Thermodynamics of a Nonlocal PNJL Model for Two and Three Flavors

Thomas Hell,
Simon Rößner, Marco Cristoforetti, and Wolfram Weise

Physik Department
Technische Universität München

INT Workshop
The QCD Critical Point
July 28—August 22, 2008

Outline

1. The Nonlocal Nambu–Jona-Lasinio Model . . .
   . . . abandoning the NJL cutoff
   - Effective Action of a Covariant Nonlocal Model for Two Flavors
   - Mean Field Approximation
   - Instantons vs. Perturbative QCD
   - Dynamical Quark Mass
2. Thermodynamics of the Nonlocal PNJL Model
   - Coupling Quarks and Polyakov Loop
   - Gap Equations in Mean-Field Approximation
   - Pressure and Mesonic Corrections
   - Finite Density Case and QCD Phase Diagram
3. The Three-Flavor Case
   - Effective Nonlocal Action and ’t Hooft Determinant
   - Thermodynamics
4. Summary and Conclusions

1. The Nonlocal Nambu–Jona-Lasinio Model

Formalism of the Nonlocal NJL Model

Nonlocal Action in Euclidean Space

$$\mathcal{S}_\mathrm{E} = \int \mathrm{d}^4x\, \bar\psi(x)(-\mathrm{i}\partial\!\!\!/ + m_q)\psi(x) - \frac{G}{2}\int \mathrm{d}^4x \int \mathrm{d}^4y\, j_a^\mu(x)\, C(x-y)\, j_\mu^a(y)$$

Color Currents

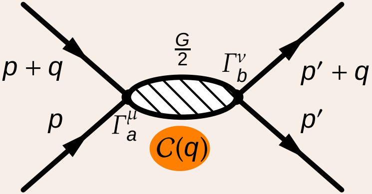
$$j_a^\mu(x) = \bar\psi(x)\gamma^\mu t_a \psi(x)$$

➡ Fierz transformation

$$j_a^\mu(x) = \bar\psi(x)\Gamma_a^\mu\psi(x)$$

$$\Gamma_a = (1, \mathrm{i}\gamma_5\vec\tau\,)$$

Effective Interaction



G. Ripka, *Quarks bound by chiral fields.* (1997) [1]

G. Dumm, D. Grundfeld and N. N. Scoccola, Phys. Rev. D (2006) [2]

Formalism of the Nonlocal NJL Model

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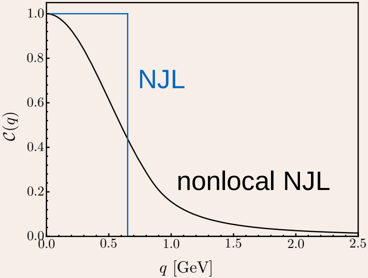
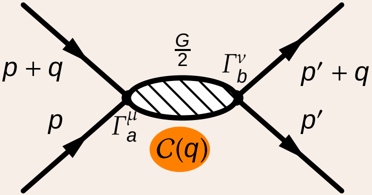
Distribution Function



Effective Interaction



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Improvements on the Standard NJL Model

- Nonlocality arises naturally in the instanton liquid model and in Dyson-Schwinger calculations [3, 4]
- Underlying Lagrangian preserves $SU_L(2) \otimes SU_R(2)$ chiral symmetry
- Axial anomalies are incorporated, current algebra relations are preserved [5]

$$f_\pi g_{\pi qq} = \bar{\sigma} + \mathcal{O}(m_q) \qquad \text{(Goldberger–Treiman)}$$

$$m_\pi^2 f_\pi^2 = -m_q \langle \bar{\psi}\psi \rangle + \mathcal{O}(m_q^2) \qquad \text{(Gell-Mann–Oakes–Renner)}$$

- No UV-cutoff is needed
- No unphysical decays (e. g. $\pi \to \bar{q}q$)

