What can we learn from model studies on the chiral critical end-point?

– Old and New Perspectives –

Kenji Fukushima

Yukawa Institute for Theoretical Physics

Kyoto University

July 2008 at INT

Discovery in Old Days

- Color Confinement – Deconfinement
  - Center Symmetry
  - 2-Color – Second-Order 3-Color – First-Order
  - High Baryon Density?

- Chiral Effective Model
  - Chiral Symmetry
  - 2-Flavor – Second-Order 3-Flavor – First-Order
  - Mesons, Baryons **sigma-model**
  - Quarks **NJL-model**

Themodynamic Potential

- Thermodynamic Potential in the NJL Model
  - Condensation Energy

$$\Omega_{\rm cond} = g_{\rm S}\left(\langle\bar{u}u\rangle^2+\langle\bar{d}d\rangle^2+\langle\bar{s}s\rangle^2\right)+4g_{\rm D}\langle\bar{u}u\rangle\langle\bar{d}d\rangle\langle\bar{s}s\rangle$$

  - Zero-Point Energy (responsible for chiral sym breaking)

$$\Omega_{\rm zero} = -2N_c\sum_i\int^{\Lambda}\frac{{\rm d}^3p}{(2\pi)^3}\,\varepsilon_i(p)$$

  - Thermal Energy

$$\Omega_{\rm quark} = -\,2T\sum_i\int\frac{{\rm d}^3p}{(2\pi)^3}\left\{\ln\det\left[1+L\,{\rm e}^{-(\varepsilon_i(p)-\mu)/T}\right]\right.$$
$$\left.+\ln\det\left[1+L^\dagger{\rm e}^{-(\varepsilon_i(p)+\mu)/T}\right]\right\}.$$

Why First-Order ?

- Some Numerics

$$M_{ud} = 336\,\mathrm{MeV}\,,\ \mu_c = 345\,\mathrm{MeV}\,,\ T = 0$$

- Increasing μ
  - μ < 336 MeV $\quad n_B = 0 \quad$ nothing changed
  - 336 MeV < μ < 345 MeV $\quad n_B \neq 0$

$$p \propto (\mu^2 - M^2)^2, \quad f \propto 2\mu^2 M^2 - M^4$$

on top of $f = -c_2 M^2 + c_4 M^4 + c_6 M^6$

  - μ > 345 MeV

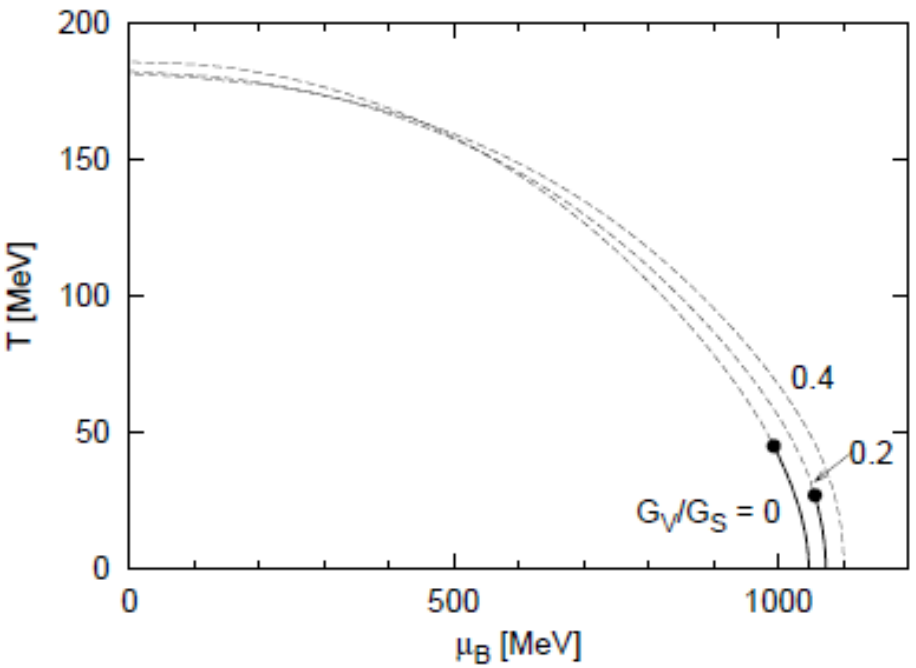
$$n_B \neq 0\,, \quad p \propto (\mu^2 - m^2)^2$$

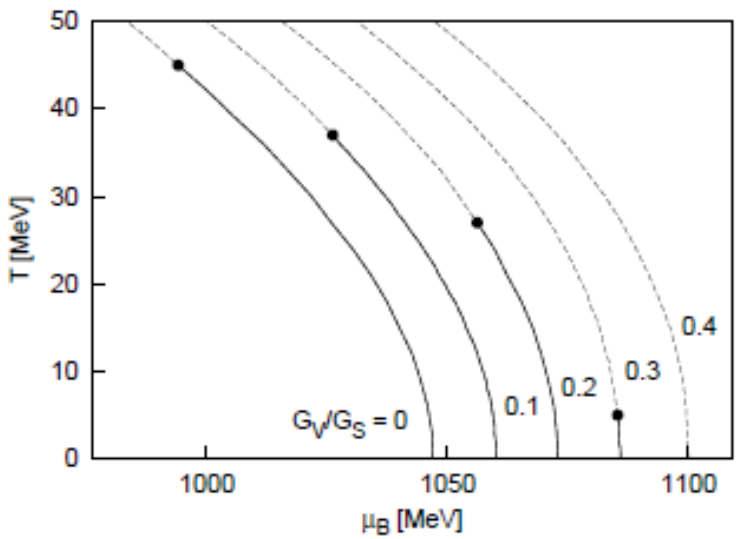
Some People Claim

- Density-Density Interaction Changes

$$\mathcal{L}_V = -g_V\,(\bar{\psi}\gamma_\mu\psi)^2 \quad \rightarrow \quad f \sim g_V n_q^2 \quad \propto \quad \mu^2(\mu^2 - M^2)^2$$

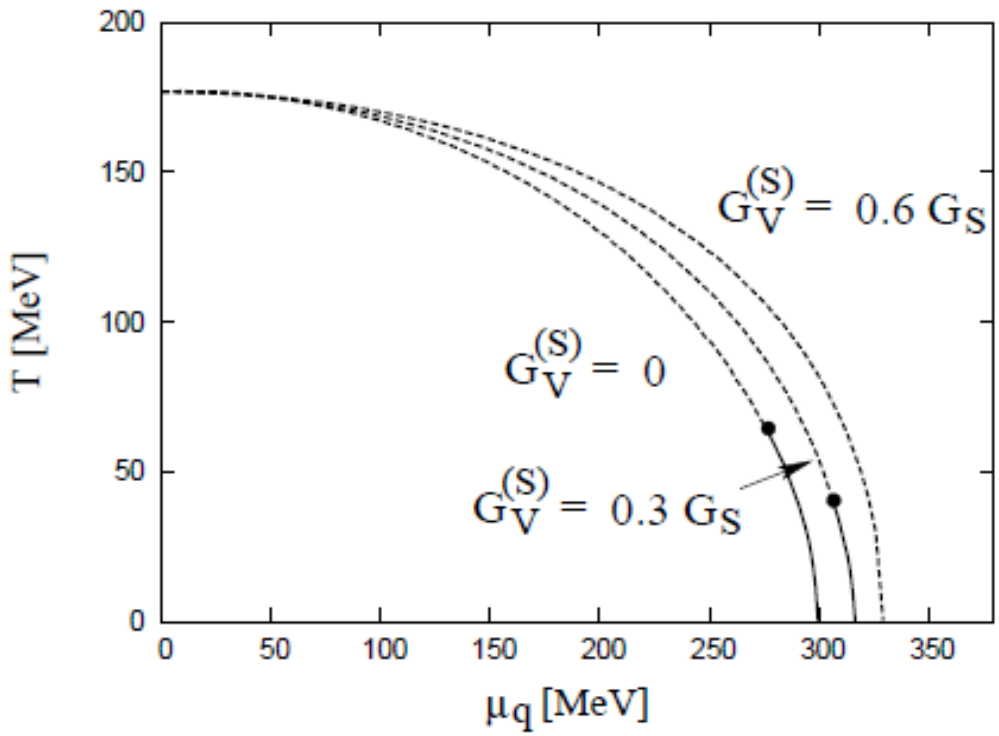
- NJL Results **Kitazawa-Koide-Kunihiro-Nemoto (2002)**





Some More from NJL

- NJL Results **Sasaki-Friman-Redlich (2006)**



So far all the discussions are limited to two-flavor quark matter in the NJL model.

Degrees of Freedom

- Quark Degrees of Freedom
  - Two-Flavor Case $$(7/8)\times 3\times 2\times 4=21$$
  - Three-Flavor Case $$(7/8)\times 3\times 3\times 4=31.5$$
- Gluon Degrees of Freedom $$8\times 2=16$$

The NJL model description is not bad, but the gluon contribution is comparable also.

PNJL Model

- Quick History (only limited milestones)
  - *Fukushima 2003*
    Simultaneous crossover of deconfinement and chiral restoration
  - *Ratti-Thaler-Weise 2005*
    Fitting the thermodynamic quantities on the lattice
  - *Fu-Zhang-Liu 2007*
    Ciminale-Gatto-Ippolito-Nardulli-Ruggieri 2007
    Fukushima 2008
    2+1 flavor with 't Hooft interaction

- It works much better than I thought...

