Johann Wolfgang Goethe-Universität Frankfurt
Institut für Theoretische Physik

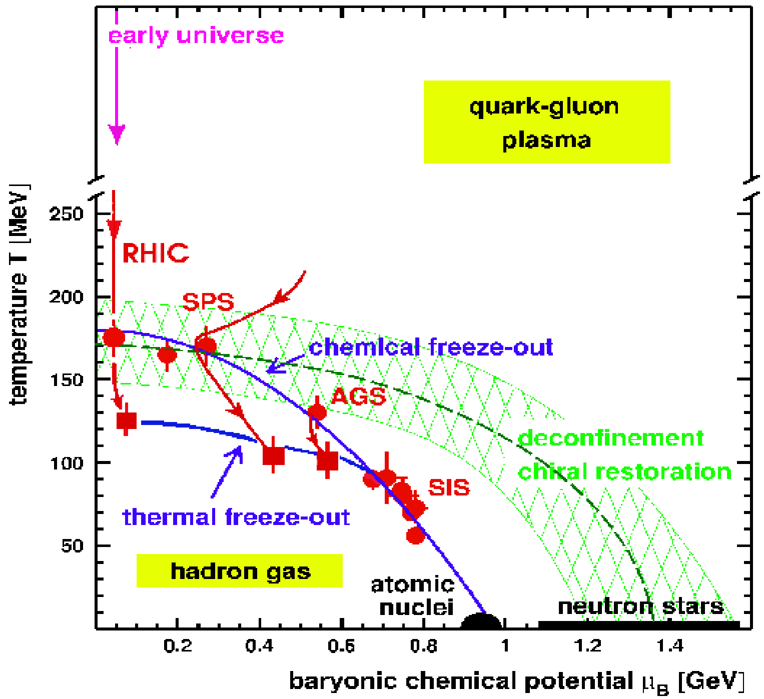
Fast chemical equilibration of hadrons -
the importance of multiparticle collisions in heavy ion reactions

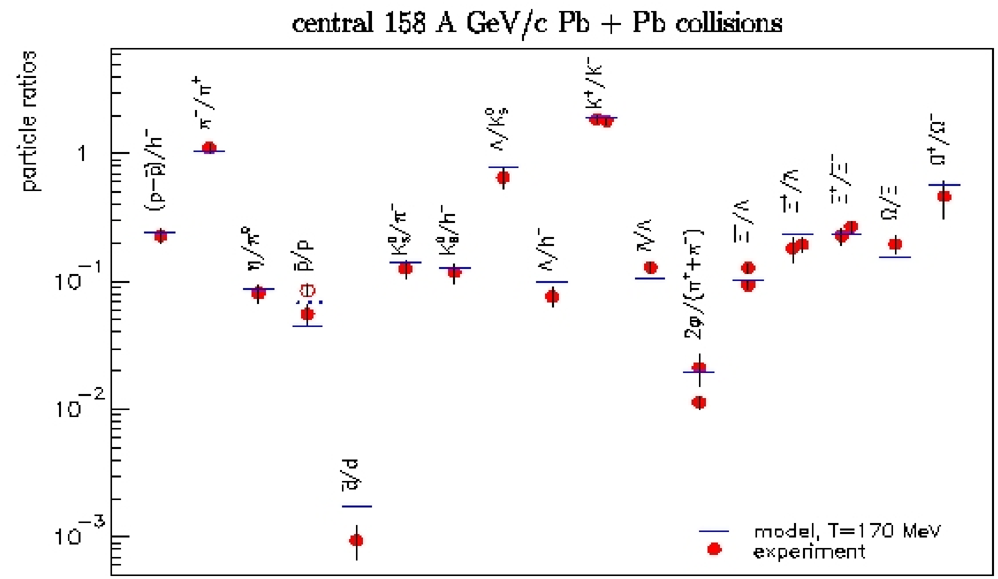
C. Greiner

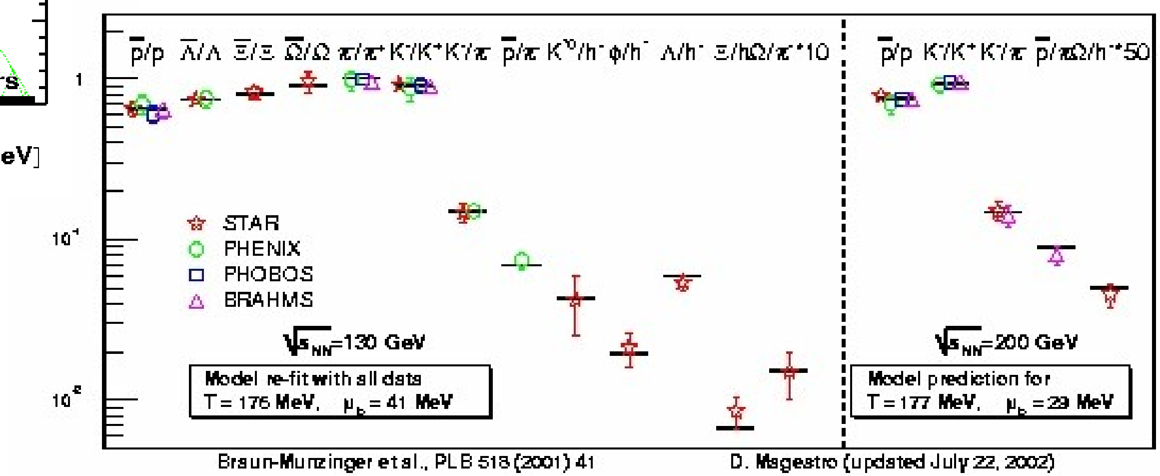
INT, Seattle , august 2008

- Motivation: chemical equilibrium of anti-baryons
- Chemical equilibration at the Hagedorn temperature
- Outlook

Exploring the phases of nuclear matter



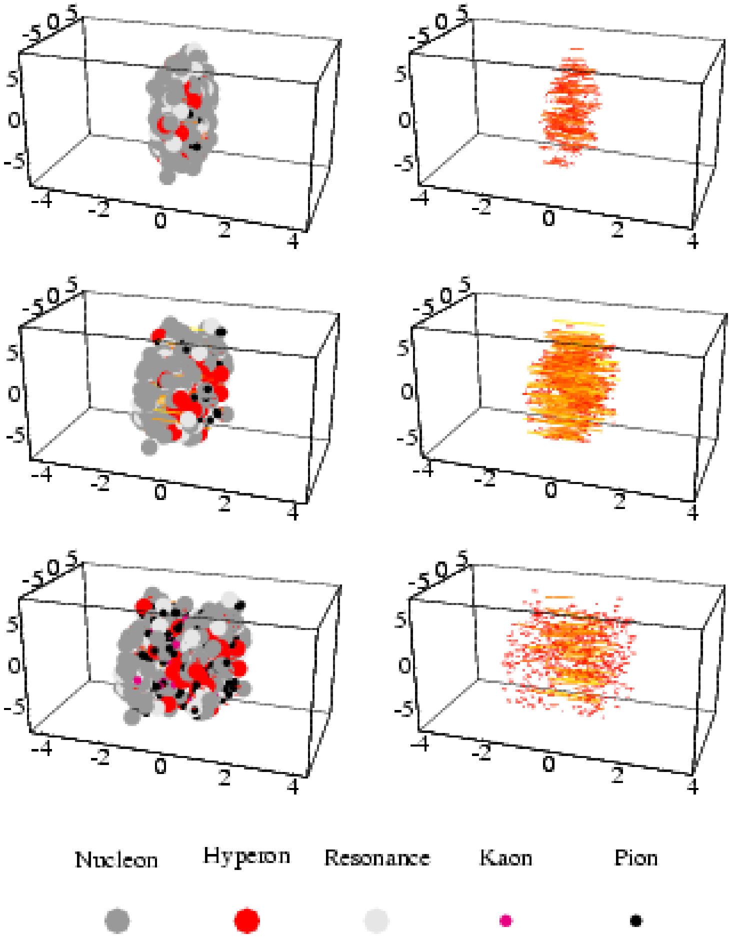




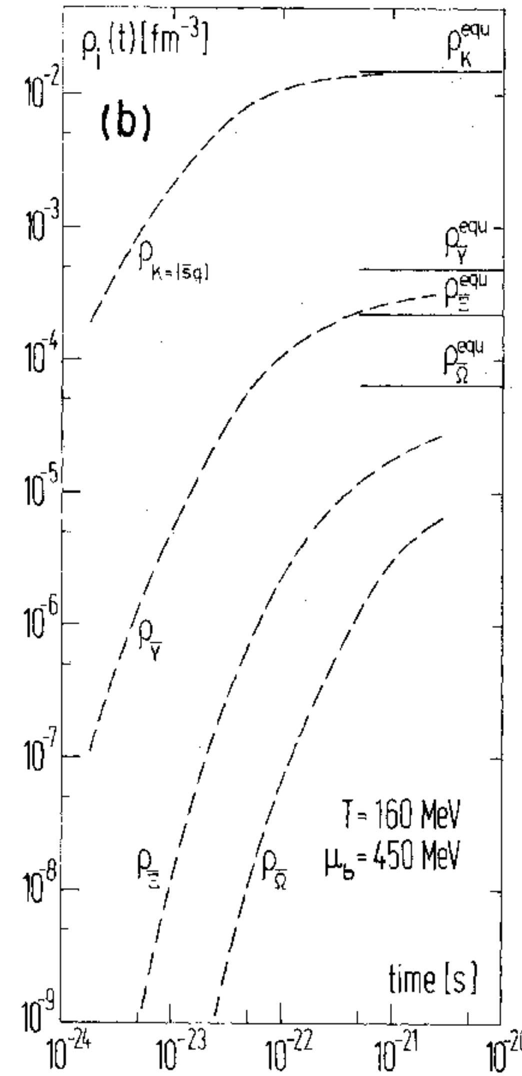
Strangeness production at SpS energies

Production of Antihyperons: QGP signature…?