Bosonization of the Euclidean Action

Bosonized Euclidean Action

Introduction of bosonic fields $\varphi_a(x) = (\sigma(x), \vec{\pi}(x))$

$$Z = \int \mathscr{D}\bar{\psi}\mathscr{D}\psi\, \mathrm{e}^{-\mathcal{S}_\mathrm{E}} \quad \rightarrow \quad Z = \int \mathscr{D}\sigma\mathscr{D}\vec{\pi}\, \mathrm{e}^{-\mathcal{S}_\mathrm{E}^\mathrm{bos}},$$

where

$$\mathcal{S}_\mathrm{E}^\mathrm{bos} = -\ln\det\hat{A} + \frac{1}{2G}\int\frac{\mathrm{d}^4 p}{(2\pi)^4}\phi_a(p)\phi_a^*(p)$$

with

$$A(p,p') = \left(-\not{p} + m_q\right)(2\pi)^4\delta^{(4)}(p-p') + C\left(\frac{p+p'}{2}\right)\Gamma_a\,\phi_a(p-p')$$

Power Series Expansion of the Action

Mean Field (MF) Approximation

Expansion of $\mathcal{S}_{\mathrm{E}}^{\mathrm{bos}}$ about MF-values $\langle\sigma\rangle_{\mathrm{MF}} = \bar{\sigma}$ and $\langle\vec{\pi}\rangle_{\mathrm{MF}} = \vec{0}$

$$\mathcal{S}_{\mathrm{E}}^{\mathrm{bos}} = \mathcal{S}_{\mathrm{E}}^{\mathrm{MF}} + \mathcal{S}_{\mathrm{E}}^{(2)} + \ldots$$

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$$\mathcal{S}_{\mathrm{E}}^{\mathrm{bos}} = \mathcal{S}_{\mathrm{E}}^{\mathrm{MF}} + \mathcal{S}_{\mathrm{E}}^{(2)} + \dots$$

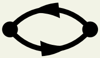
MF-Approximation of the Action

$$\frac{\mathcal{S}_{\mathrm{E}}^{\mathrm{MF}}}{V^{(4)}} = -2N_{\mathrm{f}}N_{\mathrm{c}} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \ln\left[p^2 + M^2(p)\right] + \frac{\bar{\sigma}^2}{2G},$$

with $M(p) = m_q + C(p)\bar{\sigma}$

Mesonic Fluctuations

$$\mathcal{S}_{\mathrm{E}}^{(2)} = \frac{1}{2}\int \frac{\mathrm{d}^4 p}{(2\pi)^4}\left[G^{+}(p^2)\,\delta\sigma(p)\,\delta\sigma(-p) + G^{-}(p^2)\,\delta\vec{\pi}(p)\cdot\delta\vec{\pi}(-p)\right],$$

$$G^{\pm}(p^2) = \frac{1}{G} - \Gamma^{\pm}$$  $$\Gamma^{\pm}$$

Determination of the Distribution Function C

Constituent Quark Mass in MF-Approximation

Constituent quark mass M given by pole of fermion propagator

$$0 = S_{\mathrm{F,\,nl}}^{-1}(p) = \left(\frac{\delta^2 \mathcal{S}_{\mathrm{E}}}{\delta\bar{\psi}(p)\,\delta\psi(0)}\right)_{p^2=-M^2} \Longleftrightarrow M(p^2) = m_q + \mathcal{C}(p)\bar{\sigma}$$

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Perturbative QCD

Comparison to QCD mass formula H. D. Politzer, Nucl. Phys. B (1976) [7]

$$\mathcal{C}(p^2) \propto \frac{2\pi}{3}\frac{\alpha_{\mathrm{s}}(p^2)}{p^2} \quad \text{for } p^2 \geq \Gamma^2$$

with

$$\alpha_{\mathrm{s}}(p^2) = \frac{4\pi}{\beta_0 \ln\left(\frac{p^2}{\Lambda_{\mathrm{QCD}}^2}\right)} + \ldots$$