Neat Properties

- Degrees of freedom are different!
  - 16 gluons at high T should be transverse.
  - Polyakov loop is A_0 that is longitudinal.

Polyakov loop saturates the thermodynamics near T_c.

- No clear way to fix the Polyakov-loop potential?
  - Pressure, entropy, internal energy fitting.
  - Simple parametrization with T_c fixing.

Little sensitivity to the choice of the potential.

Model Ingredients

- Thermal Energy

$$\Omega_{\rm quark} = -2T\sum_i \int \frac{{\rm d}^3p}{(2\pi)^3}\left\{\ln\det\left[1+L\,{\rm e}^{-(\varepsilon_i(p)-\mu)/T}\right]\right.$$
$$\left.+\ln\det\left[1+L^\dagger{\rm e}^{-(\varepsilon_i(p)+\mu)/T}\right]\right\}.$$

- Polyakov-Loop Potential

$$\ell = \frac{1}{N_c}\langle {\rm tr}L\rangle\,, \quad \bar\ell = \frac{1}{N_c}\langle {\rm tr}L^\dagger\rangle$$

$$\Omega_{\rm Polyakov} = -b\cdot T\left\{54\,{\rm e}^{-a/T}\,\ell\,\bar\ell\right.$$
$$\left.+\ln\left[1-6\,\ell\,\bar\ell-3(\ell\,\bar\ell)^2+4(\ell^3+\bar\ell^3)\right]\right\}$$

a **determines the pure-gluonic** T_c **= 270MeV**

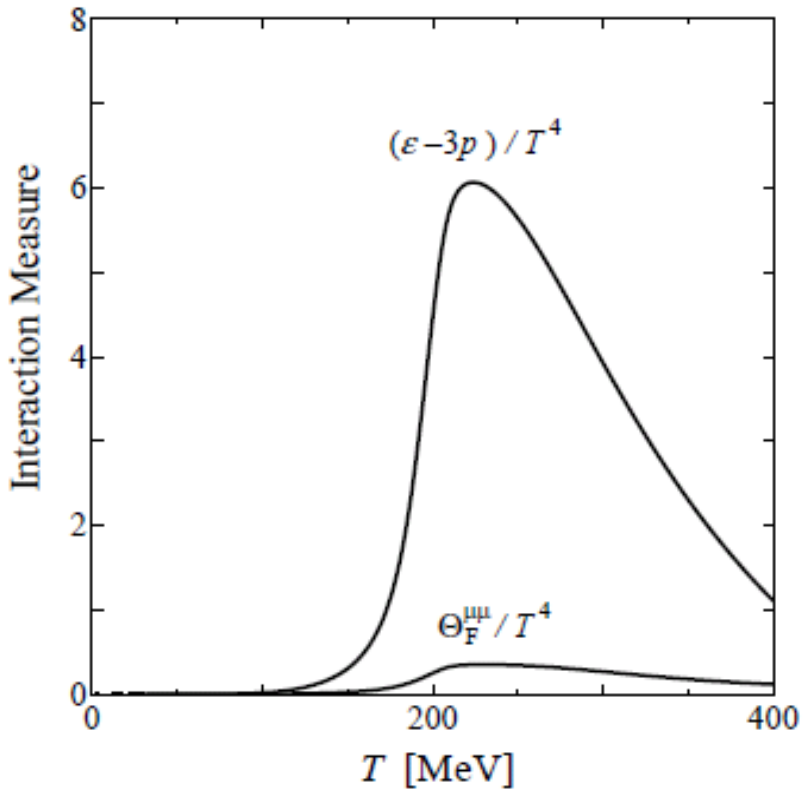
b **determines the simultaneous crossover** T_c **= 200MeV**

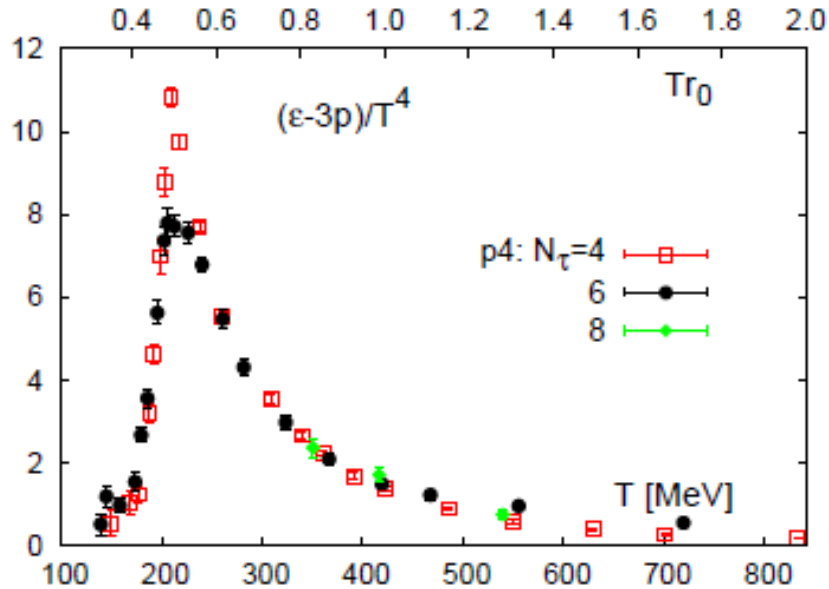
No information on bulk thermodynamic quantities

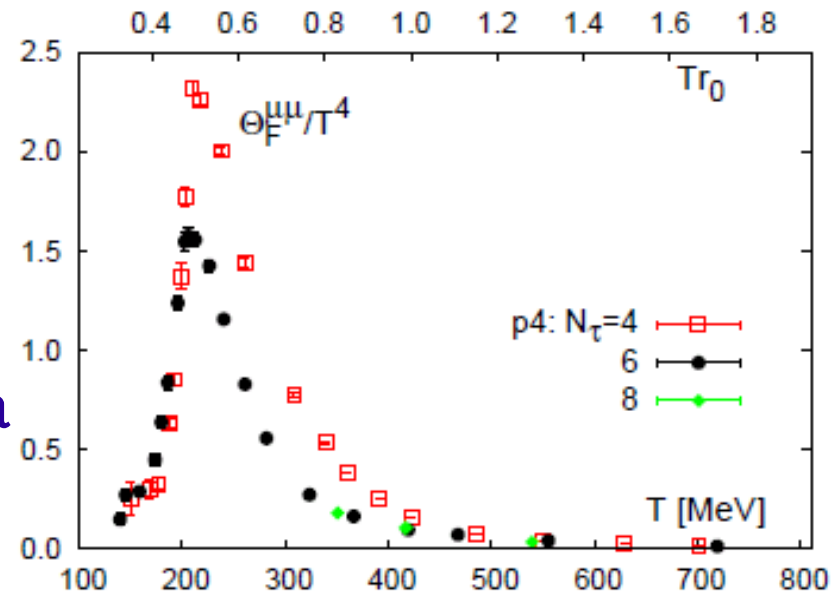
Thermodynamics at Zero Density

- Interaction Measure

2+1 flavor PNJL







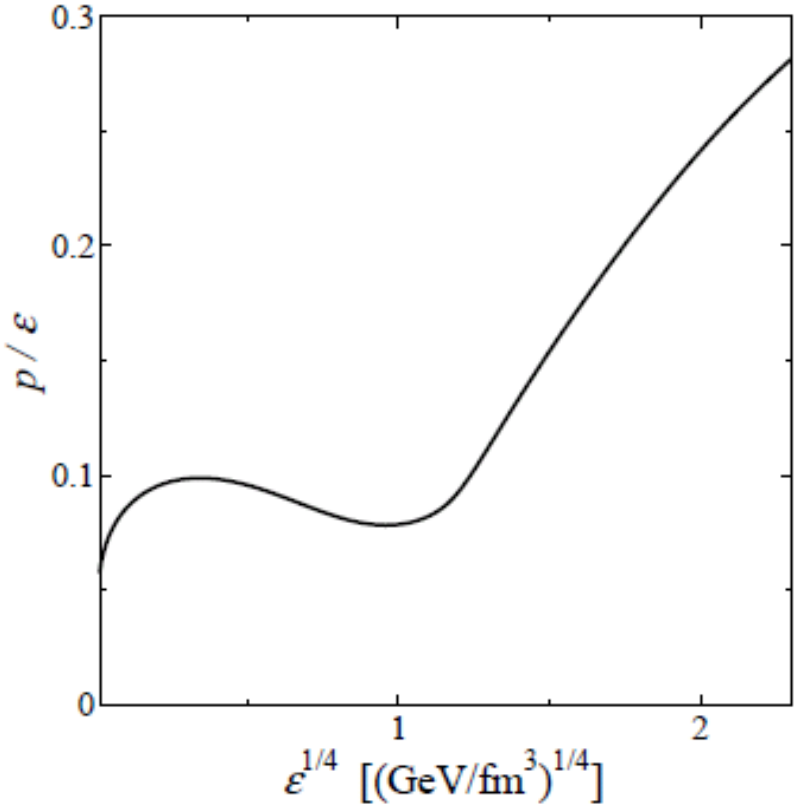
Lattice Data
Cheng.et al.