J. Geiss



P. Koch, B. Müller, J. Rafelski



$$\tau_{\bar{Y}}^{\mathrm{chem}} \gtrsim 1000\ \mathrm{fm/c}$$

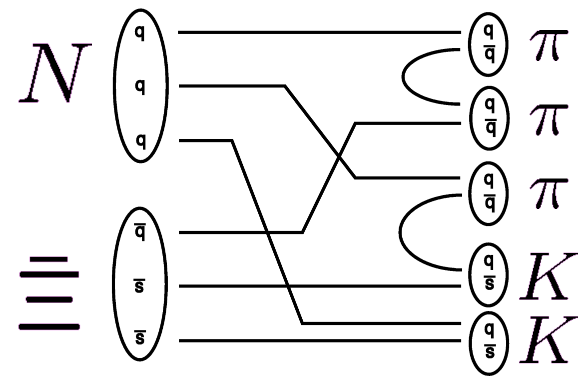
Production of Anti-Baryons
Multimesonic channels

$\bar{p}$-production: $\bar{p} + N \leftrightarrow n\pi$

R.Rapp and E. Shuryak, *Phys.Rev.Lett.* ***86*** (2001) 2980

$\bar{Y}$-production: C.Greiner and S.Leupold, *J.Phys. G* ***27*** (2001) L95

$$\bar{\Lambda} + N \leftrightarrow n\pi + K$$
$$\bar{\Xi} + N \leftrightarrow n\pi + 2K$$
$$\bar{\Omega} + N \leftrightarrow n\pi + 3K$$



$$\bar{Y} + N \leftrightarrow n\pi + n_Y K$$

detailed balance

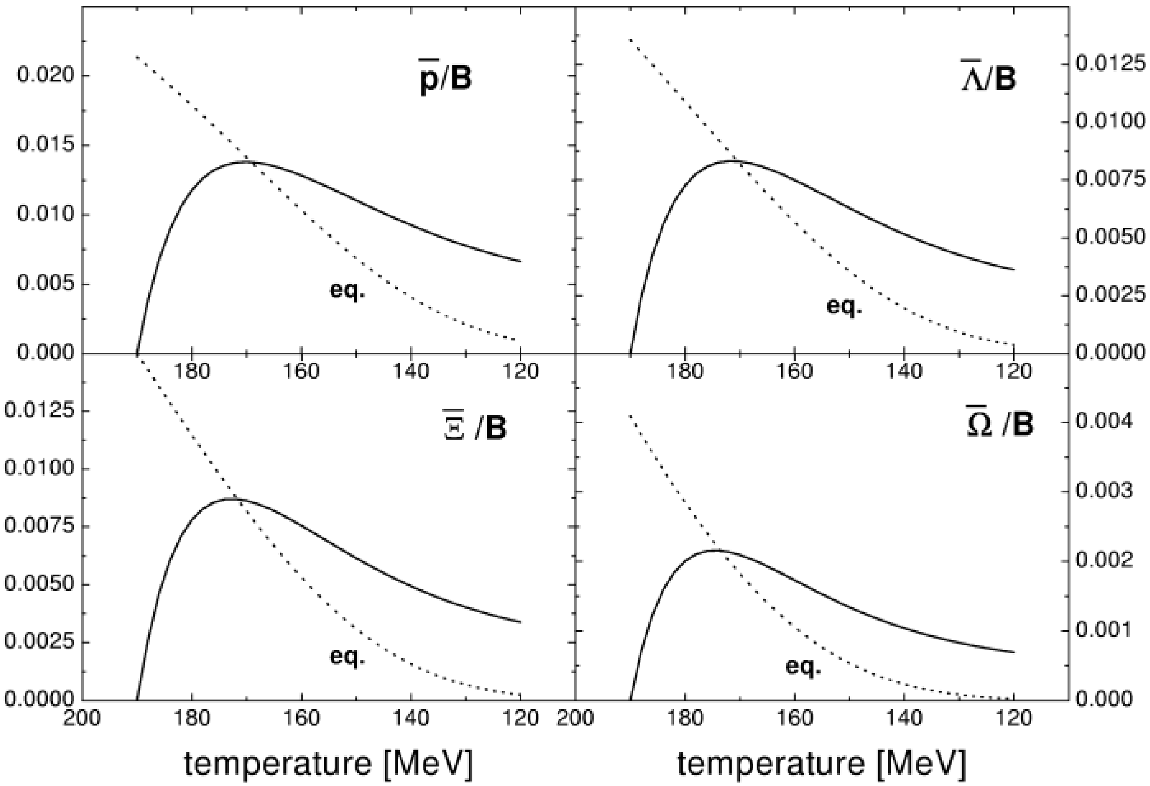
$$\sigma_{N\bar{Y}} \approx \sigma_{N\bar{p}} \approx 50 \text{ mb}$$

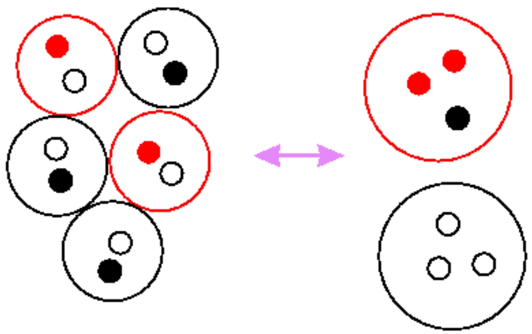
annihilation rate $\Rightarrow$ chemical equilibration rate

$$\tau_{\bar{Y}} := (\Gamma_{\bar{Y}})^{-1} = \frac{1}{\langle\langle \sigma_{N\bar{Y} \to n\pi + n_Y K} v_{\bar{Y}N} \rangle\rangle \rho_B} \approx 1 - 3\,\mathrm{fm/c}$$

master equation:

$$\frac{d}{dt}N_{\bar{Y}} = -\Gamma_{\bar{Y}}(t)\left\{N_{\bar{Y}} - N_{\bar{Y}}^{eq.}\sum_n p_n\left(\frac{N_\pi}{N_\pi^{eq.}}\right)^n\left(\frac{N_K}{N_K^{eq.}}\right)^{n_Y}\right\}$$





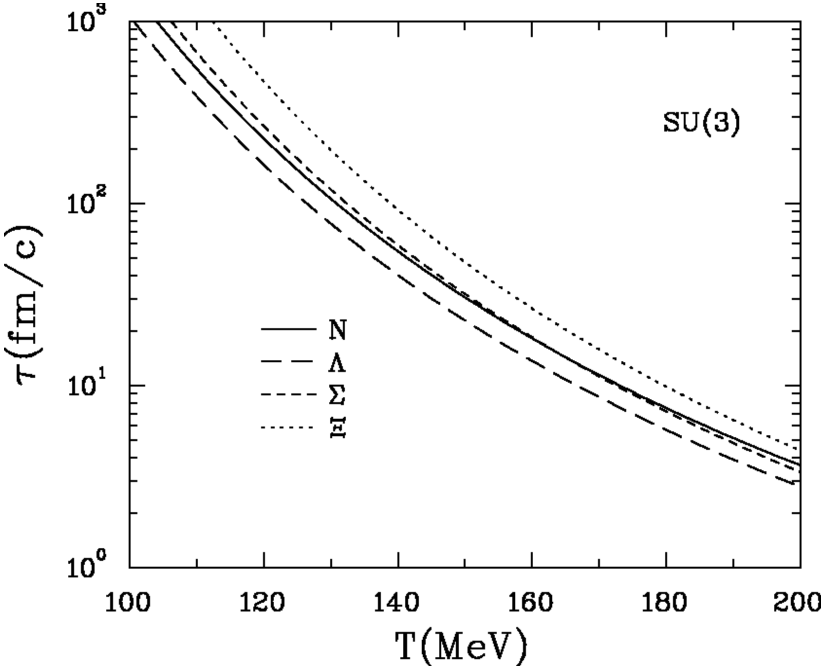
C.Greiner, AIP Conf. Proc. 644:337 (2003)

universal behavior:

$$T_{eff} = 155 - 165 \text{ MeV}$$

$B - \bar{B}$ production at RHIC

Thermal rates within
chiral SU(3) description

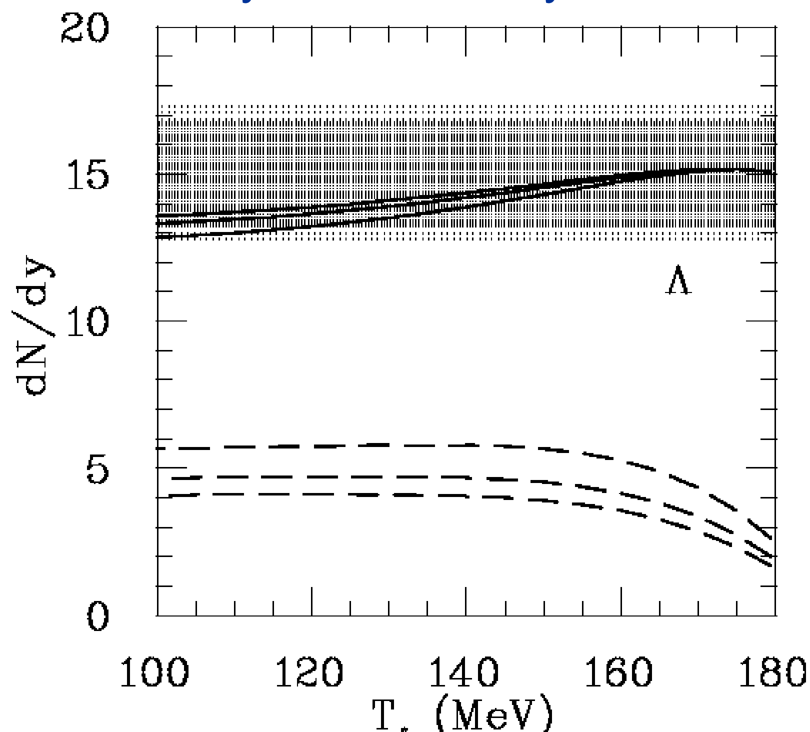


I. Shovkovy, J. Kapusta

$$\tau_{\Omega} \approx \left(\langle \sigma_{\Omega\bar{B}} v_{\Omega\bar{B}} \rangle n_{\bar{B}}\right)^{-1}$$

$$\xrightarrow{\text{RHIC}} 10\,[fm/c]$$

Chemical population of
baryons / anti-baryons:



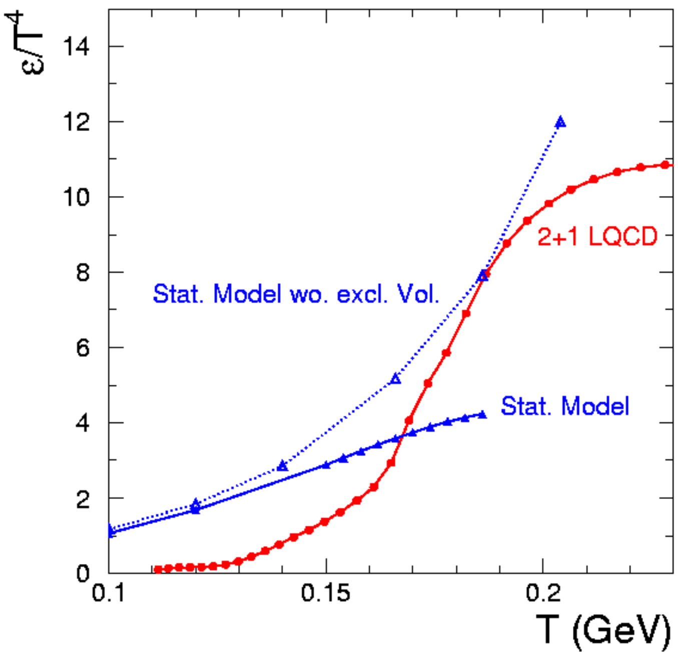
P. Huovinen, J. Kapusta

Insufficient by a factor of 3 to 4

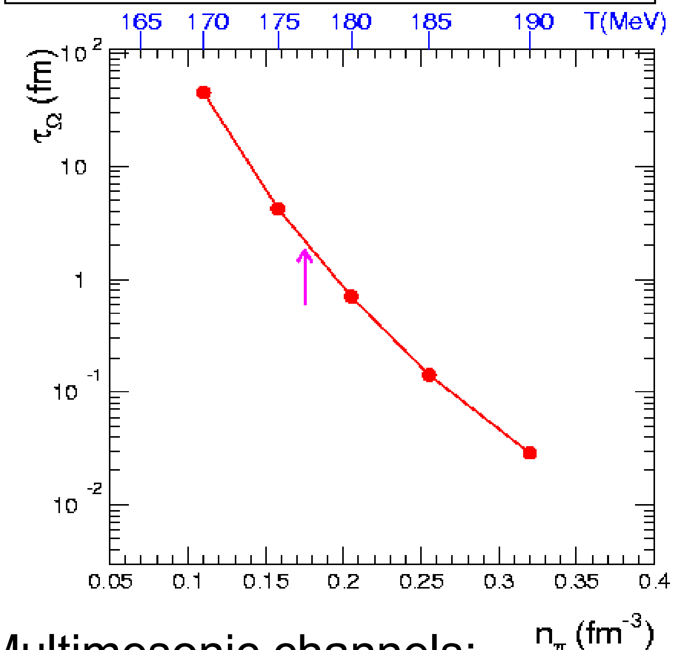
Chemical Freeze-out and T_{critical} of QCD

(P. Braun-Munzinger, J. Stachel, C. Wetterich, *Phys.Lett.* ***B***596:61-69 (2004))

Hadronic resonance gas vs. lattice:



Chemical equilibration of baryon / anti-baryons:



$$n_{\pi,K} \to 2n^{\text{eq}}_{\pi,K}$$

$$\Rightarrow n_{\Omega,\bar{B}} \to (2)^{5/2} n^{\text{eq}}_{\Omega,\bar{B}}$$

$$= 5.6 n^{\text{eq}}_{\Omega,\bar{B}}$$

Multimesonic channels:

$$2\pi + 3K \leftrightarrow \Omega(sss) + \bar{N}(\bar{q}\bar{q}\bar{q})$$

$\rightarrow$ tremendous production rate close to T_c

... but too many antibaryons

Possible solution by Hagedorn states

C. Greiner, P. Koch, F. Liu,
I. Shovkovy, H. Stöcker
J.Phys.G ***31*** (2005)

- $B\bar{B}$ annihilation at LEAR
  - statistical description works well
  - intermediate doorway meson states

- We propose Hagedorn States
  as intermediate, highly unstable states,
  which decay statistically, for counting and
  generating multi-particle collisions .

$$\pi + \pi \to \pi + \pi$$

$$\rho$$

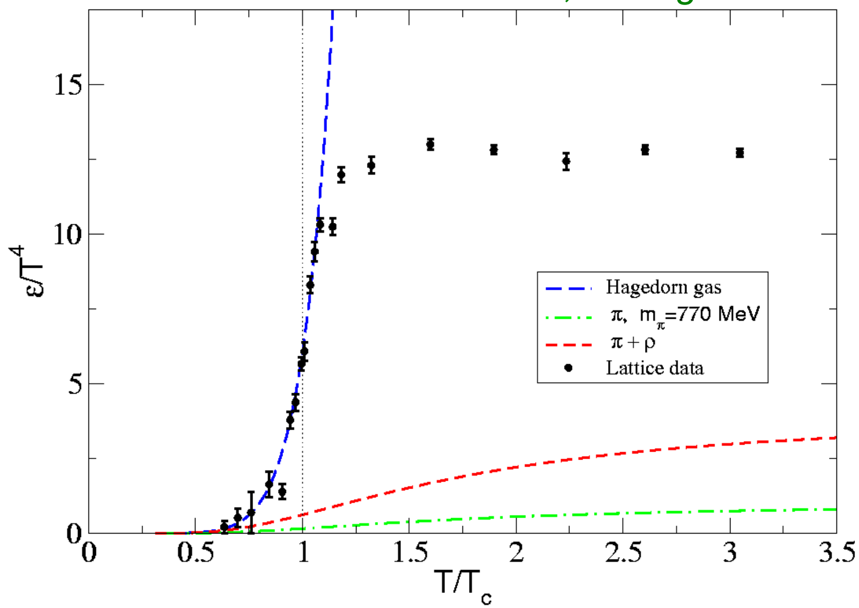
$$\begin{aligned} HS &\leftrightarrow n_1 \cdot \pi + n_2 \cdot K + n_3 \cdot \bar{K} \\ &\leftrightarrow B + \bar{B} \\ &\leftrightarrow B + \bar{B} + n_1 \cdot \pi + n_2 \cdot K + n_3 \cdot \bar{K} \end{aligned}$$

The last multi-particle decay will dominate over direct $B\bar{B}$ production.

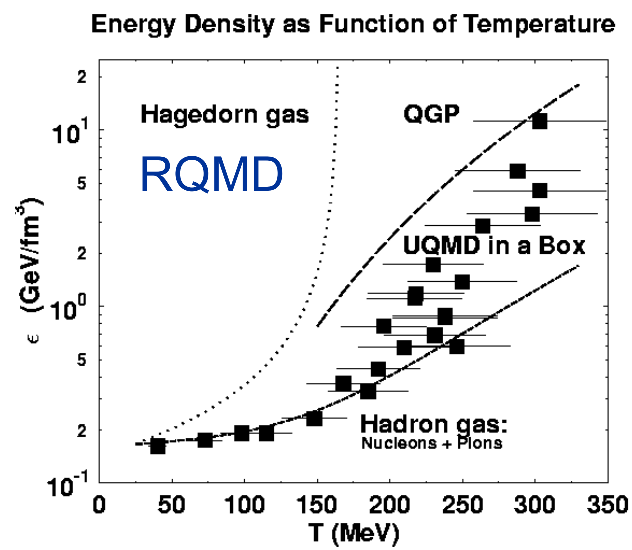
Hagedorn gas close to T_{critical}

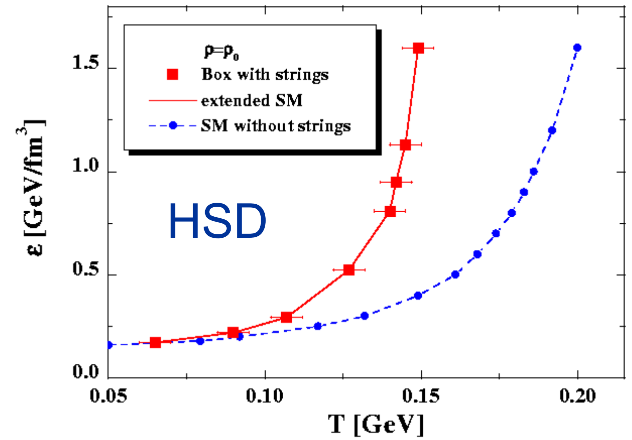
- Hagedorn spectrum: $\rho_{HS} \sim m^{-a} \exp[m/T_H]$

K. Redlich et al, K. Bugaev et al

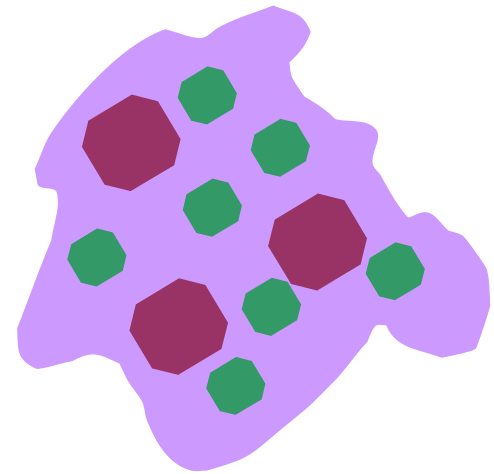


- Hagedorn like excitations in transport models:





Estimate for baryon/antibaryon production



- properties of HS

$M_{HS} \approx 3-6\ \text{GeV}$

$\Delta\epsilon_{BSW} \equiv \epsilon_{HS} \approx 0.3-0.5\,[GeV/fm^3]$

$\Rightarrow\ n_{HS} \approx \frac{\Delta\epsilon_c}{<M_{HS}>} \approx 0.05-0.15\,[fm^{-3}]$

- $HS \rightarrow B\bar{B} + X$ (eg $n\cdot\pi$)

  assume: $\Gamma_{HS\rightarrow B\bar{B}+X} \approx 100\,MeV$;

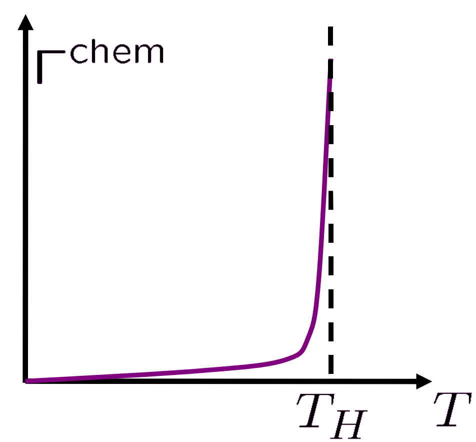
  $\Gamma^{tot}_{HS} \approx 0.5-1\,GeV$

$\Rightarrow\ \frac{dN_{B\bar{B}}}{d^4x} = \Gamma^{prod}_{B\bar{B}} = n_{HS}\cdot\Gamma_{HS\rightarrow B\bar{B}+X} \approx 0.05\,[fm^{-4}]$

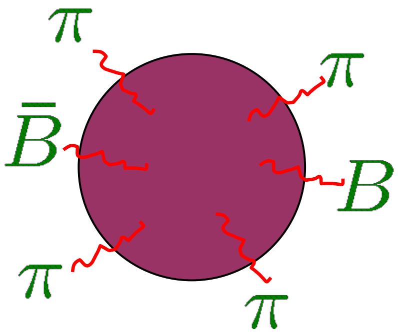
with $n^{RHIC}_{\bar{B}} \approx \underline{0.04\,[fm^{-3}]}$ one has

$\Gamma^{chem}_{B\bar{B}} = \frac{\Gamma^{prod}_{B\bar{B}}}{n^{RHIC}_{\bar{B}}} \approx 1.25\,fm^{-1}$