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Non-perturbative QCD

Comparison to the Instanton Liquid Model

T. Schäfer, E. V. Shuryak, Rev. Mod. Phys (1998) [6]

$$C(p^2) \propto \mathrm{e}^{-p^2 d^2/2} \quad \text{for } p^2 < \Gamma^2$$

with the instanton size $d = 0.35\,\mathrm{fm} = (0.56\,\mathrm{GeV})^{-1}$

Perturbative QCD

Comparison to QCD mass formula H. D. Politzer, Nucl. Phys. B (1976) [7]

$$C(p^2) \propto \frac{2\pi}{3} \frac{\alpha_{\mathrm{s}}(p^2)}{p^2} \quad \text{for } p^2 \geq \Gamma^2$$

with

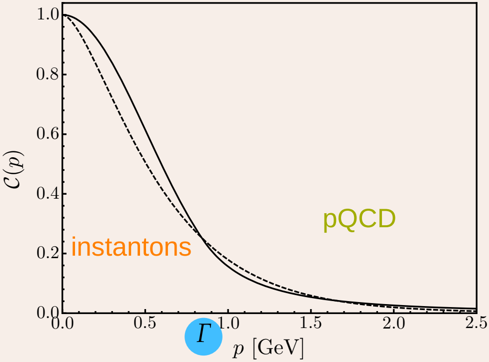
$$\alpha_{\mathrm{s}}(p^2) = \frac{4\pi}{\beta_0 \ln\left(\frac{p^2}{\Lambda^2_{\mathrm{QCD}}}\right)} + \dots$$

Determination of the Distribution Function C

Distribution Function

$$C(p^2) \propto \begin{cases} \frac{2\pi}{3}\frac{\alpha_s(p^2)}{p^2} & \text{for } p^2 \geq \Gamma^2 \\ e^{-p^2 d^2/2} & \text{for } p^2 < \Gamma^2 \end{cases}$$

Distribution Function C



Gap Equation and Constituent Quark Mass

- MF-value $\bar{\sigma}$ can be found by the principle of least action $\frac{\delta \mathcal{S}_{\mathrm{E}}^{\mathrm{MF}}}{\delta \sigma} = 0$

Gap Equation (MF-Approximation)

$$\bar{\sigma} = 4 N_{\mathrm{f}} N_{\mathrm{c}} G \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \mathcal{C}(p) \frac{M(p)}{p^2 + M^2(p)}$$

with $M(p) = m_q + \mathcal{C}(p)\bar{\sigma}$

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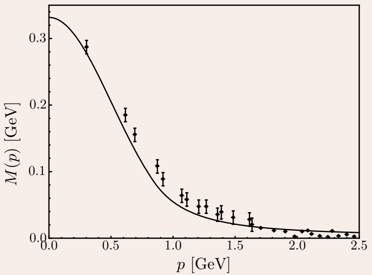
Lattice data from

P. O. Bowman *et al.*,
Nucl. Phys. Proc. Suppl. (2003) [8]

Constituent Quark Mass



2 Thermodynamics of the Nonlocal PNJL Model

Coupling Quarks and Polyakov Loop

Definition (Polyakov Loop)

$$\langle \Phi(\vec{x}\,) \rangle = \frac{1}{N_c} \langle \mathrm{tr}_c \left[L(\vec{x}\,) \right] \rangle$$

$$L(\vec{x}\,) = \mathcal{P} \exp \left\{ \mathrm{i} \int_0^\beta \mathrm{d}\tau \, A_4(\tau, \vec{x}\,) \right\}$$

- Polyakov Loop serves as an order parameter for confinement-deconfinement phase transition [9, 10]

$$\langle \Phi \rangle = \mathrm{e}^{-\beta \mathscr{F}_q}$$

Confinement-Deconfinement Phase Transition

$$\text{confinement} \Longleftrightarrow \langle \Phi \rangle = 0 \quad \Longleftrightarrow Z(3) \text{ unbroken}$$

$$\text{deconfinement} \Longleftrightarrow \langle \Phi \rangle \neq 0 \quad \Longleftrightarrow Z(3) \text{ spontaneously broken}$$