July 2008

More on Thermodynamics

- (A part of) Sound Velocity

2+1 flavor PNJL



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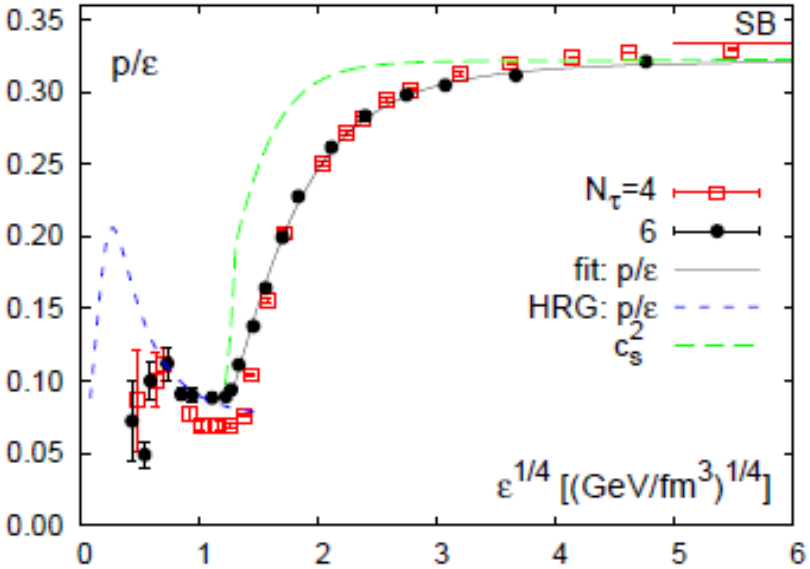
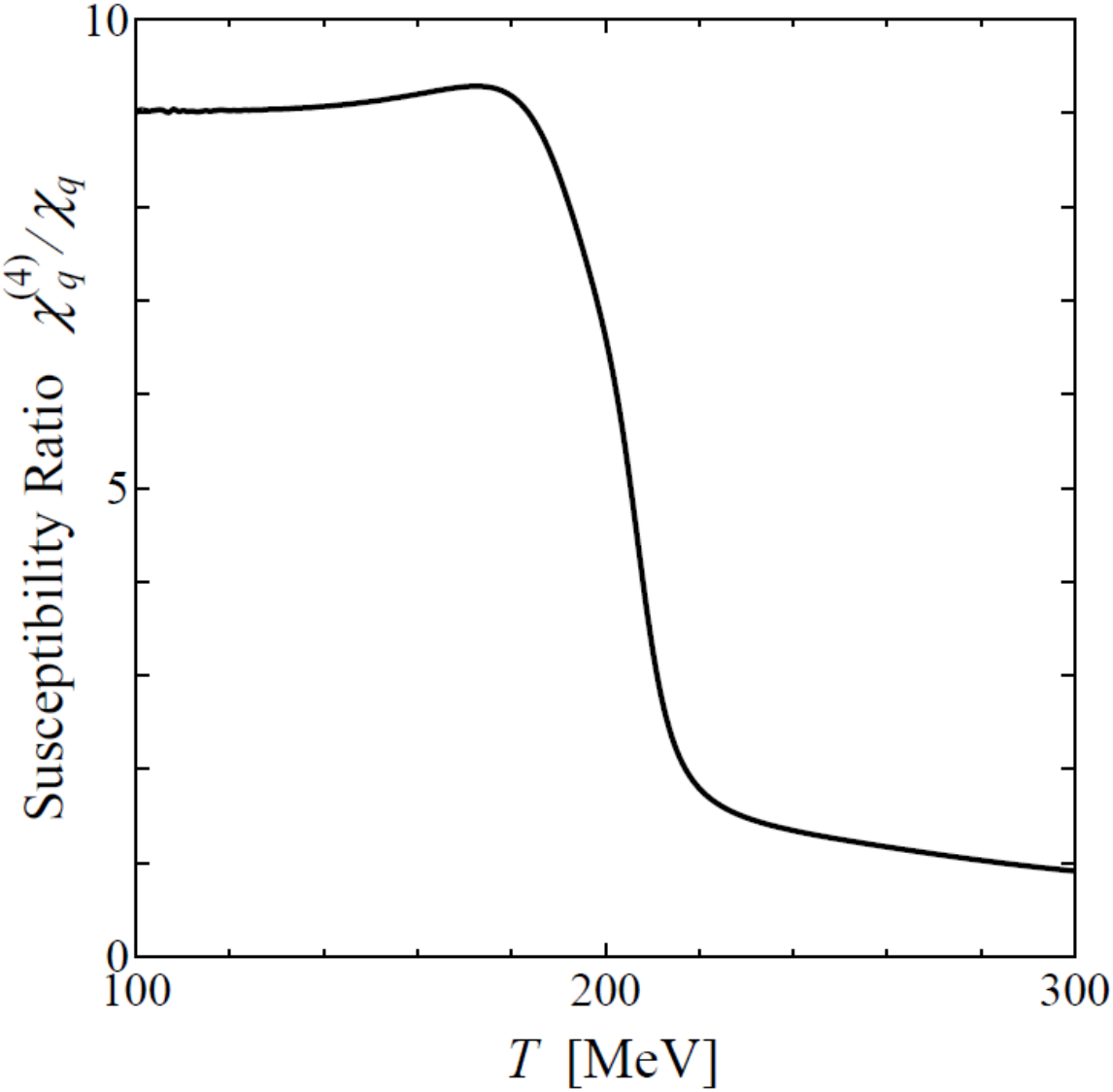


c.f. Ghosh et al (2006)

