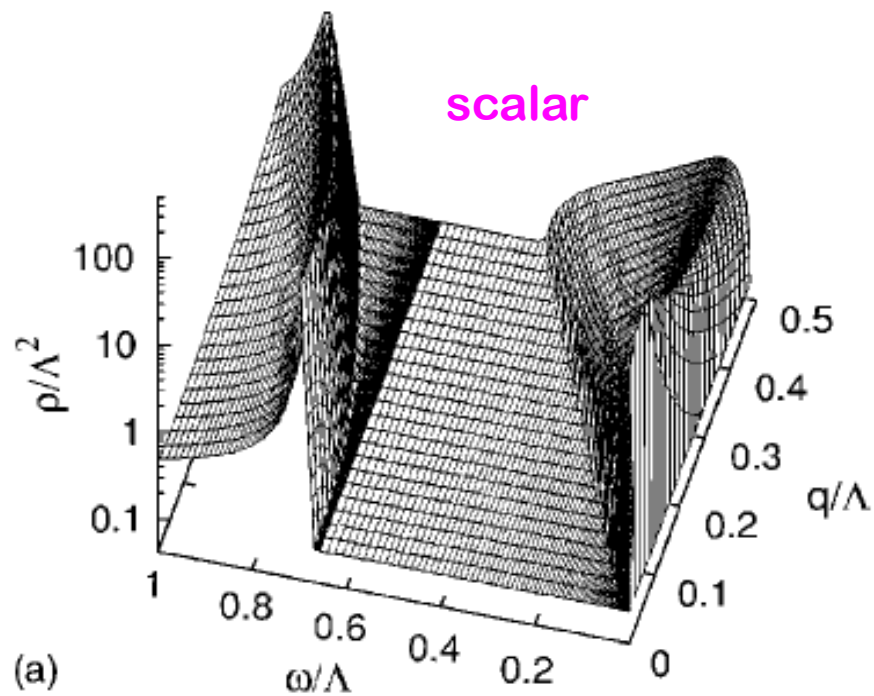
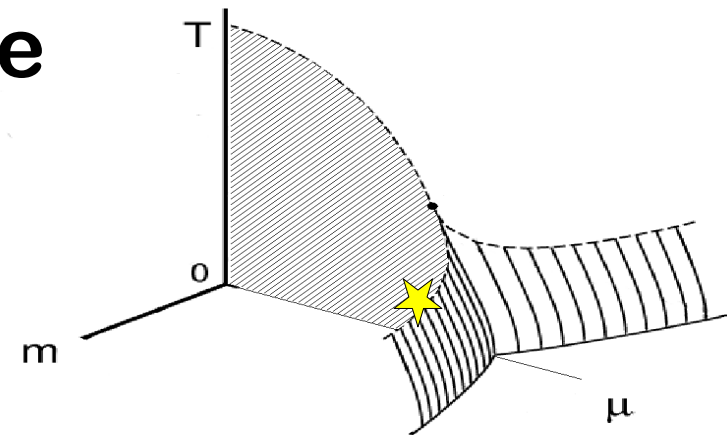