$\Rightarrow\ \boxed{\tau^{chem}_{B\bar{B}} = 0.8\,fm/c}$



Microcanonical decay of HS



(Fuming Liu)

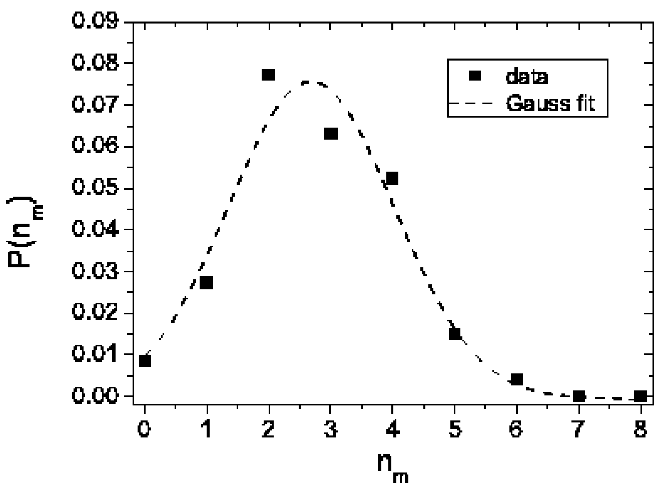
- (anti-)baryon decay probability

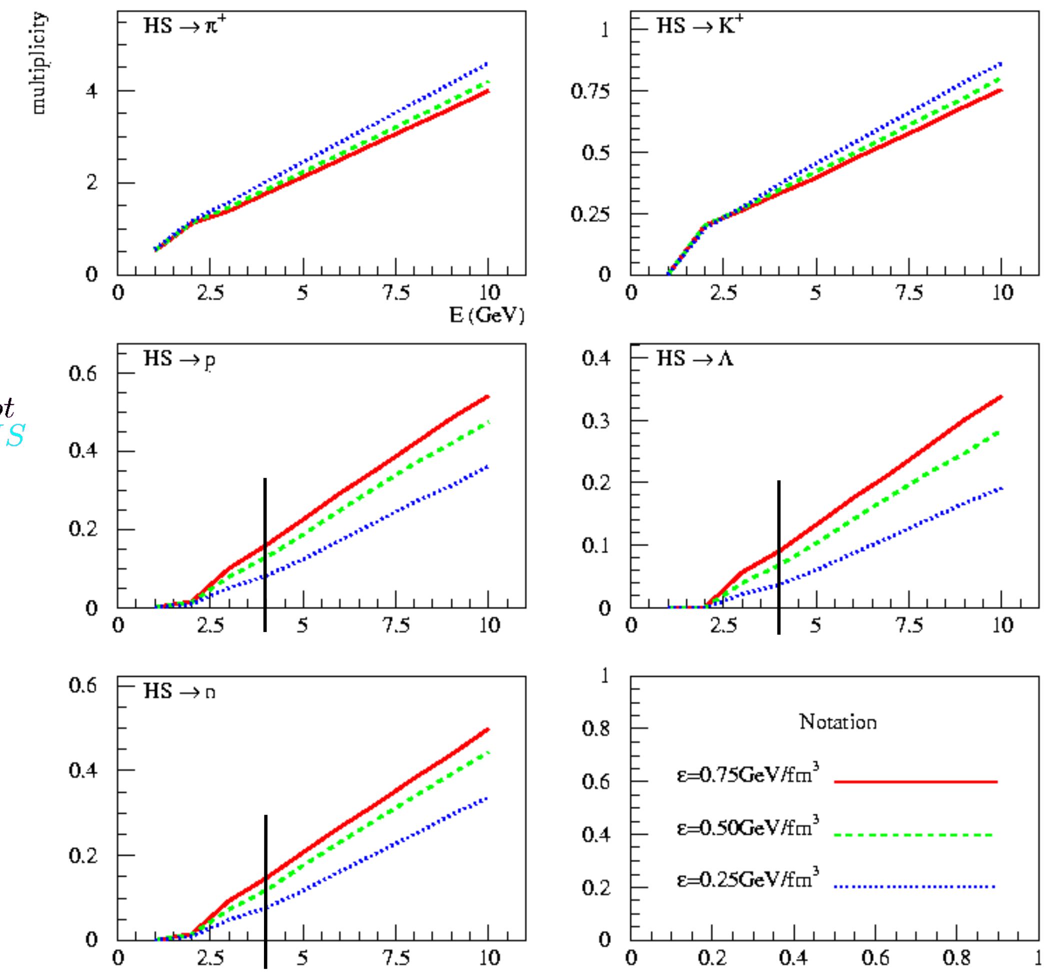
$$HS \leftrightarrow B + \bar{B} + X$$

$$\Gamma_{B\bar{B}X} \approx \langle B \rangle \cdot \Gamma_{HS}^{tot} \approx 0.2 - 0.4 \cdot \Gamma_{HS}^{tot}$$

$$\approx \boxed{100 - 400\, MeV}$$

- distribution of $X \equiv n_m$ mesons





$m_{HS} = 4$ GeV

Rate Equations

$$n\pi \leftrightarrow HS \leftrightarrow n'\pi + B\bar{B}$$

J. Noronha-Hostler, C. Greiner, I. A. Shovkovy, PRL 100:252301, 2008.

$$\begin{aligned}
\dot{\lambda}_i &= \Gamma_{i,\pi}\left(\sum_{n=2}^{\infty} B_{i,n}\lambda_\pi^n - \lambda_i\right) + \Gamma_{i,B\bar{B}}\left(\lambda_\pi^{\langle n_{i,b}\rangle}\lambda_{B\bar{B}}^2 - \lambda_i\right), \\
\dot{\lambda}_\pi &= \sum_i \Gamma_{i,\pi}\frac{N_i^{eq}}{N_\pi^{eq}}\left(\lambda_i\langle n_i\rangle - \sum_{n=2}^{\infty} B_{i,n} n \lambda_\pi^n\right) \\
&+ \sum_i \Gamma_{i,B\bar{B}}\langle n_{i,b}\rangle\frac{N_i^{eq}}{N_\pi^{eq}}\left(\lambda_i - \lambda_\pi^{\langle n_{i,b}\rangle}\lambda_{B\bar{B}}^2\right), \\
\dot{\lambda}_{B\bar{B}} &= \sum_i \Gamma_{i,B\bar{B}}\frac{N_i^{eq}}{N_{B\bar{B}}^{eq}}\left(\lambda_i - \lambda_\pi^{\langle n_{i,b}\rangle}\lambda_{B\bar{B}}^2\right)
\end{aligned}$$

$\lambda = \dfrac{N}{N^{eq}}$ where N is the total number of particles

Decay Widths

- Linear fit (PDG) $\Gamma_i = 0.15 m_i - 58 = 250 - 1000 \text{ MeV}$

- Baryon anti-baryon decay (microcanonical)

$$\Gamma_{i,B\bar{B}} = \langle B \rangle \Gamma_i$$

$$\Gamma_{i,\pi} = \Gamma_i - \Gamma_{i,B\bar{B}}$$

Where the average baryon number is

$$\langle B \rangle \approx 0.06 \text{ to } 0.4$$

- Analogously for kaon anti-kaon pairs, i.e.

$$\Gamma_{i,K\bar{K}} = \langle K \rangle \Gamma_i$$

$$\langle K \rangle = 0.4 \text{ to } 0.5$$

Time Scale Estimate

Assuming $N_\pi \approx N_\pi^{eq}$ and $N_i \approx N_i^{eq}$

$$\dot{\lambda}_{B\bar{B}} = \sum_i \Gamma_{i,B\bar{B}} \frac{N_i^{eq}}{N_{B\bar{B}}^{eq}} \left(\lambda_i - \lambda_\pi^{\langle n_{i,b} \rangle} \lambda_{B\bar{B}}^2\right)$$

$$= \sum_i \Gamma_{i,B\bar{B}} \frac{N_i^{eq}}{N_{B\bar{B}}^{eq}} \left(1 - \lambda_{B\bar{B}}^2\right)$$

$$\lambda_{B\bar{B}} = \frac{\left(\frac{\phi+1}{\phi-1}\right) \exp\left(\frac{2t}{\tau_{B\bar{B}}}\right) + 1}{\left(\frac{\phi+1}{\phi-1}\right) \exp\left(\frac{2t}{\tau_{B\bar{B}}}\right) - 1}$$

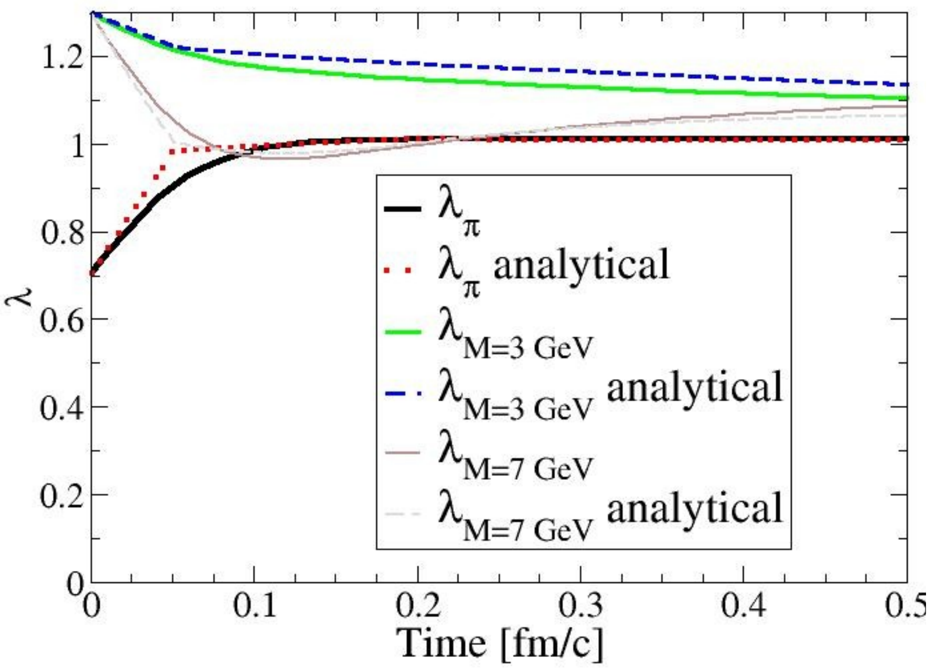
where $\phi := \lambda_{B\bar{B}}(0)$ and