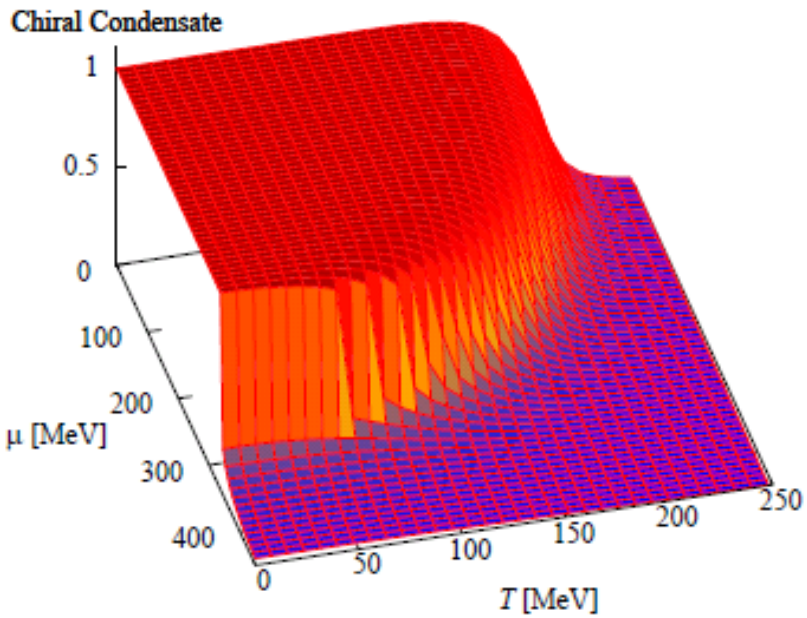
More and More

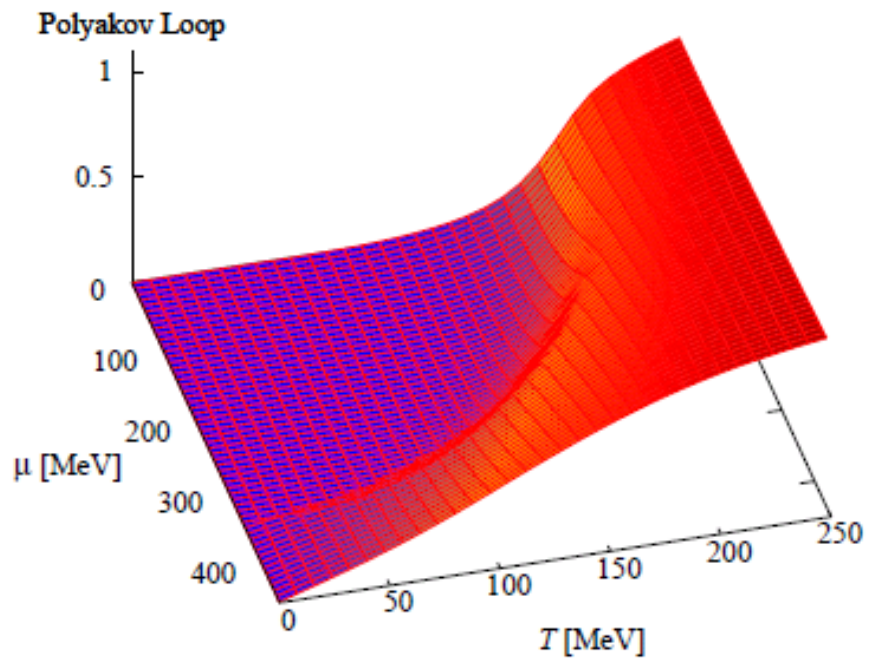
- 4th order Cumulant



Finite-Density Study

- 3D Plots – Order Parameters

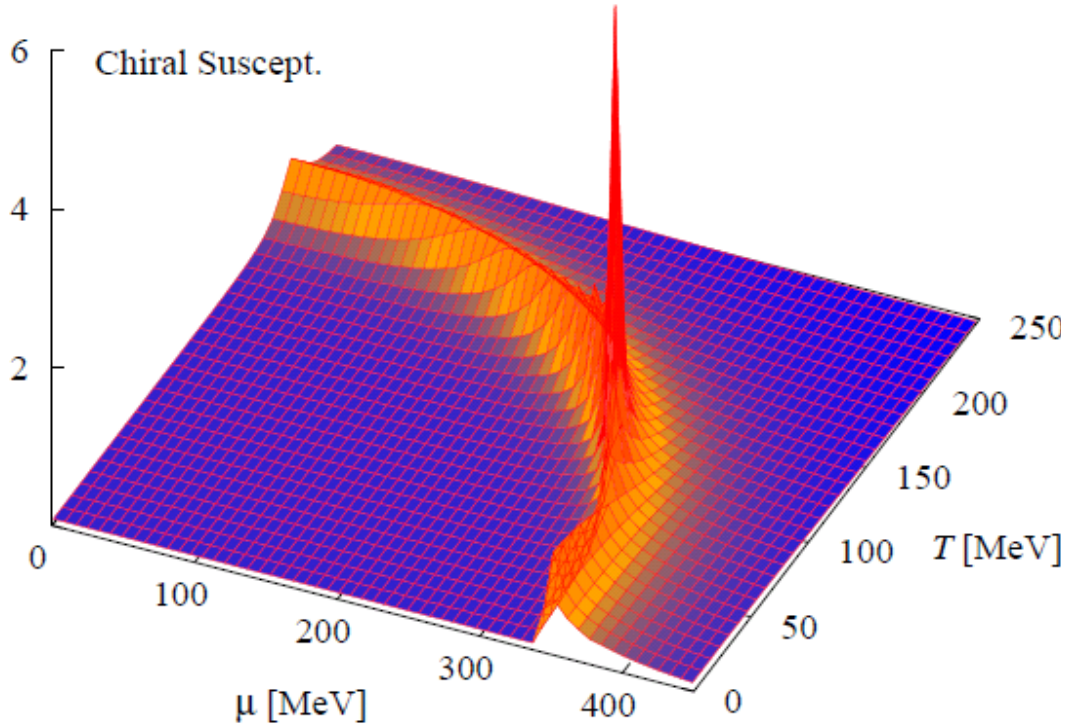


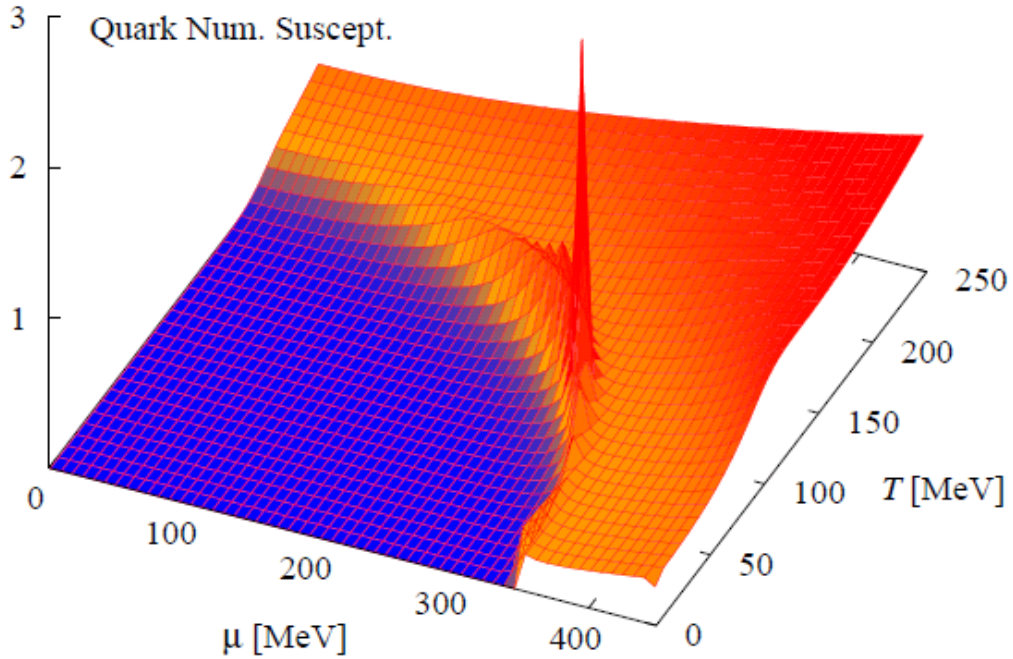


Strong correlation is evident, but!
the Polyakov loop stays small at low temperature.

Susceptibility

- (Light-Quark) Chiral and Quark Number Susceptibility





Prediction from the PNJL

- Zero Polyakov Loop at Zero Temperature

$$\left\langle \det\left[1 + L\,\mathrm{e}^{-(\varepsilon-\mu)/T}\right]\right\rangle = 1 + \mathrm{e}^{-3(\varepsilon-\mu)/T} + 3\,\ell\,\mathrm{e}^{-(\varepsilon-\mu)/T} + 3\,\bar{\ell}\,\mathrm{e}^{-2(\varepsilon-\mu)/T}$$

dominant!

$$\left\langle \det\left[1 + L^{\dagger}\,\mathrm{e}^{-(\varepsilon+\mu)/T}\right]\right\rangle = 1 + \mathrm{e}^{-3(\varepsilon+\mu)/T} + 3\,\bar{\ell}\,\mathrm{e}^{-(\varepsilon+\mu)/T} + 3\,\ell\,\mathrm{e}^{-2(\varepsilon+\mu)/T}$$

Degenerate quark matter with all colors equally occupied does not break global center symmetry.

- No Deconfinement at High Density

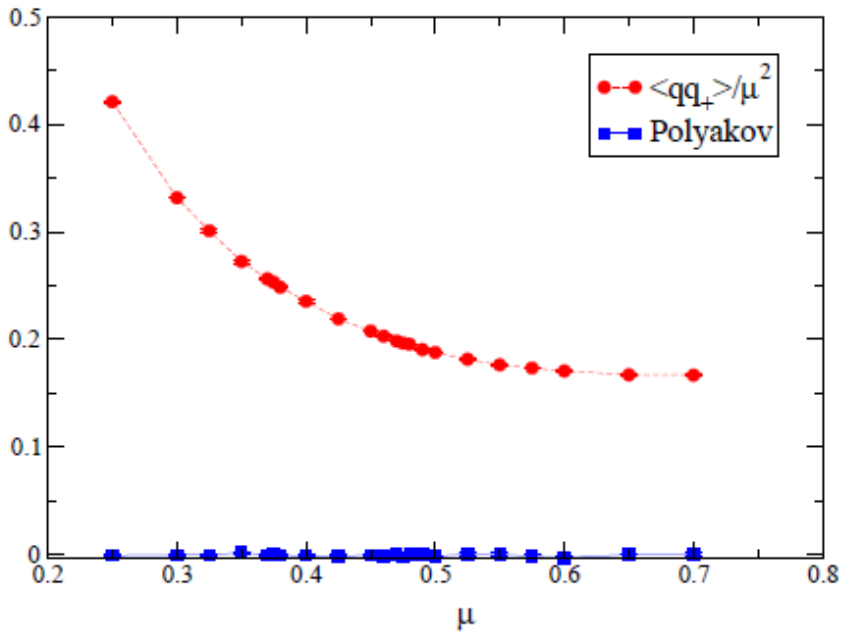
Quark matter with $\langle \bar{\psi}\,\psi \rangle \approx 0,\ l \approx 0,\ n \neq 0$

Quarkyonic Matter
McLerran-Pisarski (2007)

Evidence ?

- Talk by Simon Hands at XQCD (2-color simulation)

Order Parameters



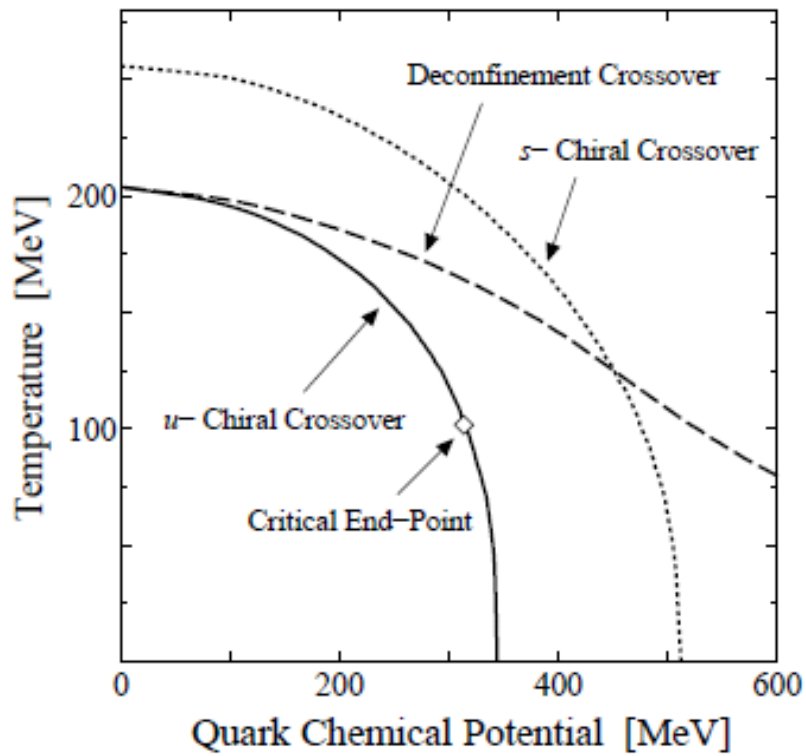
Superfluid condensate approaches BCS scaling as μ increases

Polyakov loop ≈ 0 throughout – no deconfinement!