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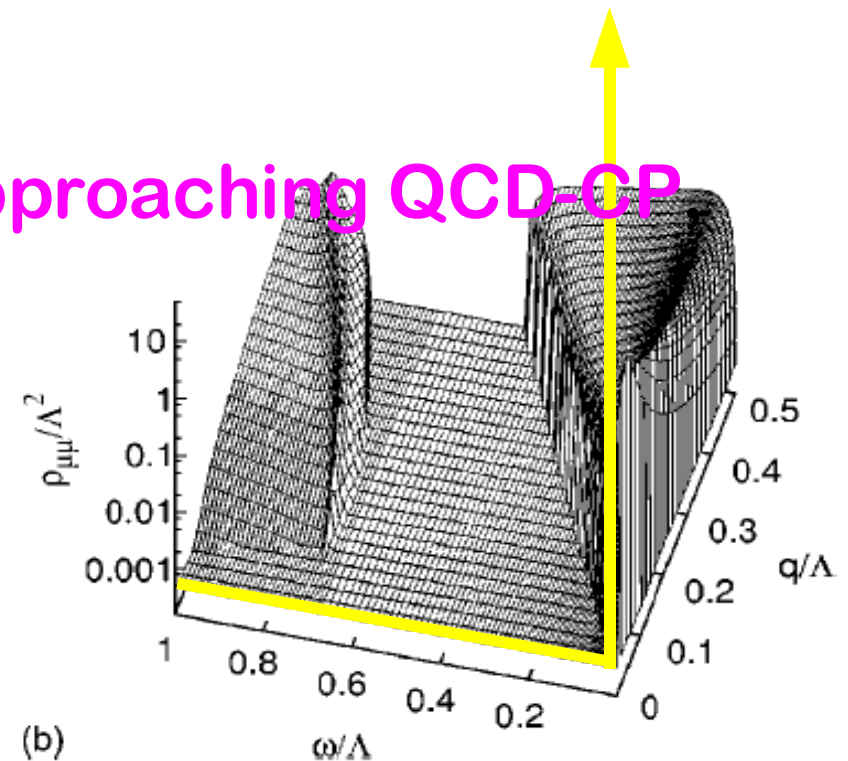
CP with nonzero m

- Divergent χ 's due to p-h mode
 - no dropping mass!



Why is p - h mode enhanced?

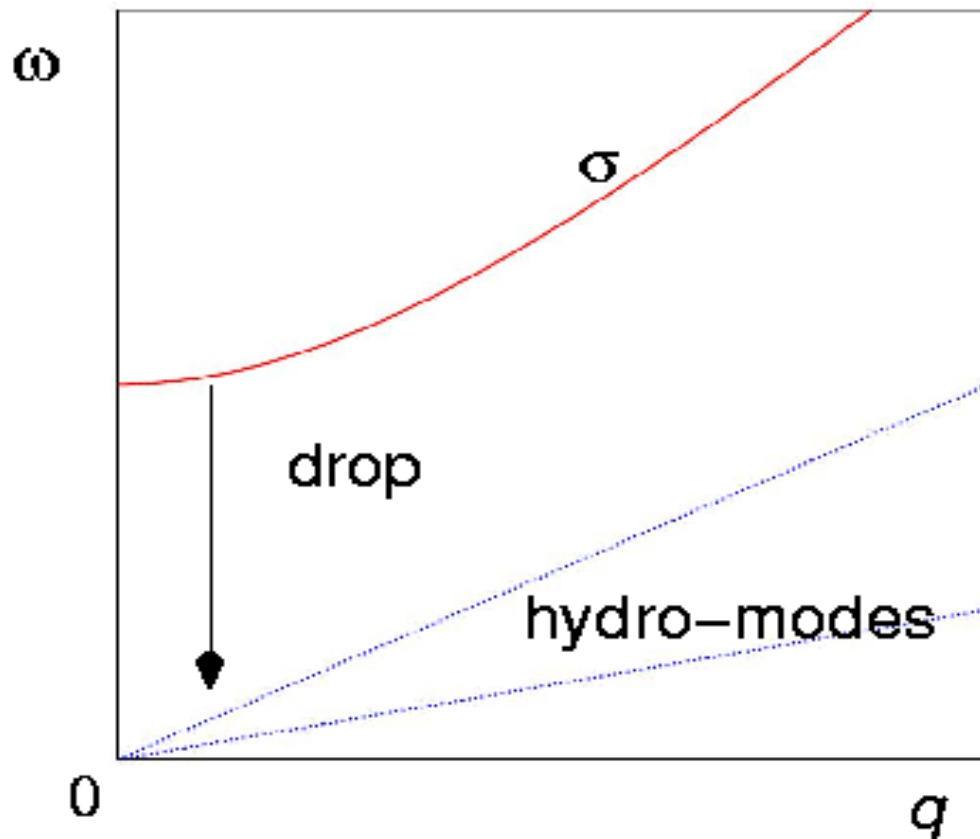
- $\chi_{\mu\mu}$ = fluctuation of conserved density
 - cannot fluctuate at $q=0$ (total charge)
 - $\omega \sim D q^2$
 - $\lim_{q \rightarrow 0} \rho_{\mu\mu}(\omega, q) \propto \omega \delta(\omega)$
 - strength diverges as approaching QCD-CP