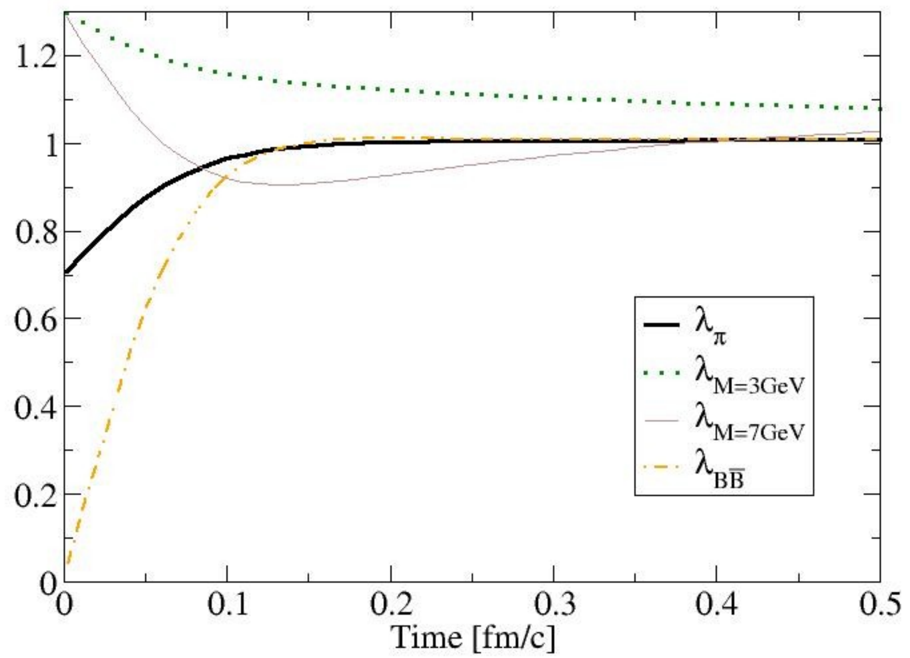
$$\boxed{\tau_{B\bar{B}} := \frac{N_{B\bar{B}}^{eq}}{\sum_i \Gamma_{i,B\bar{B}} N_i^{eq}} = 0.2 - 0.7 \frac{fm}{c}}$$

Results: Analytical vs. Numerical Results

$\lambda_\pi(0) = 0.7$ $\lambda_i(0) = 1.3$ $\lambda_{B\bar{B}}(0) = 0$

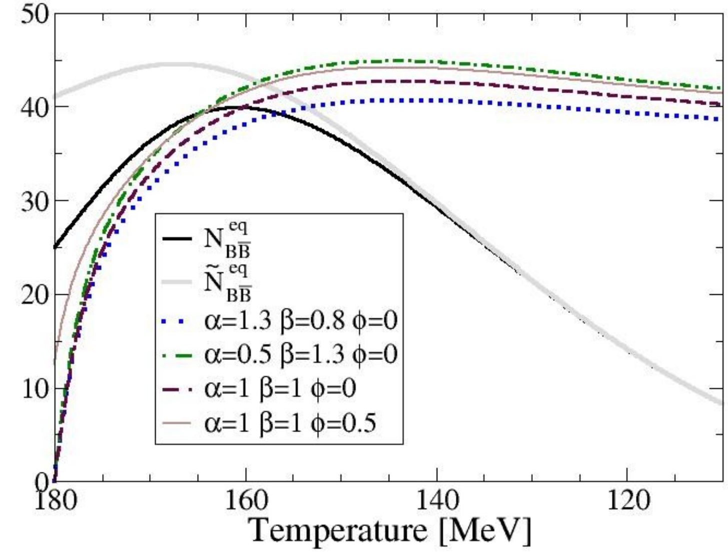
$T = 175\,\mathrm{MeV}$

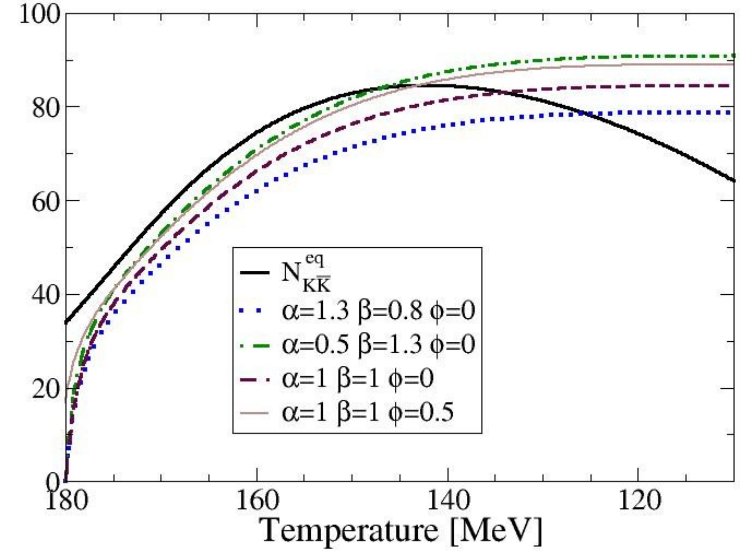


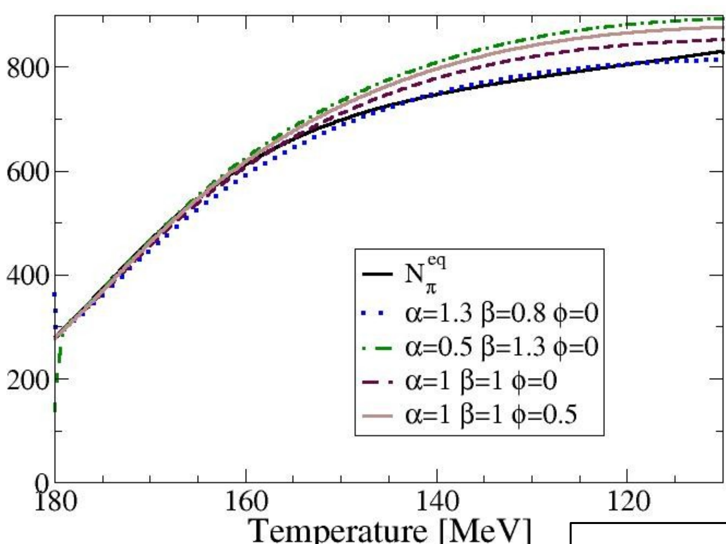


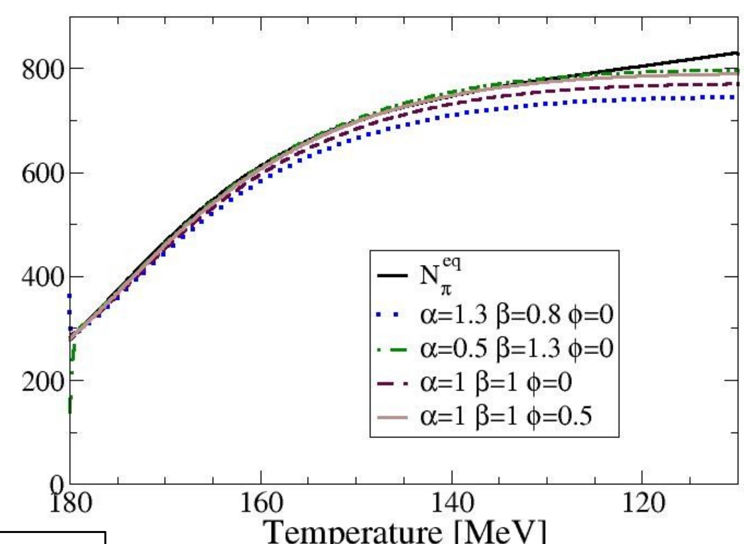
Results: Expansion

Varying IC's has only small effect!





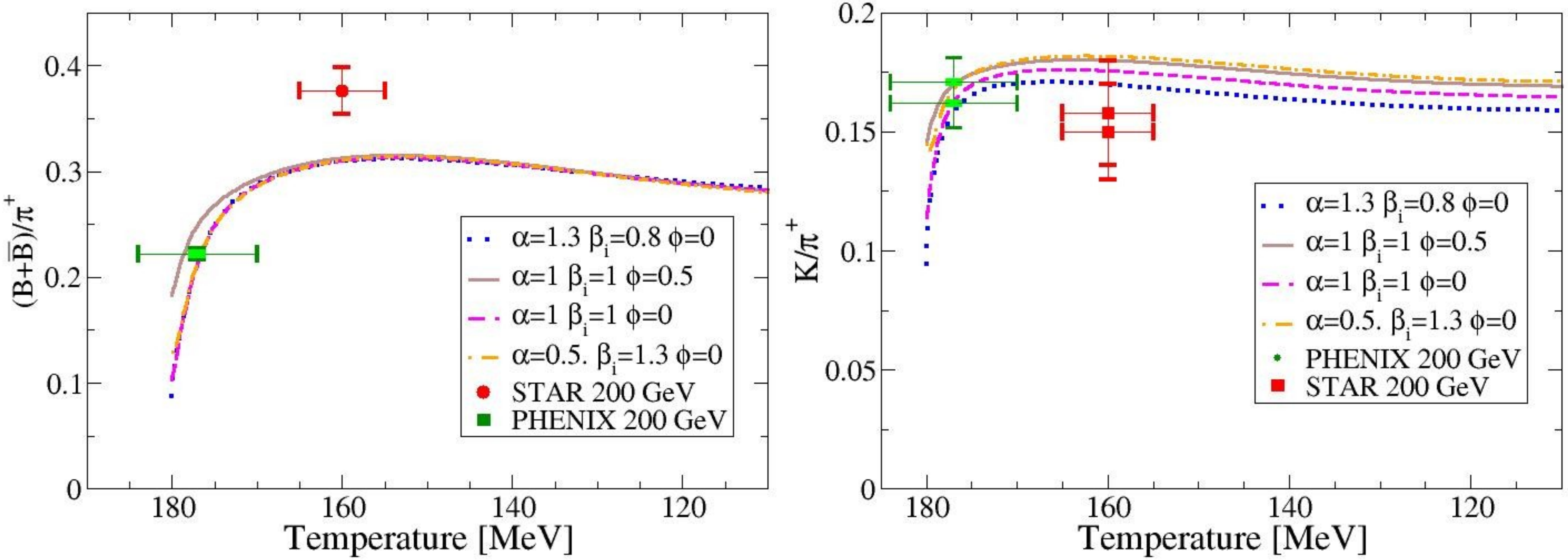




$$\Delta t = 2 - 3 \ \frac{fm}{c}$$

J. Noronha-Hostler, C. Greiner, I. A. Shovkovy, PRL 100:252301, 2008.

Results: Particle Ratios



•J. Noronha-Hostler, C. Greiner, I. A. Shovkovy, PRL 100:252301, 2008.

•STAR DATA: O. Y. Barannikova [STAR Collaboration], arXiv:nuclex/0403014; J. Adams et al. [STAR Collaboration],Nucl. Phys. A 757, 102 (2005).