Due to poor signal:noise? Or has it disappeared?

Phase Diagram

- Without Color Superconductivity

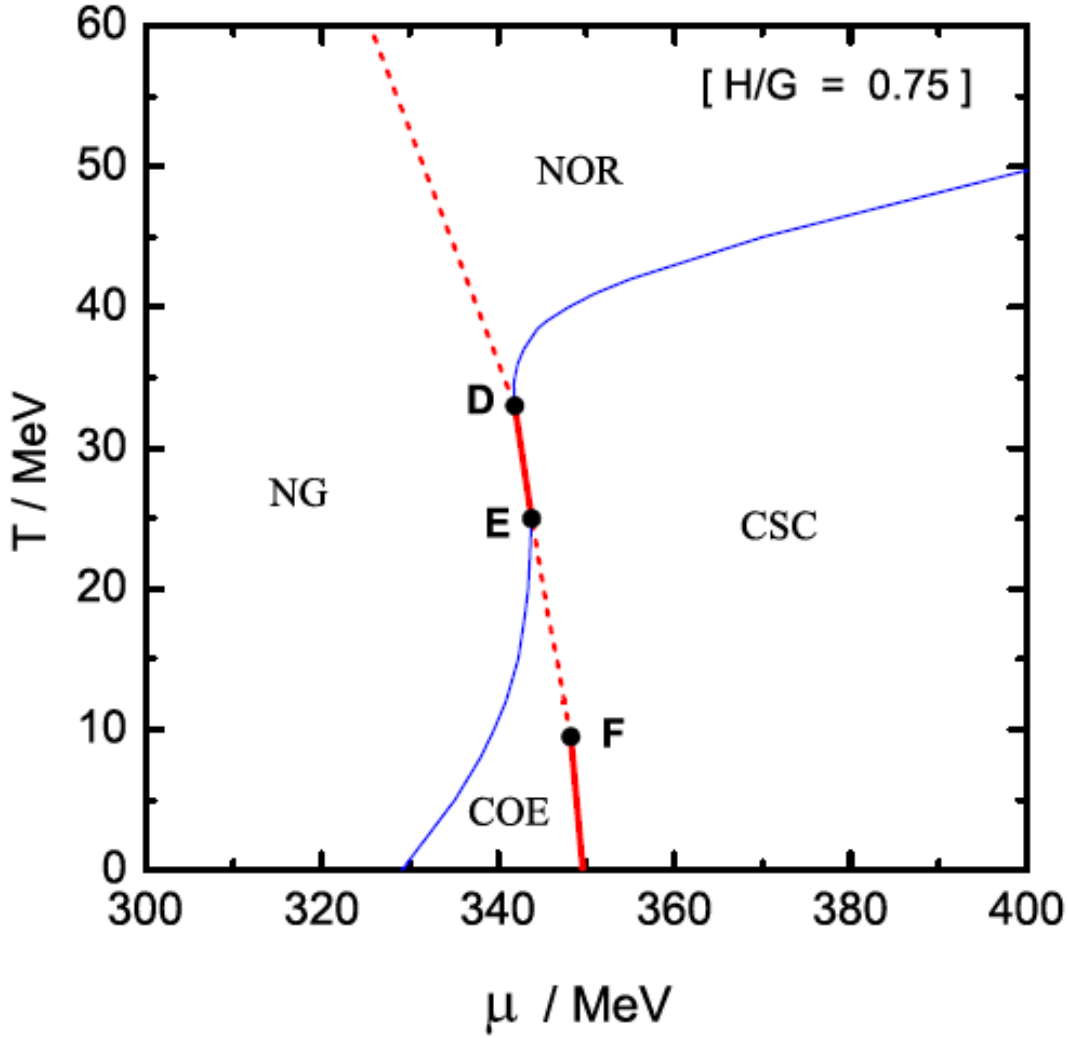


- With Color Superconductivity

Z. Zhang will surprise you with three critical-end-points.

Do not ask me a question...

- Zhang-Fukushima-Kunihiro (coming soon)



Model Assumptions

- Polyakov Loop treated as a Mean Field.
  - Cannot describe the spatial correlations correctly.
  - Sign Problem not completely resolved...

  Bulk thermodynamics is OK.

- Parameters fixed at $T = \mu = 0$
  - No new interaction induced ?
  - No coupling running with T and μ.

  Four-coupling as a result of dressed (heavy) gluons is OK unless at high T or μ.
  There are two dangerous parts...

Dangerous Parts

- Interaction Directly Coupled to Density

$$\mathcal{L}_V = -g_V \left(\bar{\psi}\gamma_\mu\psi\right)^2$$

Fierz trans generates various interaction channels.
Wave-function may have a larger component at finite density.

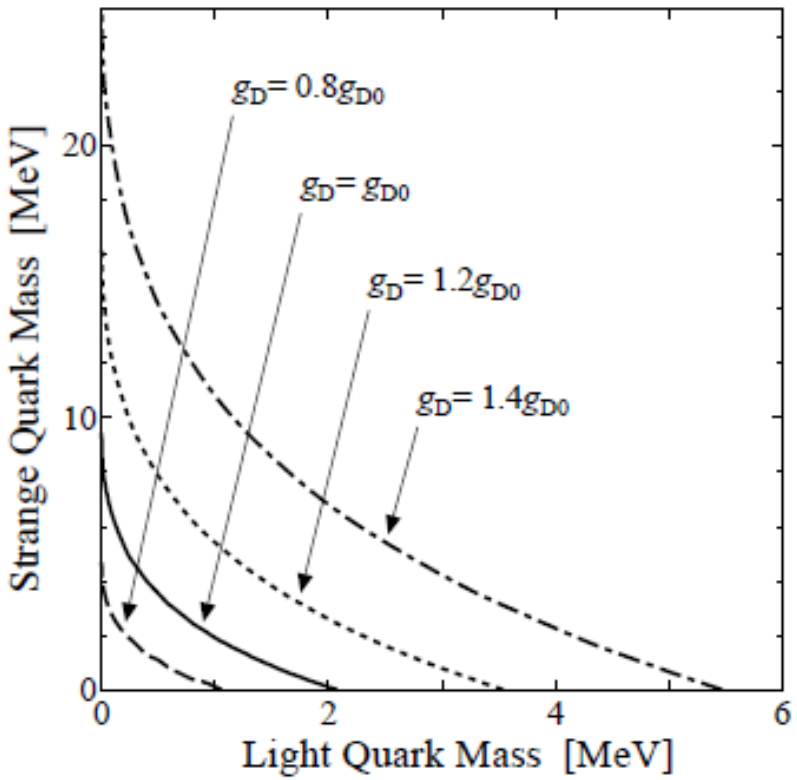
- Interaction Induced by Anomaly (Instanton)

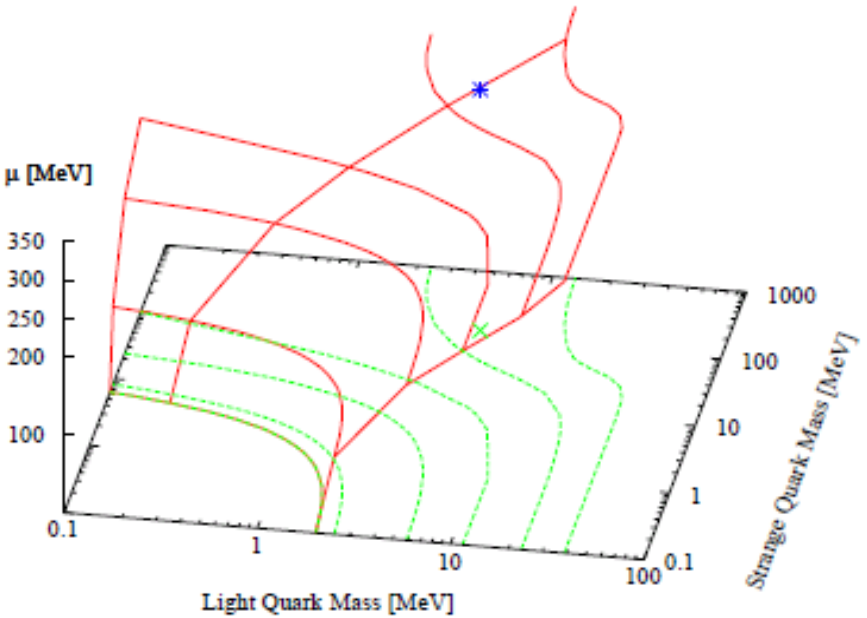
$$\mathcal{L}_A = g_D \left[\det \bar{\psi}(1-\gamma_5)\psi + \text{h.c.}\right]$$

Instanton excitation should be suppressed at finite T and μ.
Effective $U_A(1)$ symmetry restoration.