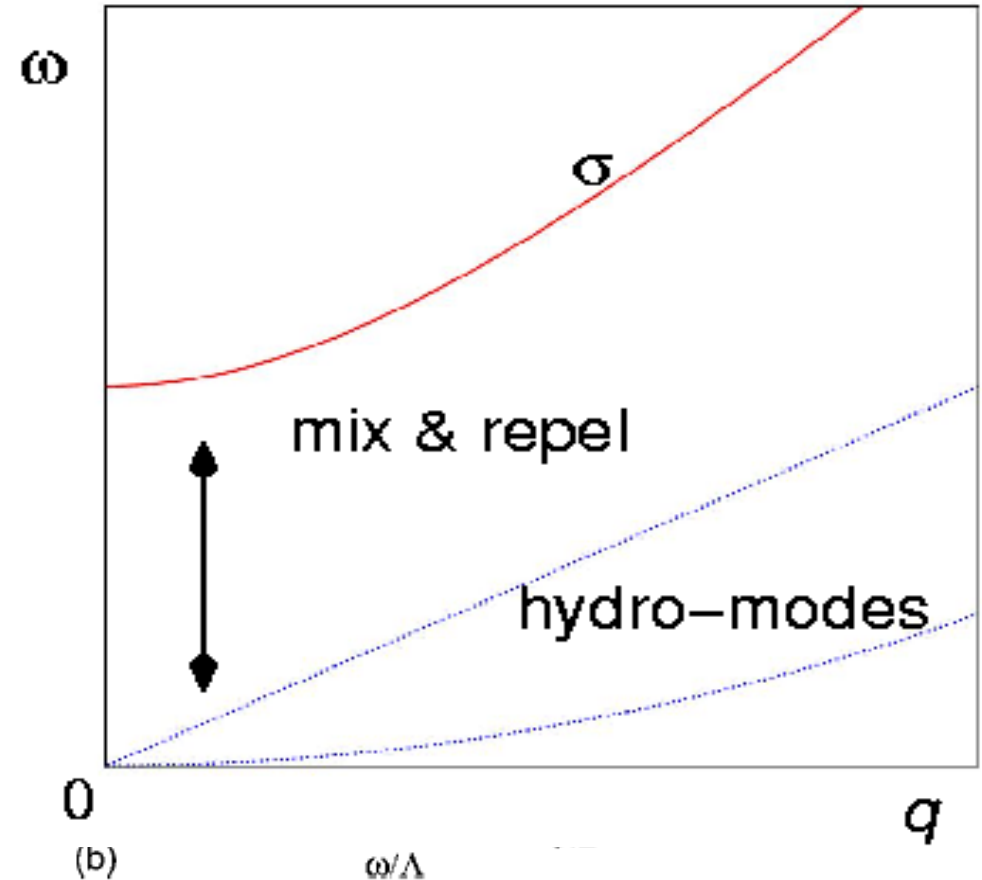
Why is p -h mode enhanced?