Coupling Quarks and Polyakov Loop

- Introduction of gluons through minimal coupling

$$p_\mu \to p_\mu + A_\mu$$

- 3- and 8-components of gluon fields; no spatial fluctuations

$$A_\mu = \delta_\mu^0 (A_0^3 \lambda_3 + A_0^8 \lambda_8)$$

- Integrate out non-diagonal components of $SU(3)_c$

Polyakov Potential

K. Fukushima, Phys. Lett. B (2004) [11]

$$\frac{\mathcal{U}}{T^4} = -\frac{1}{2} b_2(T) \Phi^* \Phi + b_4(T) \ln\left[1 - 6\Phi^*\Phi + 4\left(\Phi^{*3} + \Phi^3\right) - 3(\Phi^*\Phi)^2\right]$$

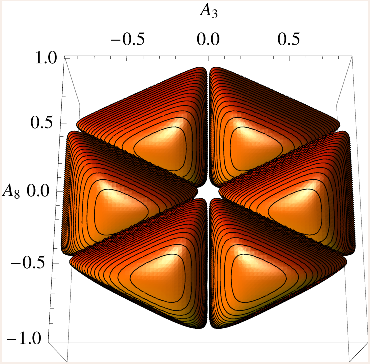
with $$\Phi = \tfrac{1}{3} \mathrm{tr}_c \left[\exp\left(\mathrm{i}\left(\beta A_3 \lambda_3 + \beta A_8 \lambda_8\right)\right)\right],$$

coefficients $b_2(T)$, $b_4(T)$ and parameters from

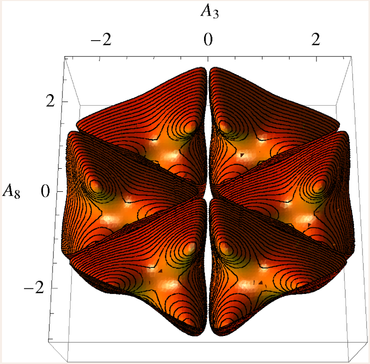
S. Rößner, C. Ratti and W. Weise, Phys. Rev. D (2007) [12]

Polyakov Potential

$T < T_c$



$T > T_c$



- Each minimum is “populated” equiprobably

$\Rightarrow$ Color neutrality preserved if one sums over all six minima!

Thermodynamic Potential

Partition Function

$$Z = \int \mathscr{D}\varphi \int \mathscr{D}A \, \mathrm{e}^{\ln \widetilde{\det} \tilde{A}} \exp\left\{ - \int_0^\beta \mathrm{d}\tau \int \mathrm{d}^3 x \, \frac{\varphi^2(x)}{2G} \right\}$$

Thermodynamic Potential

Partition Function

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Thermodynamic Potential

$$\Omega = -\frac{T}{V} \ln Z = -\ln \widetilde{\det}\left[\beta \tilde{S}^{-1}\right] + \frac{\bar{\sigma}^2}{2G} + \mathcal{U}(\Phi, \Phi^*, T)$$

with the Nambu-Gor'kov propagator in Matsubara space

$$S^{-1}(\mathrm{i}\omega_n, \vec{p}) = \begin{pmatrix} \mathrm{i}\omega_n\gamma_0 - \vec{\gamma}\cdot\vec{p} - \widetilde{M} - \mathrm{i}A_4\gamma_0 & 0 \\ 0 & \mathrm{i}\omega_n\gamma_0 - \vec{\gamma}\cdot\vec{p} - \widetilde{M}^* + \mathrm{i}A_4\gamma_0 \end{pmatrix}$$