Columbia Plots

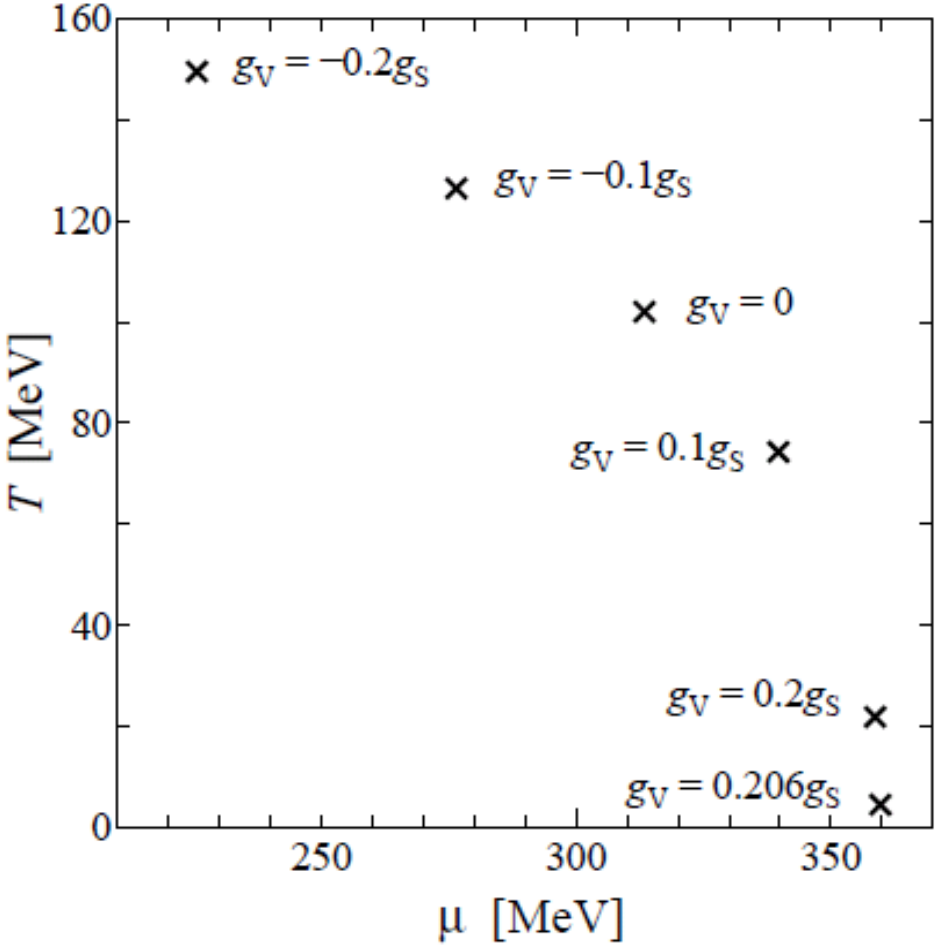
- First-Order Regions

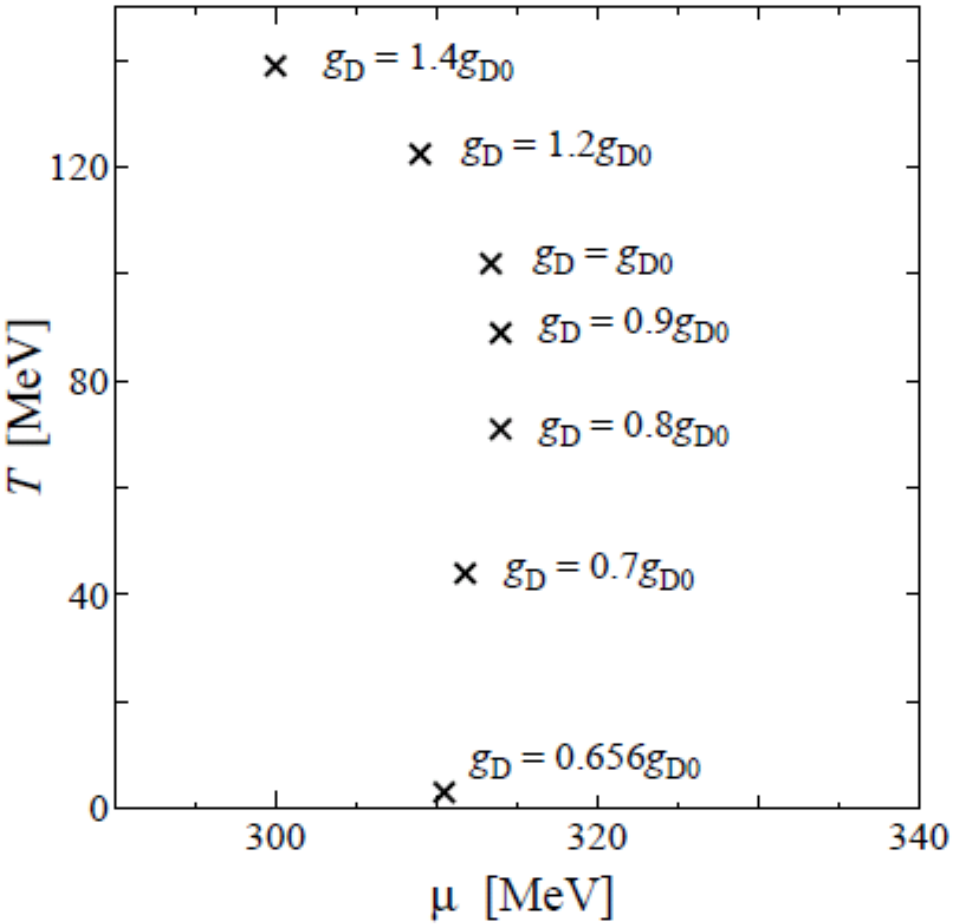




Their Effects

- Shift of the Critical Point





What Model Studies Can Say

- There IS a critical end-point.
  - Some reasonable model parameters lead to existence of a critical end-point on the phase diagram.
- There IS NOT a critical end-point.
  - Some other model parameters within reasonable range lead to non-existence of a critical point.
- There ARE more critical end-points.
  - Some other dynamics at high density lead to more than one (up to three!) critical end-points.

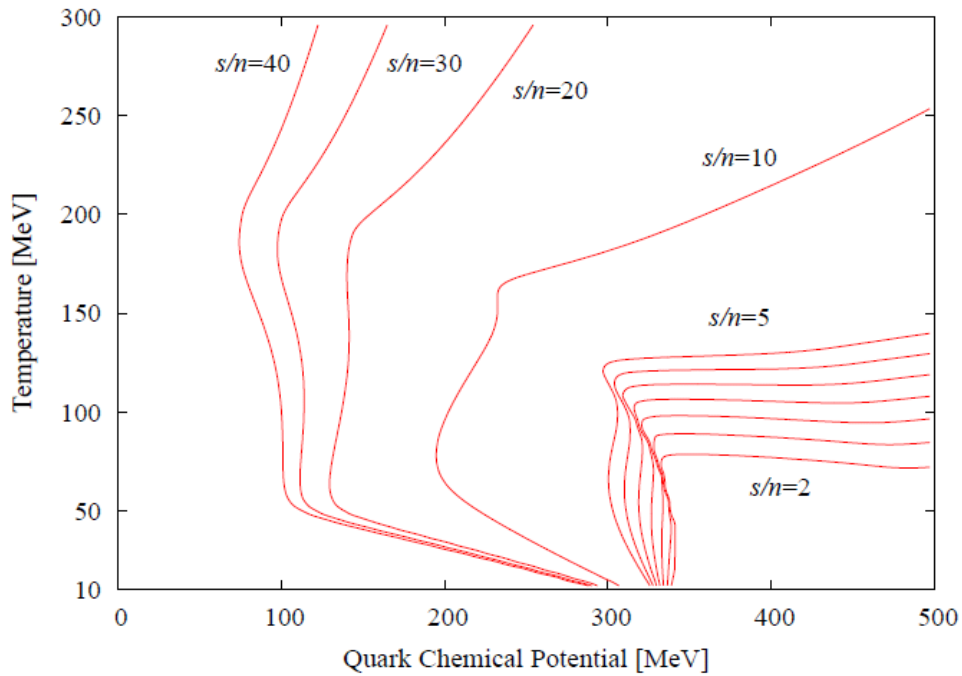
Still Useful Way to Go

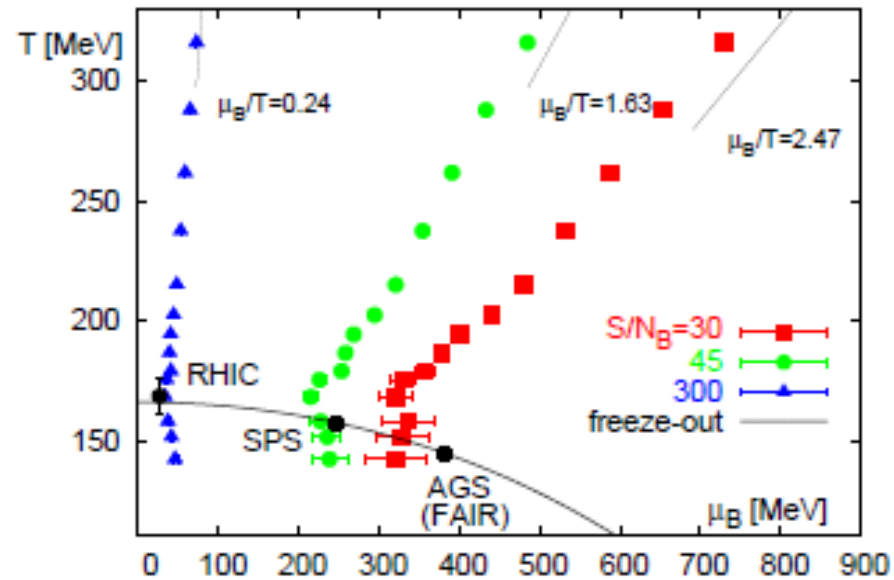
- Meaningful Approach (Attitude)
  - Let us assume one of them.
  - We can see what happens in the model.
  - Model prediction based on a scenario.