- Schematic sketch

w/ exact chiral symmetry

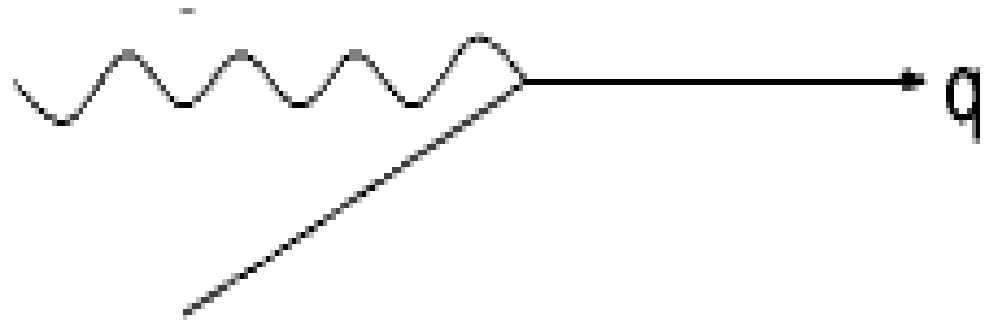


w/ non-zero quark mass



Implications to HIC

- **Critical softening in p-h mode**
 - decoupling of σ meson (ω meson as well)
 - no direct access to dileptons, $\pi\pi$
 - particle scattering may produce γ^*
 - low pT dist. of scattered particles
 - standard ...



Dynamic Universality

- **Static universality**
 - 3D Z(2) Ising
- **Dynamics constrained by conservation laws**
 - possible slow modes near QCD-CP
 - σ (π massive due to $m_q \neq 0$)
 - $T^{0\mu}$, $n \dots$ 5 densities
 - decoupling of σ due to finite m
 - slow modes = sound (2), shear (2), heat (1) = liq.gas

Hints to phenomenology

- within critical region of gas-liquid CP
 - mode-coupling between hydro-modes important
 - shear viscosity diverges, but only very weakly
 - $\eta \sim \xi^{(1/19)\epsilon}$,
 - thermal conductivity diverges
 - $\lambda \sim \xi^{(18/19)\epsilon}$, $\omega \sim \xi^{-3}$
- But, finite size and time in HIC set the limit to these critical effects?