- $\widetilde{M} = \mathrm{diag_c}(M(\omega_n^-, \vec{p}), M(\omega_n^+, \vec{p}), M(\omega_n^0, \vec{p}))$
- $\omega_n = (2n+1)\pi T$, $\omega_n^\pm = \omega_n \pm A_4, \omega_n^0 = \omega_n$

Gap Equations in MF Approximation

Gap Equations ($A_8^4 = 0$ S. Rößner, Diploma Thesis, Technische Universität München (2006) [13])

$$\frac{\partial\Omega}{\partial\bar{\sigma}} = \frac{\partial\Omega}{\partial A_3^4} = \frac{\partial\Omega}{\partial A_8^4} = 0$$

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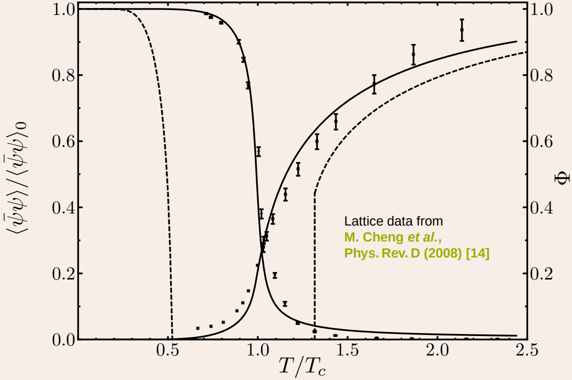
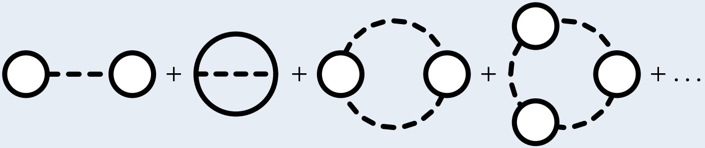
Chiral Condensate and Polyakov Loop ($T_c = 207\,\mathrm{MeV}$)



Pressure Contributions in Random Phase Approximation

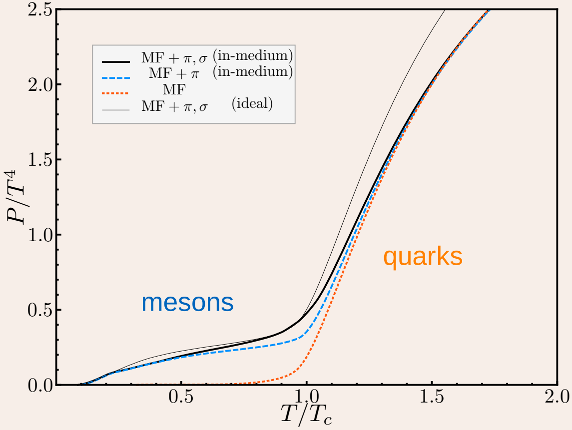
J. Hüfner, S. P. Klevansky, P. Zhuang, H. Voss, Annals Phys. (1994) [15]



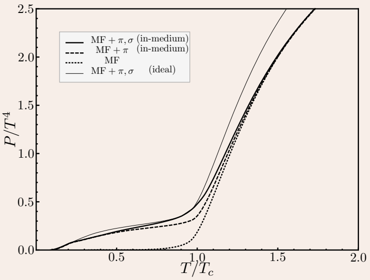
$$P_{\mathrm{MF}}(T) = 2N_{\mathrm{f}} \sum_{i=0,\pm} T \sum_{n\in\mathbb{Z}} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \ln\left[{\omega_n^i}^2 + \vec{p}^{\,2} + M(\omega_n^i, \vec{p}\,)^2\right] - \frac{\bar{\sigma}^2}{2G} - \mathcal{U}$$

$$P_{\mathrm{meson}}(T) = -\sum_{M=\pi,\sigma} \frac{d_M}{2} T \sum_{m\in\mathbb{Z}} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \ln\left[1 - G \;\; \nu_m, \vec{p} \;\;\right]$$