- One Critical End-Point (standard picture)
  - Let us accept "reasonable model parameters."
  - We can see the isentropic trajectories on the phase diagram.
  - Model prediction for the "hydrodynamical" evolution.

Isentropic Trajectories

- s/n = constant lines



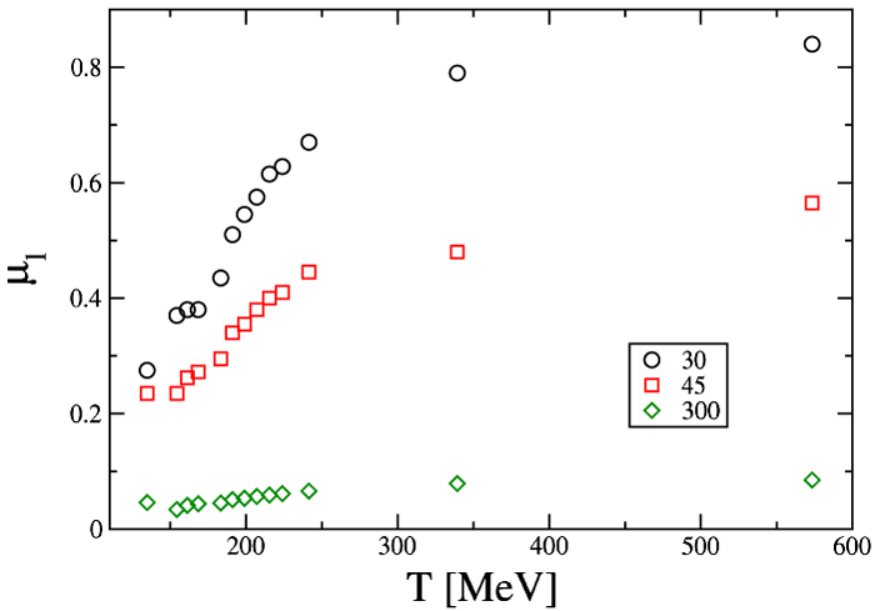


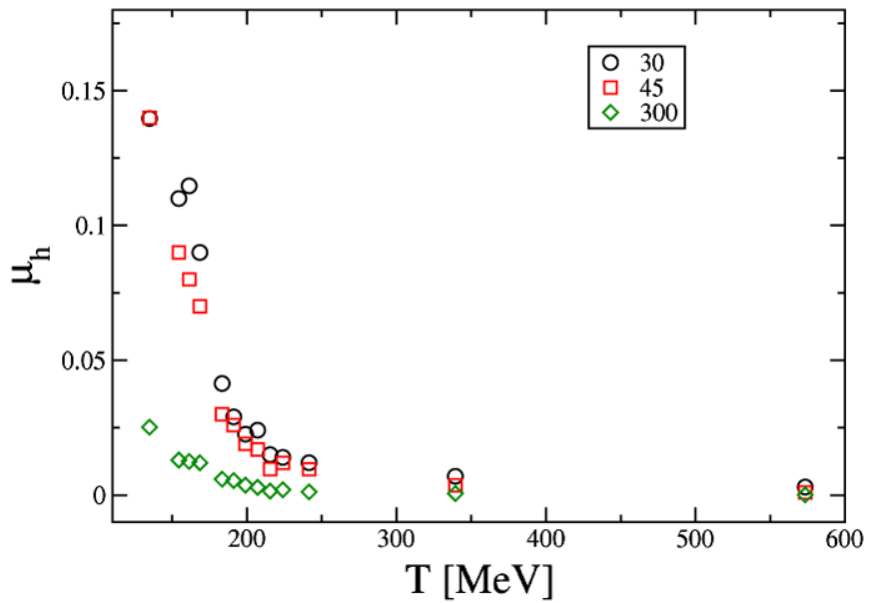
Lattice Data
Ejiri-Karsch-Laermann-Schmidt 05

Critical region is very small and neglected.
Then, nothing is special around the end-point.

Strangeness-free Isentropic Trajectories

- n_s must be zero in the heavy-ion collision



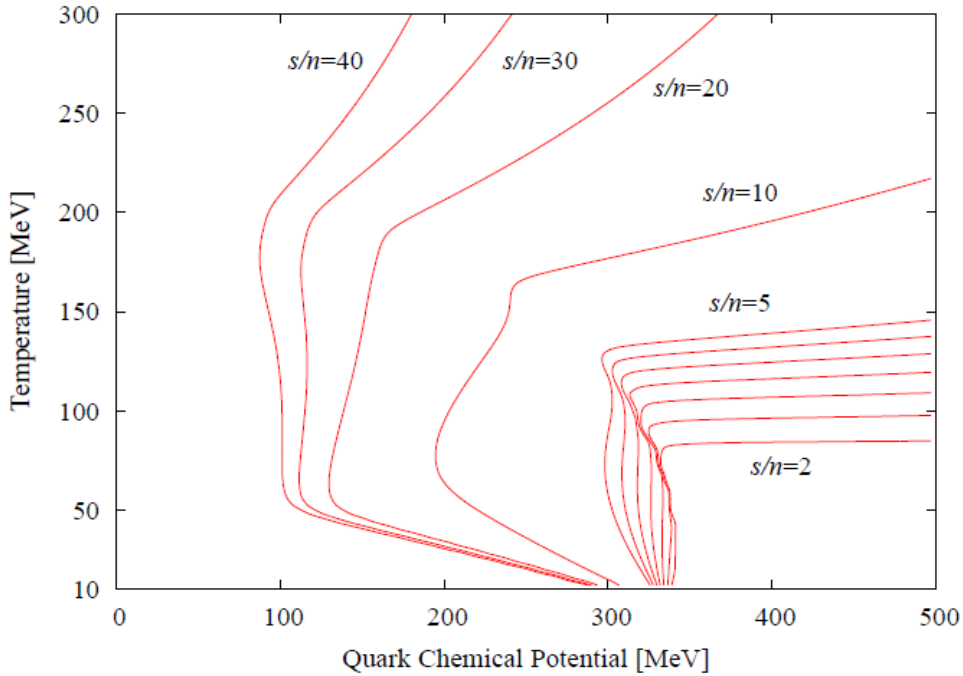


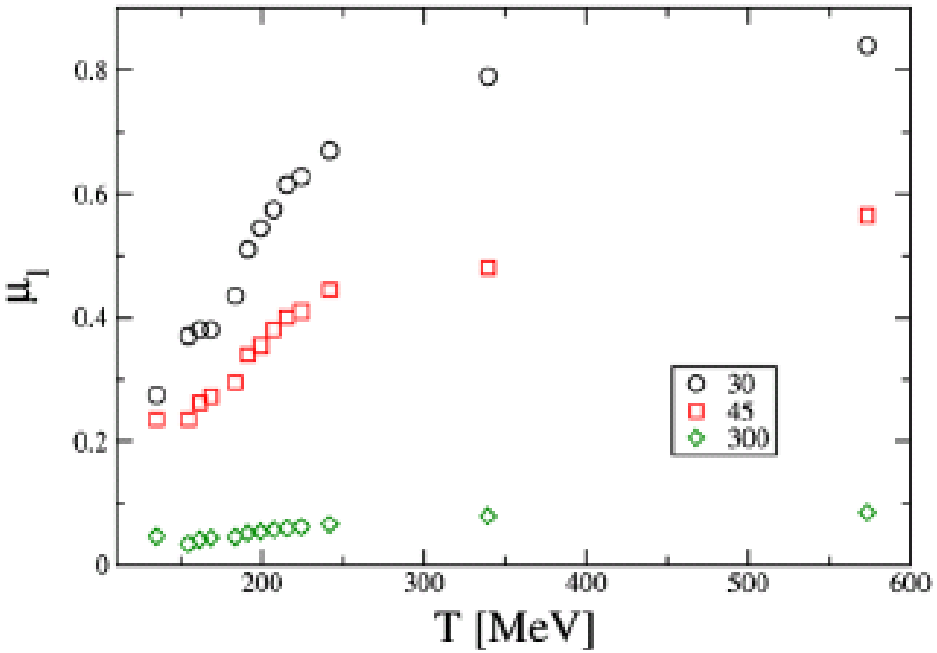
Lattice Data
MILC 2008

Positive μ_s is necessary, why so?
PNJL model can answer!