•PHENIX DATA: S. S. Adler et al. [PHENIX Collaboration], Phys. Rev. C 69, 034909 (2004).

Shear Viscosity

$$\boxed{\left(\frac{\eta}{s}\right)_{tot}} = \frac{\eta_{HG}+\eta_{HS}}{s_{HG}+s_{HS}}$$
$$\boxed{= \frac{s_{HG}}{s_{HG}+s_{HS}}\left[\left(\frac{\eta}{s}\right)_{HG}+\frac{\eta_{HS}}{s_{HG}}\right].}$$

Kinetic Theory:

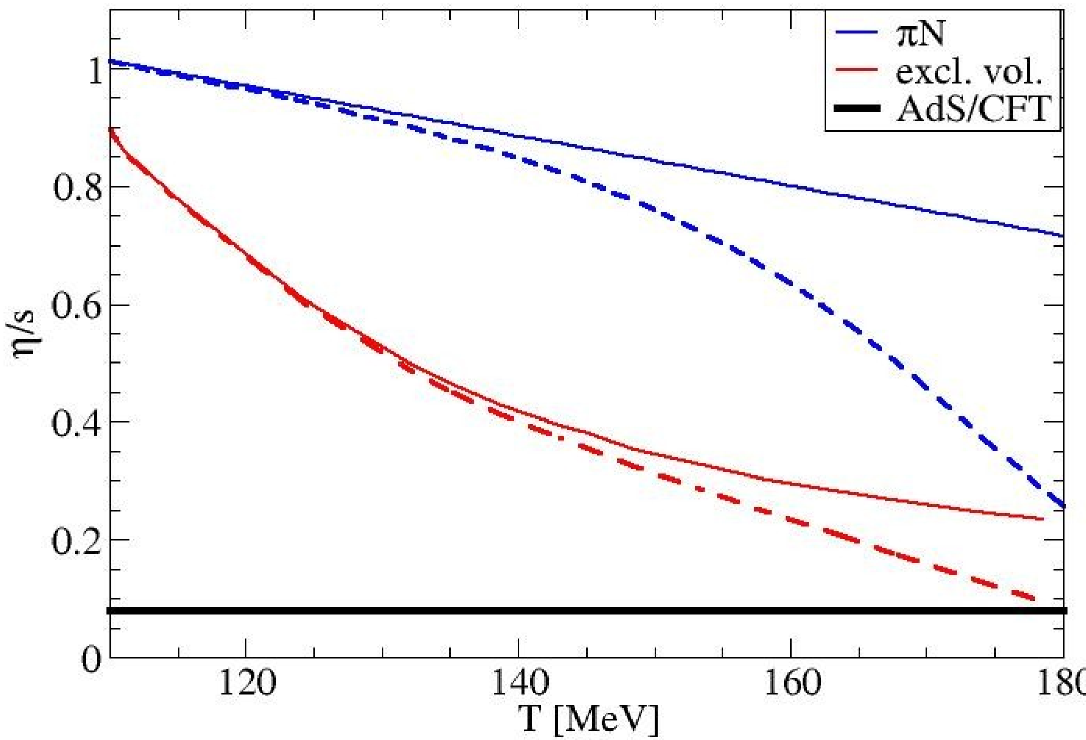
$$\eta = \frac{1}{3} mn\langle v_z\rangle\lambda$$

-non-relativistic particles

HS: $$\eta = \frac{1}{3}\sum_i m_i n_i \langle v_z\rangle_i \lambda_i$$

where $$\langle v_z\rangle_i = \sqrt{\frac{T}{m_i}}$$

$$\lambda_i = \tau_i\langle v_i\rangle = \tau_i\sqrt{\frac{3T}{m_i}}$$



$$\Rightarrow \quad \eta_{HS} = \frac{\sqrt{3}}{3}\sum_i n_i\tau_i T$$

J. Noronha-Hostler, CG, J. Noronha

$$c_s^2 = \frac{dp}{d\varepsilon}$$

Bulk Viscosity

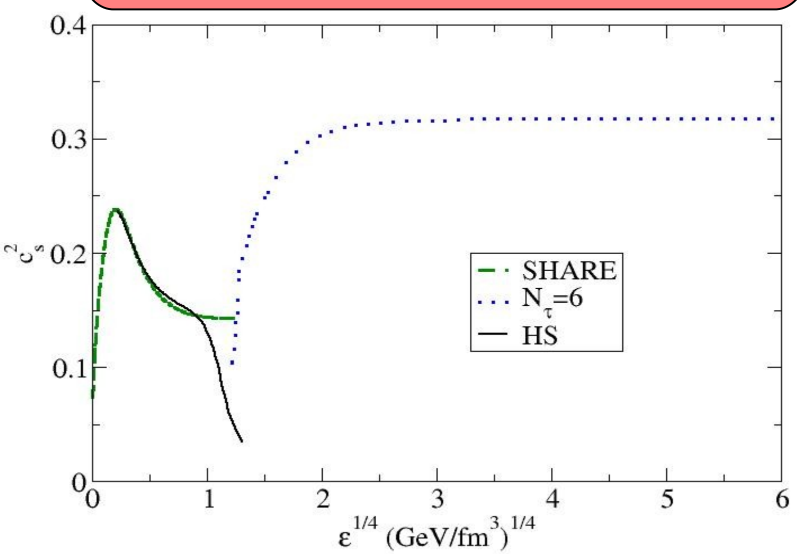
M. Cheng et al., Phys. Rev. D 77, 014511 (2008). (Lattice Results)

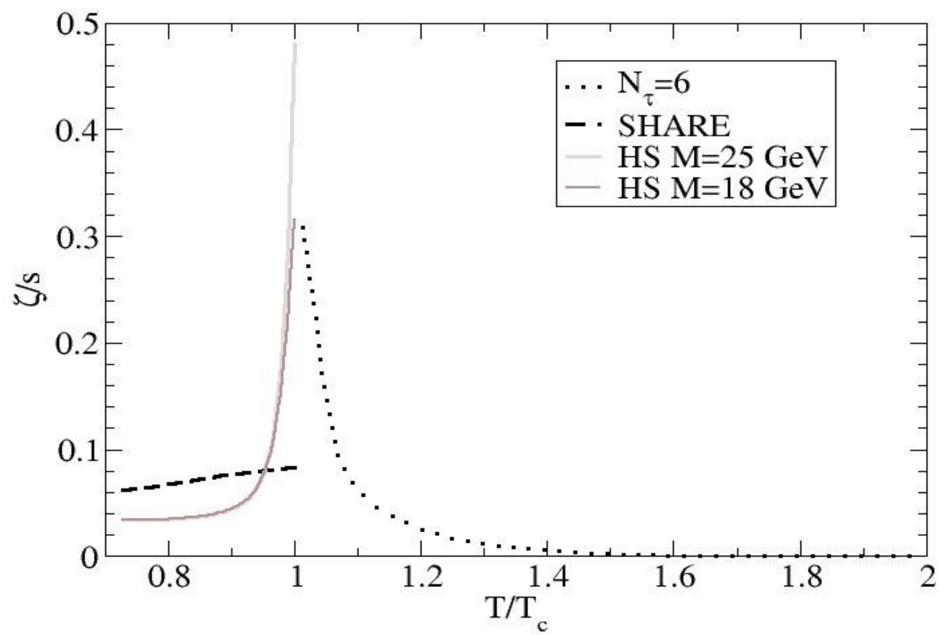
$\omega_0 \approx T$

From F. Karsch et al., Phys. Lett. B 663, 217 (2008).

$$9\omega_0\xi = Ts\left(\frac{1}{c_s^2} - 3\right) - 4(\varepsilon - 3p) + \left(T\frac{\partial}{\partial T} - 2\right)\langle m\bar{q}q\rangle^* + 16|\epsilon_v| + 6\left(m_\pi^2 f_\pi^2 + m_K^2 f_K^2\right)$$

$$16|\epsilon_v|\left(1 + \frac{3}{8}1.6\right) \approx (560MeV)^4$$





The strange sector of baryons/antibaryons

- Koch, Müller, Rafelski : 2-part. Xsections 'too' small

strangeness prod. : $\pi + N \leftrightarrow K + Y$

$$< \sigma_{\pi N} v > \approx 0.1 - 0.3\, mb$$

$$\tau_Y^{chem} \approx \left[< \sigma_{\pi N} v > \frac{n_{mes} n_B}{n_Y} \right]^{-1} \gtrsim \underline{12\, fm/c}$$

strangeness exchange: $\bar{K} + N \leftrightarrow \pi + Y$

$$< \sigma_{KN} v > \approx 1 - 3\, mb$$

$$\tau_Y^{chem} \approx \left[< \sigma_{KN} v > \frac{n_B n_K}{n_Y} \right]^{-1} \gtrsim \underline{15\, fm/c}$$

- $HS_{nonstrange} \rightarrow \Omega + \bar{B} + X$:

with $< \Omega > \equiv p_\Omega \approx 2 \cdot 10^{-4}$ $(p_\Lambda \approx 0.075)$,

$$\Gamma_\Omega^{prod} \approx 0.1 - 0.2\, MeV$$

$$n_\Omega \approx 2 \cdot 10^{-4} [fm^{-3}]$$

$$\Rightarrow \Gamma_\Omega^{chem} = \Gamma_\Omega^{prod} \frac{n_{HS}}{n_\Omega}$$

$$\approx 50 - 100\, MeV$$

$$\Rightarrow \tau_\Omega \approx 2 - 4\, fm/c$$

- $HS(sss\bar{q}\bar{q}\bar{q}) \rightarrow \Omega + \bar{B} + X$:

with $< \Omega > \equiv p_\Omega \approx 0.05$,

$$\Gamma_\Omega^{prod} \approx 25 - 50\, MeV$$

$$\Rightarrow \Gamma_\Omega^{chem} = \Gamma_\Omega^{prod} \frac{n_{HS}(sss)}{n_\Omega}$$

$$\approx 50 - 100\, MeV$$

$$\Rightarrow \tau_\Omega \approx 2 - 4\, fm/c$$

$$\frac{n_{HS}}{n_B} \approx 2.5$$

Importance of baryonic HS $\leftrightarrow$ CBM?

- At $\mu_B = 0$ one might estimate:

$$\frac{n_{BHS}}{n_{MHS}} \approx \frac{n_B}{n_M} \approx \frac{0.04\,[fm^{-3}]}{0.3\,[fm^{-3}]} \approx 0.13$$

... can be relevant at larger $\mu_B > 0$!