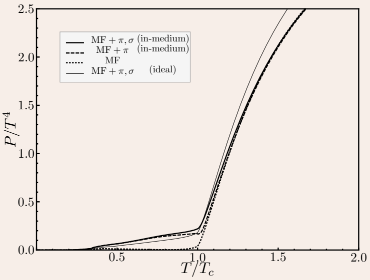
Pressure and Mesonic Contributions



$m_\pi = 140$ MeV



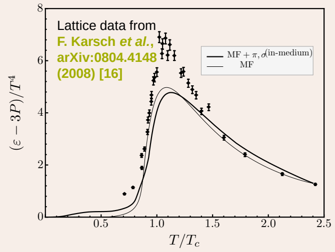
$m_\pi = 500$ MeV



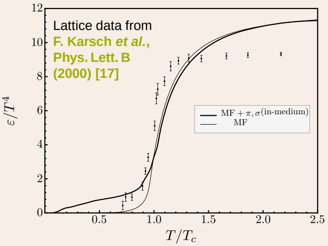
- Strong suppression of mesonic contributions for high quark masses

Related Thermodynamic Quantities

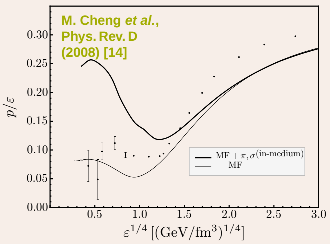
Trace anomaly



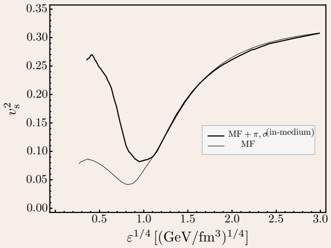
Energy density



p/ε



Sound velocity



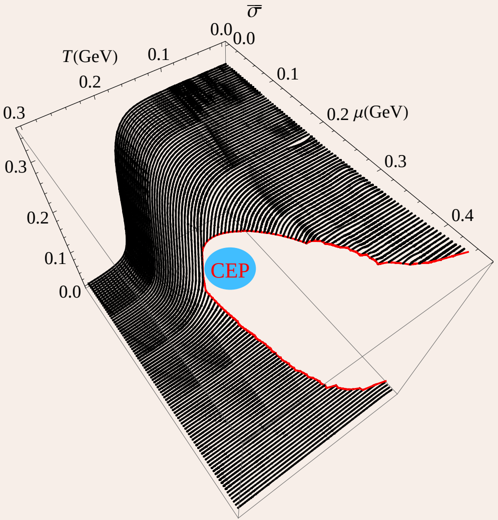
Finite Density and Phase Diagram

- Introduction of chemical potential through $\omega_n \to \omega_n - \mathrm{i}\mu + A_4$

Finite Density and Phase Diagram

- Introduction of chemical potential through $\omega_n \to \omega_n - \mathrm{i}\mu + A_4$

$\bar{\sigma}$ as a function of T and μ



- **Critical endpoint** is found at $(T_{\mathrm{CEP}}, \mu_{\mathrm{CEP}}) = (167\,\mathrm{MeV}, 175\,\mathrm{MeV})$

3. Nonlocal Quark Model for the 3-Flavor Case

Derivation of the Nonlocal Lagrangian

- Chirally invariant nonlocal two-flavor Lagrangian easily generalized to three flavors by the replacement $\tau_i \rightarrow \lambda_a$
- **Task:** Inclusion of **explicit** breaking of $\mathrm{U_A}(1)$ symmetry

Derivation of the Nonlocal Lagrangian

- Chirally invariant nonlocal two-flavor Lagrangian easily generalized to three flavors by the replacement $\tau_i \to \lambda_a$
- **Task:** Inclusion of **explicit** breaking of $U_A(1)$ symmetry