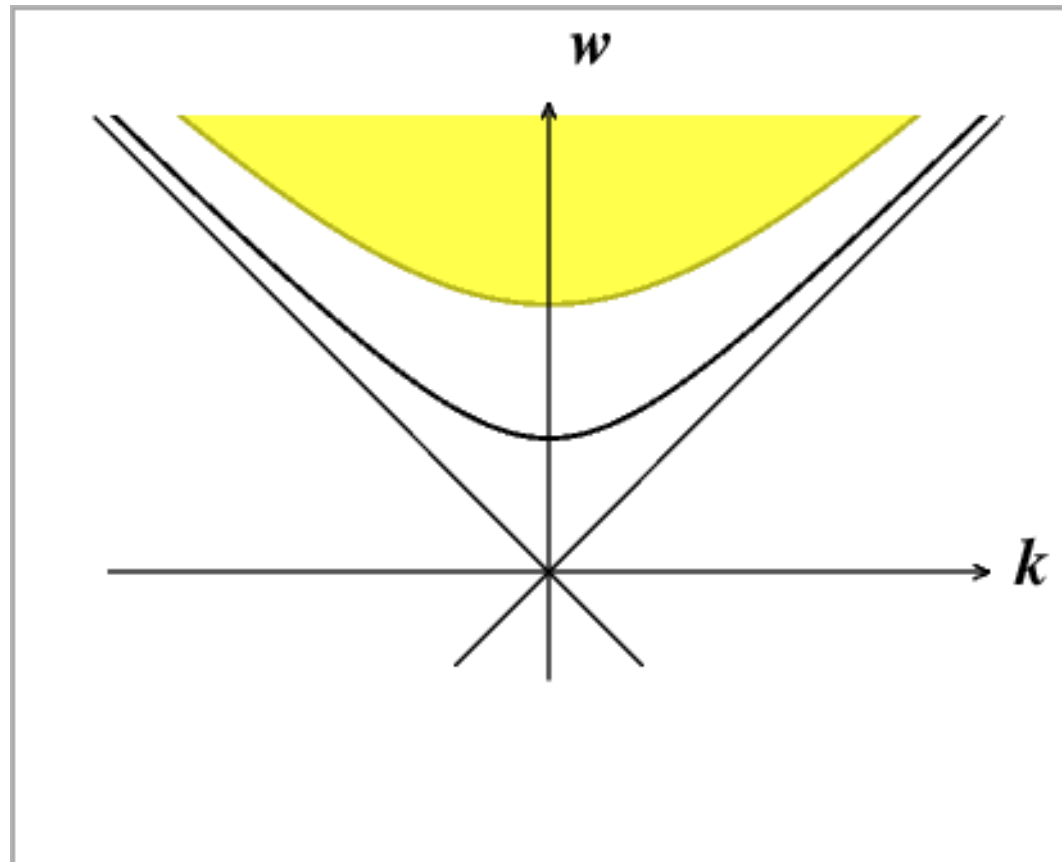
Berdnikov-Rajagopal

Summary

- **Model demonstrates:**
 - sigma softens near chiral CP, and p-h mode contributes too if approached from broken phase
 - near QCD-CP, p-h mode saturates the divergence
- **Critical mode has p-h type spectrum**
- **Dynamic Universality is the same as liq.-gas**
 - η & λ diverge
 - finite size/time may limit the critical behavior

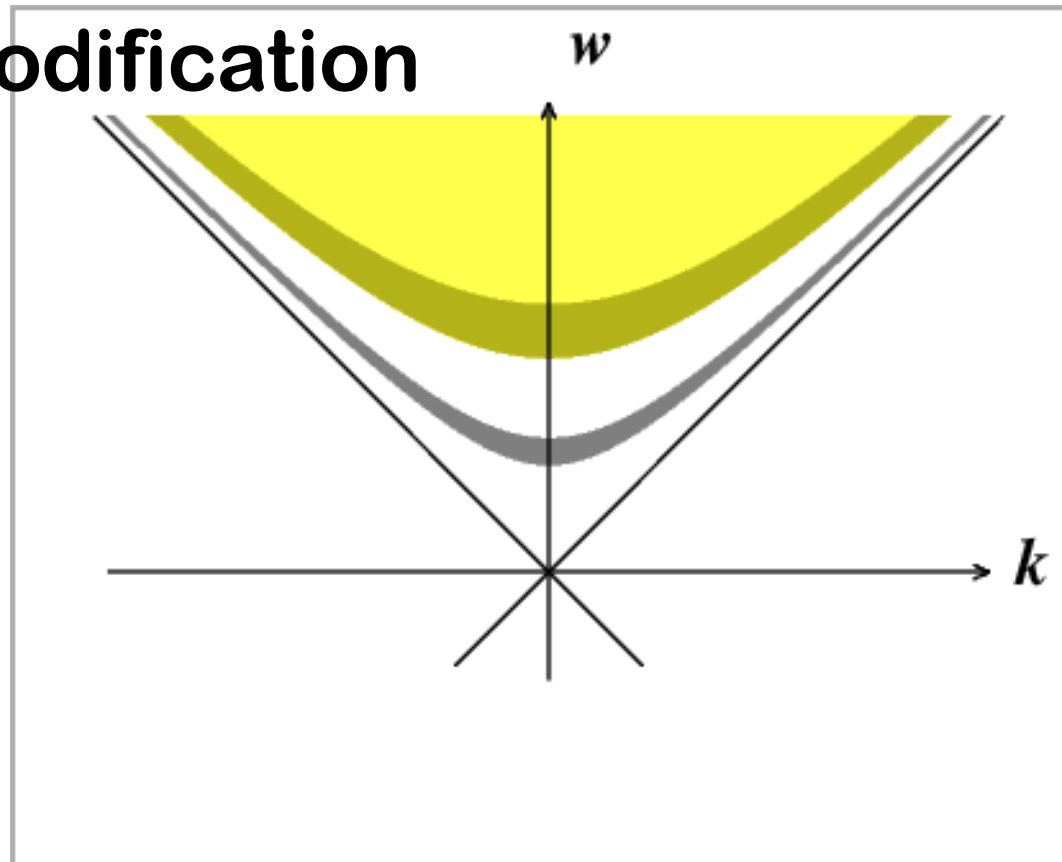
Region of spectrum

- vacuum



Region of spectrum

- mass modification



Region of spectrum

- medium fluctuation