- Consider strange (anti-)baryons

(1) $\Omega HS \leftrightarrow B\,(\neq \Omega) + X$ (2) $\Omega HS \leftrightarrow \Omega + X$

$\rightarrow$ microcanonical decay estimate for $p_{(1)}$ and $p_{(2)}$

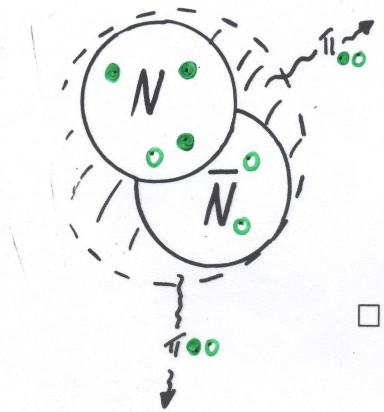
(1) $$\left(\tau^{chem}_{\Omega HS}\right)^{-1} = \Gamma_{\Omega HS \leftrightarrow B+X}$$
$$\approx p_{(1)} \cdot \Gamma^{tot} = 0.9 \cdot \Gamma^{tot}$$
$$\boxed{\approx 0.4 - 0.8\,GeV}$$

(2) $$\left(\tau^{chem}_{\Omega}\right)^{-1} = \Gamma_{\Omega HS \leftrightarrow \Omega + X}\,\frac{n_{\Omega HS}}{n_\Omega}$$
$$= 0.1 \cdot \Gamma^{tot}\,\frac{n_{\Omega HS}}{n_\Omega}$$
$$\boxed{\rightarrow \quad \tau_\Omega \lesssim 2 - 4\,fm/c}$$

$$\mathcal{O}(1) \leftrightarrow \frac{n_B}{n_p} \approx 4$$

"Duality" aspects of dense matter

HADRONIC WORLD



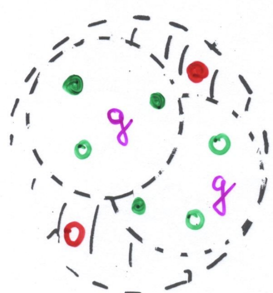
$$R_{eff} \approx 1.2\,\mathrm{fm}$$

$$\left(\pi R_{eff}^2 \approx 50\,\mathrm{mbarn}\right)$$

$$\epsilon_{eff} \sim \frac{2m_N}{4/3\pi R_{eff}^3} \approx \underline{0.3\,\mathrm{GeV/fm}^3}$$

- ☐ *multimesonic fusion processes* are effective for $\underline{\rho_B > 0.5\rho_0}$ and $\underline{\epsilon > 0.4\ \mathrm{GeV/fm}^3}$ at SPS energies

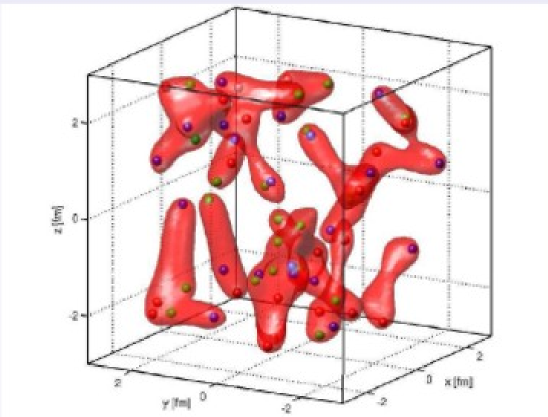
QUARK GLUON PLASMA



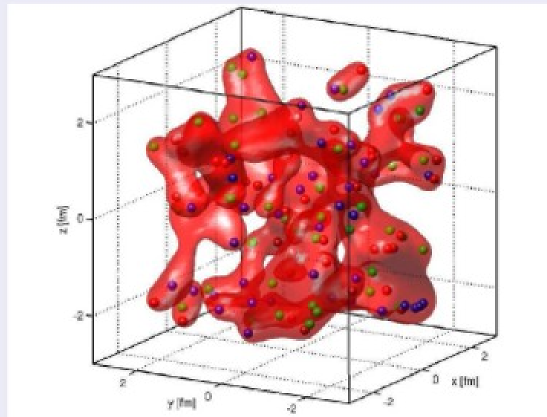
- ☐ coalescence production during hadronisation ?
- ☐ *non*perturbative phenomena

Deconfinement: transition to quark phase

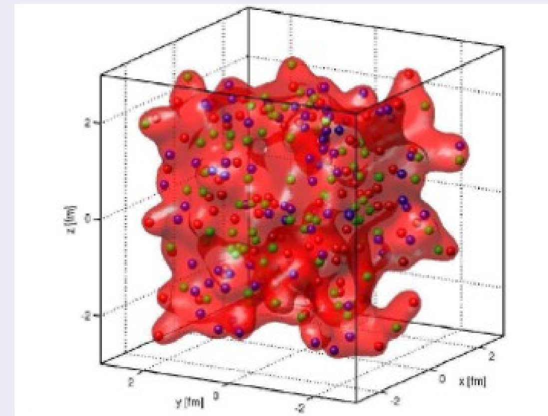
$n = 0.5\text{fm}^{-3}$



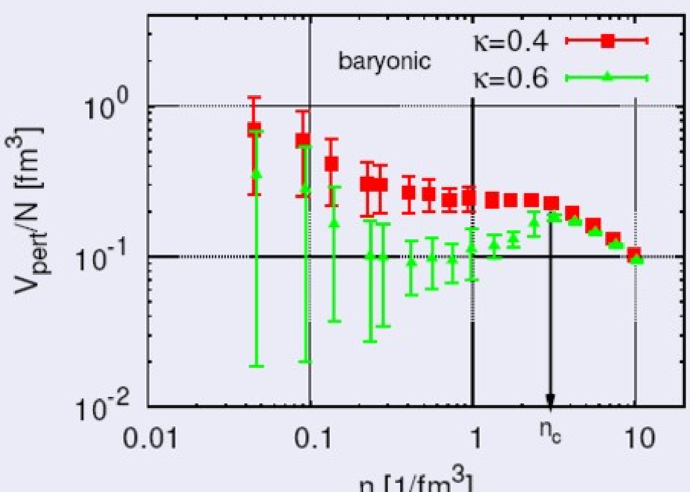
$n = 1.0\text{fm}^{-3}$



$n = 2.0\text{fm}^{-3}$



bag volume/particle



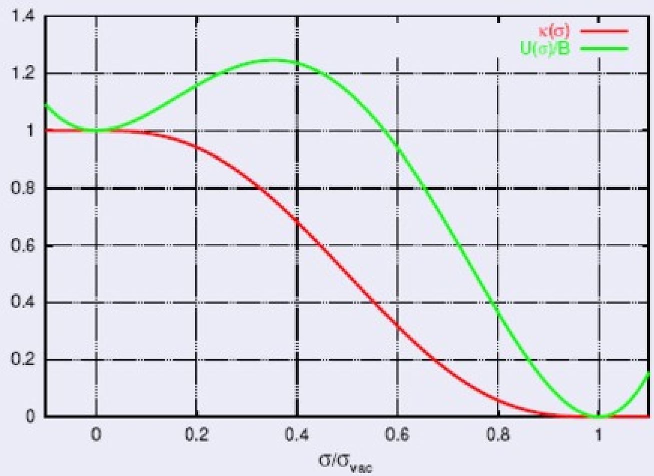
- formation of color neutral clusters at small densities
- particle number/cluster rises
- critical density at maximal overlap ($n \approx 2\text{fm}^{-3}$ or $\varepsilon \approx 1.1\text{GeV}/\text{fm}^3$)
- percolation transition

A model for QCD

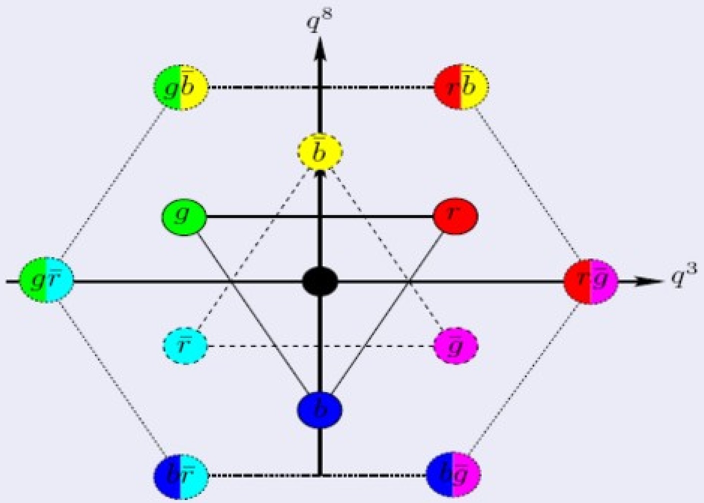
Chromodielectric model (CDM) [G.Martens et.al., Phys.Rev. D70/D73]

$$\begin{aligned}
\mathcal{L}_{\text{cdm}} &= \tfrac{1}{4}\kappa(\sigma)F^{a}_{\mu\nu}F^{\mu\nu,a} && \mathcal{L}_{glue} \\
&\quad -g\, j^{a}_{\mu} A^{\mu,a} && \mathcal{L}_{q,g} \\
&\quad +\tfrac{1}{2}\partial_{\mu}\sigma\partial^{\mu}\sigma \;-\; U(\sigma) && \mathcal{L}_{\sigma} \\
F^{\mu\nu,a} &= \partial^{\mu}A^{\nu,a} - \partial^{\nu}A^{\mu,a} && a \in \{3,8\}
\end{aligned}$$

Self interaction & Dielectric

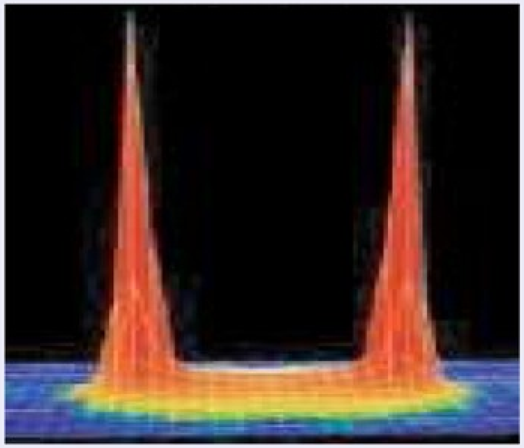


Color multipletts

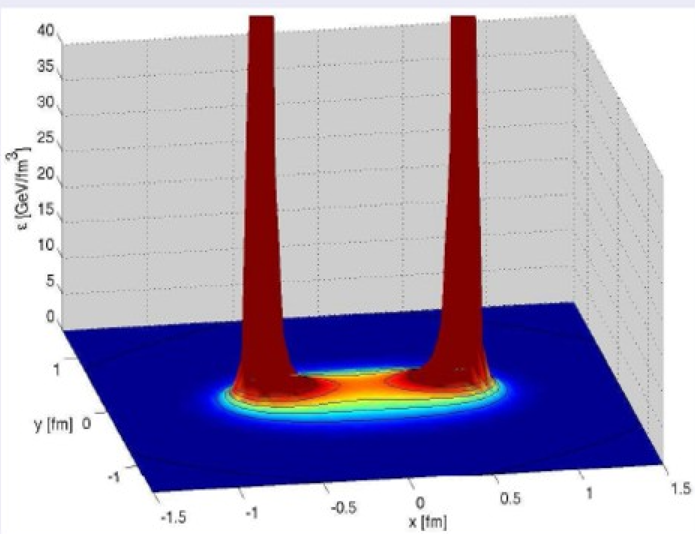


$q\bar{q}$-strings in:

lattice QCD [G.Bali, Phys.Rep. 343]