Solution: Instanton induced quark interactions T. Schäfer, E. V. Shuryak (1998) [6]

$$\mathcal{S}^{E}_{int} \propto \int d^4x \left(\det_f \mathcal{J}^+ + \det_f \mathcal{J}^- \right)$$

with

$$(\mathcal{J}^{\pm}(x))_{ij} = \int d^4y \, d^4z \, \psi_i^{\dagger}(y) \, \frac{1}{2} (1 \mp \gamma_5) \mathcal{K}(y - x, z - x) \psi_j(z)$$

- $\mathcal{S}^{E}_{int}$ invariant under $SU(3)_V \otimes SU(3)_A \otimes U(1)_V$ transformations
- **'t Hooft determinant not** invariant under $U(1)_A$ transformations

$\Rightarrow$ $U(1)_A$ problem is solved describing the η' mass correctly

Bosonized Euclidean Action N. N. Scoccola, (2004) [19]

- Introduction of 9 scalar and pseudoscalar fields $\sigma_a(x)$ and $\pi_a(x)$
- Introduction of $9 + 9 = 18$ **auxiliary fields** $\Sigma_a(x)$ and $\Pi_a(x)$

Bosonization G. Ripka, (1997) [1]; B. O. Kerbikov and Yu. A. Simonov (1995) [18]

$$
\begin{aligned}
\mathcal{S}^{\mathrm{E}}_{\mathrm{bos}} = &- \ln \det \Big\{ (-\not{p} + \hat{m}_q)(2\pi)^4 \delta^{(4)}(p - p') \\
&\qquad + \mathcal{C}\left(\frac{p + p'}{2}\right) \lambda_a \left[\sigma_a(p - p') + \mathrm{i}\,\gamma_5 \pi_a(p - p') \right] \Big\} \\
&- \int \mathrm{d}^4 x \,\Big\{ \sigma_a \Sigma_a + \pi_a \Pi_a + \frac{G}{2} \left(\Sigma_a \Sigma_a + \Pi_a \Pi_a \right) \\
&\qquad \left(\quad \right) + \frac{H}{4} \mathcal{D}_{abc} \left(\Sigma_a \Sigma_b \Sigma_c - 3 \Sigma_a \Pi_b \Pi_c \right) \Big\}
\end{aligned}
$$

- Determine Σ_a and Π_a using the **Stationary Phase Approximation (SPA)**

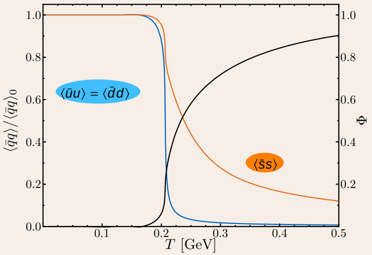
Thermodynamic Potential

Thermodynamic Potential in Mean Field Approximation

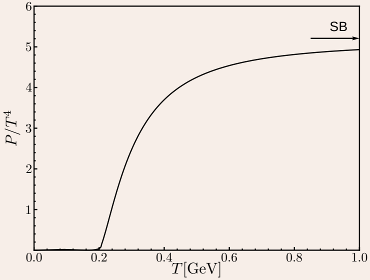
$$\Omega_{\mathrm{MF}} = -2T \sum_{\substack{f\in\{u,d,s\}\\ c\in\{\pm,0\}}} \int \frac{\mathrm{d}^3p}{(2\pi)^3} \sum_{n\in\mathbb{Z}} \mathrm{Tr}\,\ln\left[\omega_n^{c\,2} + \vec{p}^{\,2} + M_f^2(\omega_n^c, \vec{p}\,)\right] +$$

$$-\frac{1}{2}\left[\sum_{f\in\{u,d,s\}} \left(\bar{\sigma}_f \bar{S}_f + \frac{G}{2}\bar{S}_f^2\right) + \frac{H}{2}\bar{S}_u\bar{S}_d\bar{S}_s\right] + \mathcal{U}(\Phi,\Phi^*,T)$$