Qualitative Comparison I

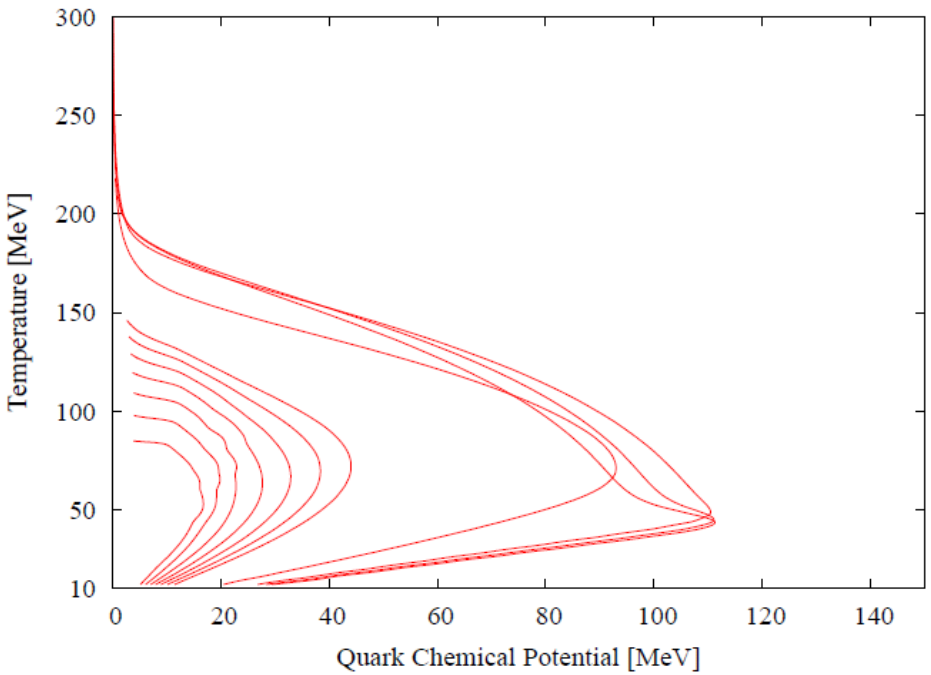
- Light Quark Sector

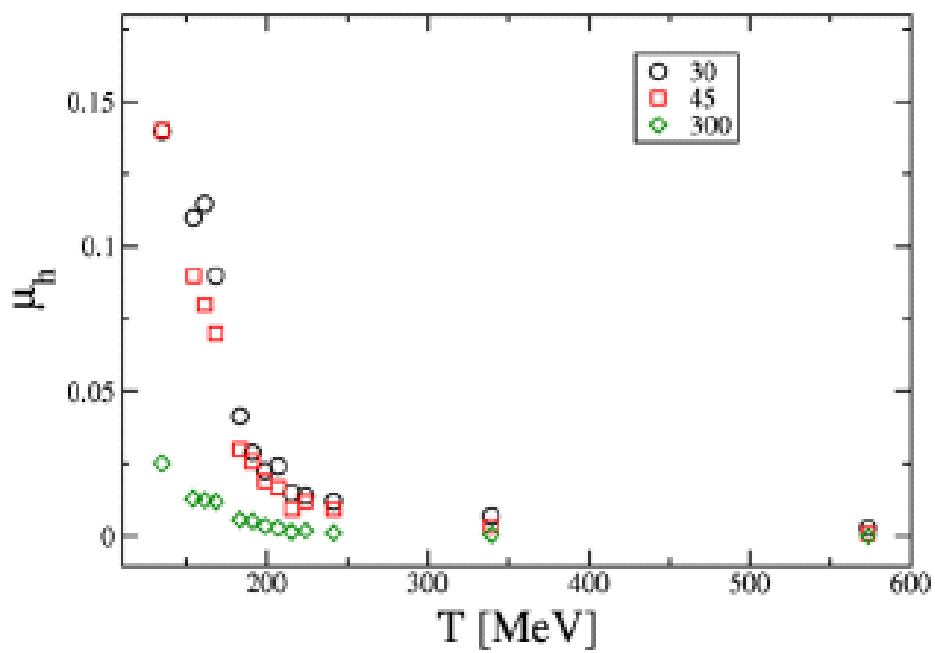




Qualitative Comparison II

- Heavy Quark Sector





Polyakov Loop Feedback

- Particle-rich state favors anti-Polyakov-loop.

$$\ell = \frac{1}{N_c}\langle \mathrm{tr} L\rangle\,, \quad \bar{\ell} = \frac{1}{N_c}\langle \mathrm{tr} L^\dagger\rangle \qquad \bar{l} > l \ \text{ for } \ \mu > 0$$

- Anti-Polyakov-loop acts as a negative chem. pot.

$$\Omega_{\mathrm{quark}} = -2T\sum_i \int \frac{\mathrm{d}^3 p}{(2\pi)^3}\left\{\ln\det\left[1 + L\,\mathrm{e}^{-(\varepsilon_i(p)-\mu)/T}\right] + \ln\det\left[1 + L^\dagger \mathrm{e}^{-(\varepsilon_i(p)+\mu)/T}\right]\right\}.$$

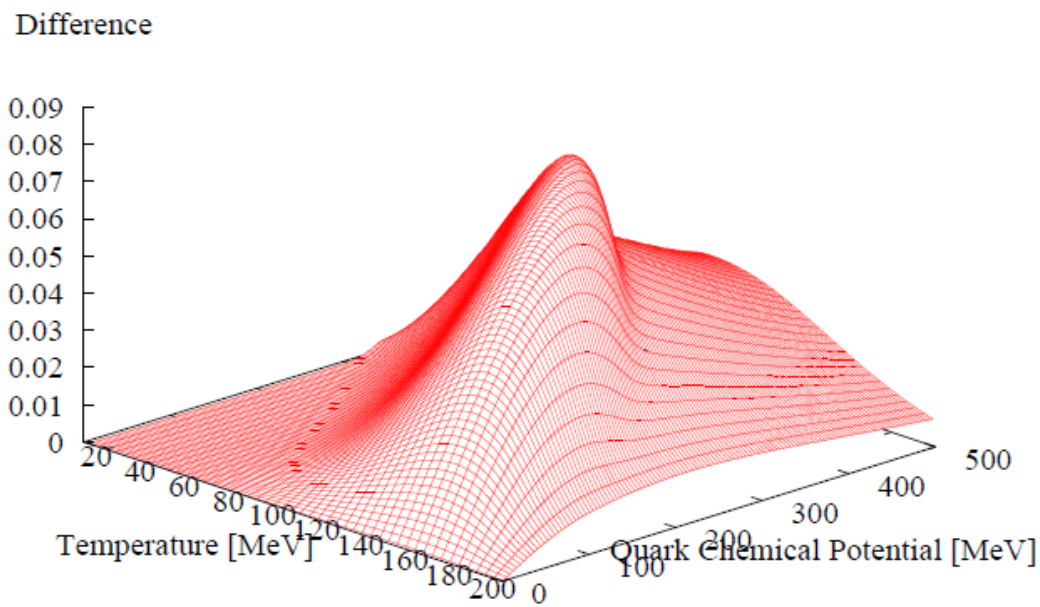
- Positive chem. pot. is necessary.

$$\log(\bar{l}/l) \approx 2\mu_s/T$$

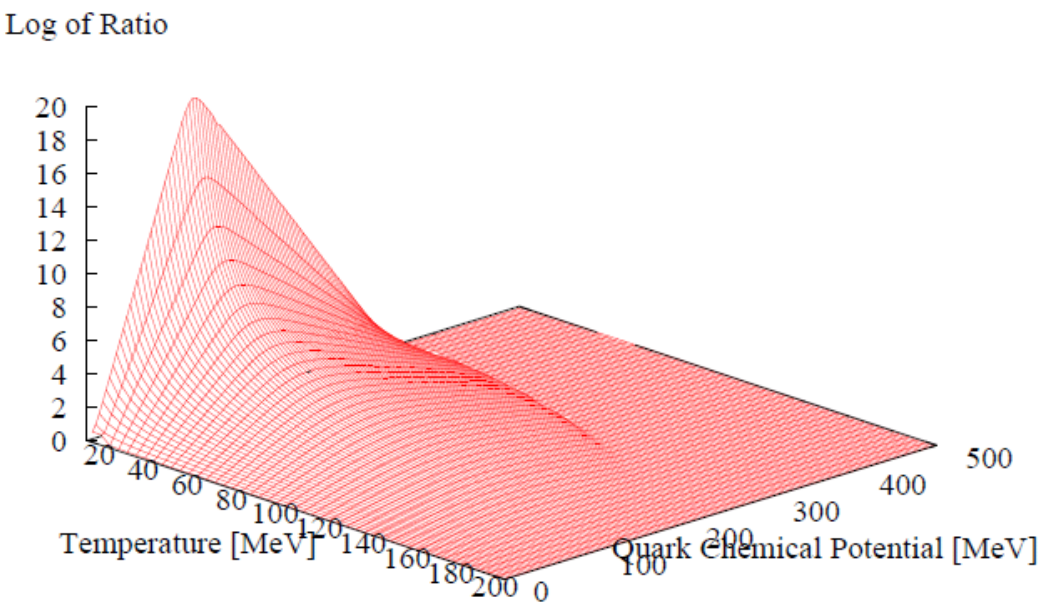
Polyakov Loop Difference

- Difference and Ratio

$$\bar{l} - l$$



$$\log(\bar{l}/l)$$



Summary

- PNJL model can reproduce bulk thermodynamic properties at zero density.
- PNJL model can give predictions for finite density.
- Two model parameters, the coupling strength of the vector-channel interaction and the magnitude of 't Hooft term, are not under theoretical control.
- Under assumption of one critical end-point, PNJL model can draw isentropic trajectories.
- Strangeness-free isentropic trajectories are close to ones with no such constraint.
- Anti-Polyakov-loop induces strange chem. pot.