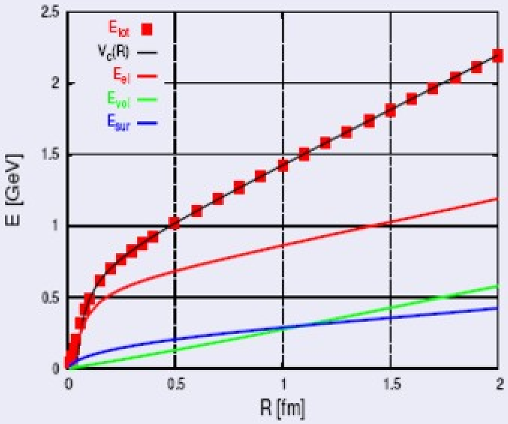


CDM

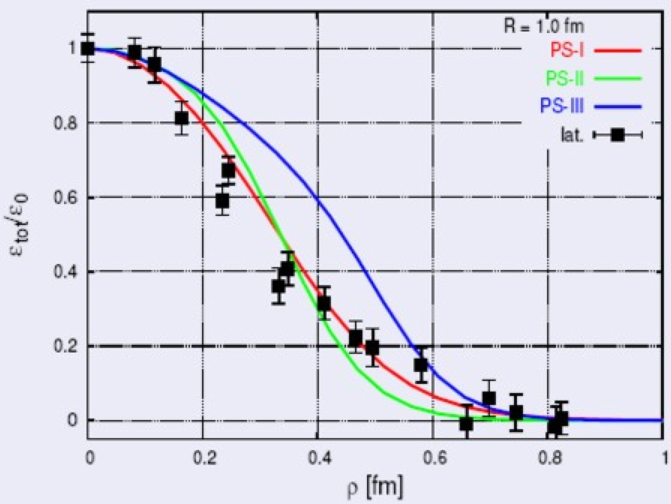


fixing the model on

$q\bar{q}$-potential

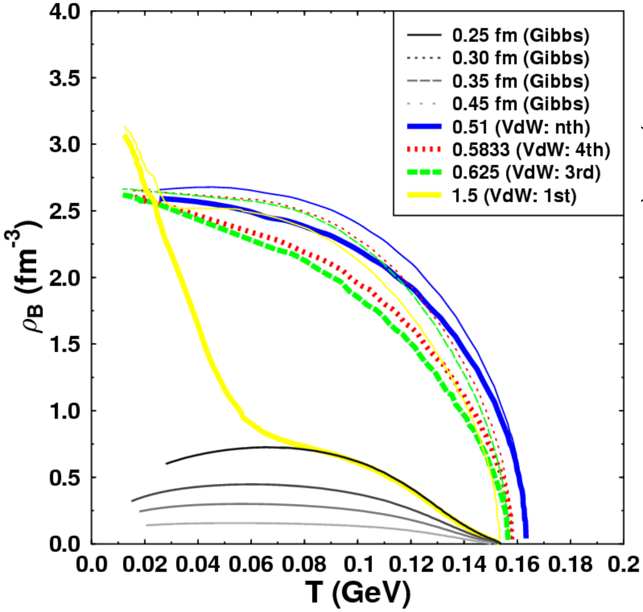


$q\bar{q}$ transverse shape



The order and shape of QGP phase transition

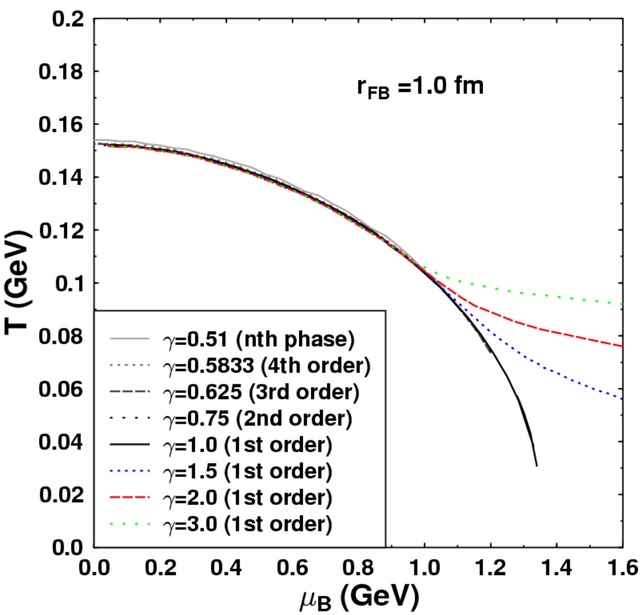
I. Zakout, CG and J. Schaffner-Bielich, NPA 781 (2007) 150

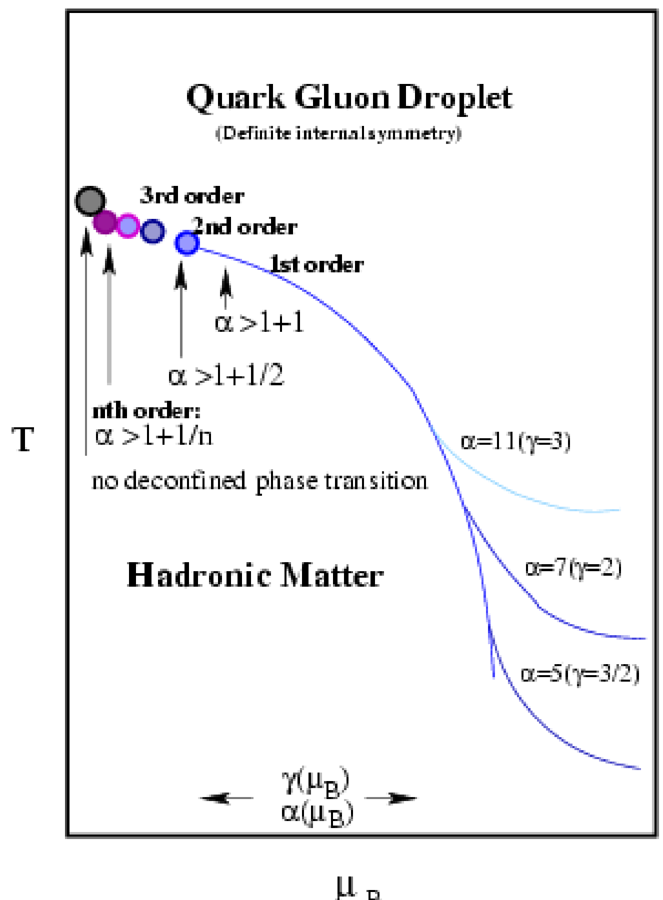


$$\gamma = \frac{\alpha + 1}{4}$$

density of states:

$$\rho(m, v) \sim c\, m^{-(\alpha+2)}\, e^{\frac{m}{T_H[B]}}\, \delta(m - 4Bv)$$





$$\alpha(\mu_B)$$

Conclusions and Outlook

- Potential Hagedorn states as additional dof can explain $B\bar{B}$ and also strange baryon production close to T_H ; (re-)population and decay are governed by detailed balance

- Three main assumptions:

  (1): $\Delta\epsilon_{HS} \approx 0.3 - 0.5\, GeV/fm^3$

  (2): $\Gamma_{HS}^{tot} \approx 0.5 - 1\, GeV$

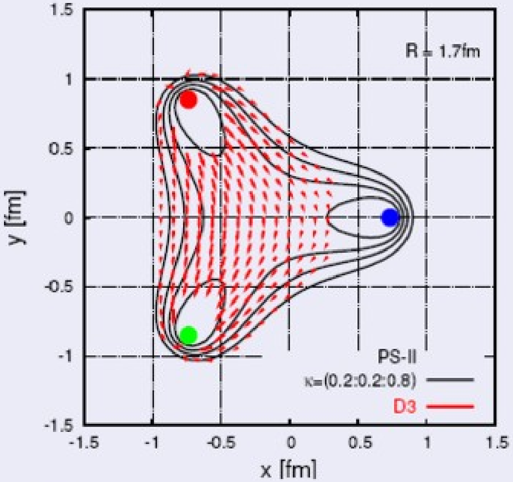
  (3): microcanonical statistical decay

- Future: Embedding into UrQMD

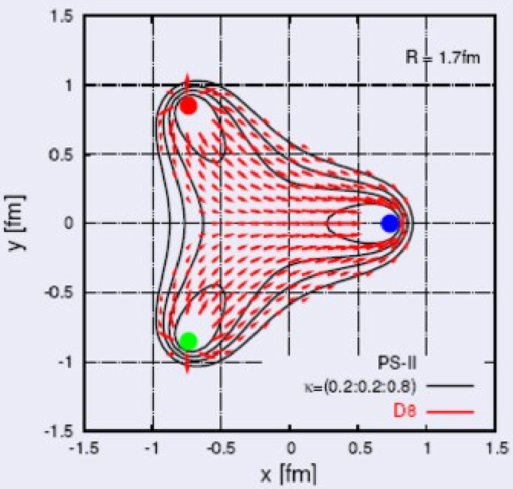
qqq-baryons

electric fields

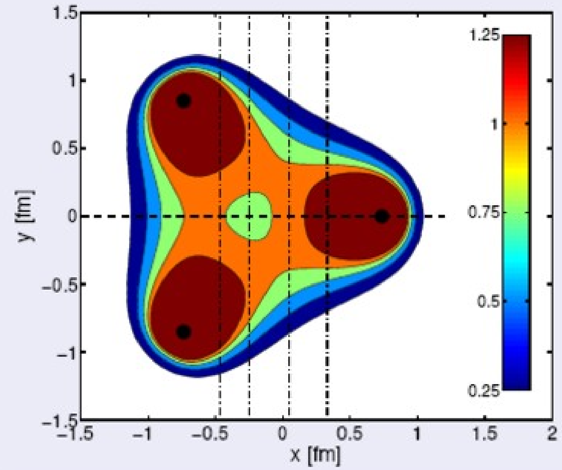
D^3



D^8



energy density



- non-trivial structure of electric fields
- Y-like flux tubes for large baryons
- color blind energy density