Chiral Condensate and Loop



Pressure



Summary and Conclusions

Summary

- ✶ Nonlocal model derived from first symmetry principles of QCD
- ✶ Coupling to gluons is possible through introduction of Polyakov Loop Φ
- ✶ Corrections beyond mean field approximation were included in the 2-flavor case

Conclusions

- ➡ “Poor man’s” Dyson-Schwinger approach to nonperturbative QCD
- ➡ Momentum dependent effective quark mass
- ➡ Thermodynamics: Entanglement of chiral and confinement/deconfinement transition
- ➡ Model applicable to finite density description of QCD
- ➡ Model leads reasonable results both for the 2- and 3-flavor case

Thank you for your attention

Literature I

- Georges Ripka.
  Quarks bound by chiral Fields.
  Oxford University Press, Oxford, 1997.
- D. Gomez Dumm, A. G. Grunfeld, and N. N. Scoccola.
  On covariant nonlocal chiral quark models with separable interactions.
  Phys. Rev., D74:054026, 2006.
- C. D. Roberts and A. G. Williams.
  Dyson-Schwinger equations and the application to hadronic physics.
  Progress in Particle and Nuclear Physics, 33:477, 1994.
- Craig D. Roberts and Sebastian M. Schmidt.
  Dyson-Schwinger equations: Density, temperature and continuum strong QCD.
  Prog. Part. Nucl. Phys., 45:S1–S103, 2000.

Literature II

E. Ruiz Arriola and L. L. Salcedo.
Chiral and scale anomalies of non local dirac operators.
Phys. Lett., B450:225–233, 1999.

Thomas Schafer and Edward V. Shuryak.
Instantons in QCD.
Rev. Mod. Phys., 70:323–426, 1998.

H. David Politzer.
Effective quark masses in the chiral limit.
Nucl. Phys., B117:397, 1976.

Patrick O. Bowman, Urs M. Heller, Derek B. Leinweber, and Anthony G. Williams.
Modelling the quark propagator.
Nucl. Phys. Proc. Suppl., 119:323–325, 2003.

Literature III

- Larry D. McLerran and Benjamin Svetitsky.
  A Monte Carlo study of SU(2) Yang-Mills theory at finite temperature.
  Phys. Lett., B98:195, 1981.
- Larry D. McLerran and Benjamin Svetitsky.
  Quark liberation at high temperature: A Monte Carlo study of SU(2) gauge theory.
  Phys. Rev., D24:450, 1981.
- Kenji Fukushima.
  Chiral effective model with the Polyakov loop.
  Physics Letters B, 591:277, 2004.
- Claudia Ratti, Michael A. Thaler, and Wolfram Weise.
  Phases of QCD: Lattice thermodynamics and a field theoretical model.
  Phys. Rev., D73:014019, 2006.

Literature IV

Simon Rößner.
Field theoretical modelling of the QCD phase diagram.
Diploma Thesis, 2006.

M. Cheng et al.
The QCD Equation of State with almost Physical Quark Masses.
Phys. Rev., D77:014511, 2008.

J. Hufner, S. P. Klevansky, P. Zhuang, and H. Voss.
Thermodynamics of a quark plasma beyond the mean field: A generalized Beth-Uhlenbeck approach.
Annals Phys., 234:225–244, 1994.

Frithjof Karsch.
Equation of state and more from lattice regularized QCD.
2008.

F. Karsch, E. Laermann, and A. Peikert.
The pressure in 2, $2+1$ and 3 flavour QCD.
Phys. Lett., B478:447–455, 2000.

Literature V

- B. O. Kerbikov and Yu. A. Simonov.
  Path integral bosonization of three flavor quark dynamics.
  1995.
- A. Scarpettini, D. Gomez Dumm, and N. N. Scoccola.
  Light pseudoscalar mesons in a nonlocal SU(3) chiral quark model.
  Phys. Rev., D69:114018, 2004.