The QCD Phase Diagram: The Large N Limit

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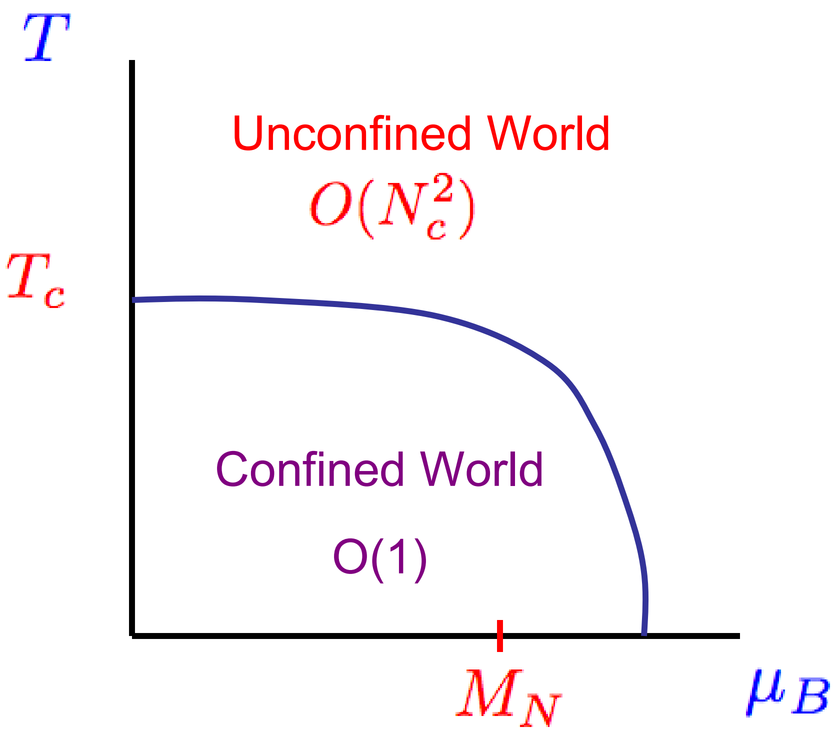
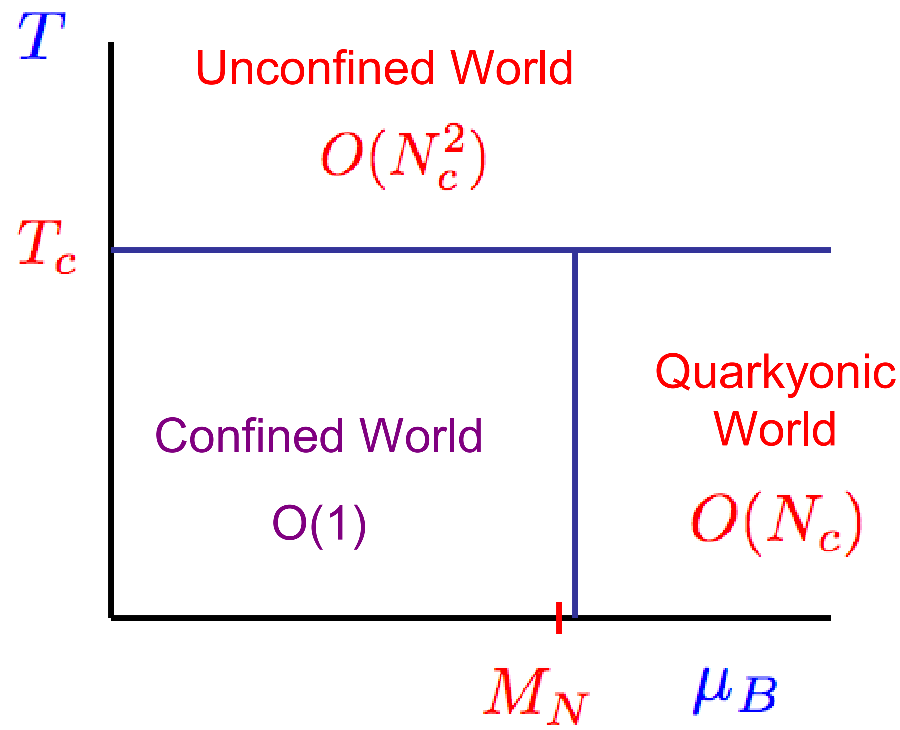


Conventional Wisdom



Large N

Will argue real world looks more like large N world

Brief Review of Large N

$$N_c \rightarrow \infty \qquad g^2 N_c \; finite$$

Mesons: quark-antiquark, noninteracting, masses ~ Λ_{QCD}

Baryons: N quarks, masses ~ $N_c \Lambda_{QCD}$, baryon interactions ~ N_c

Spectrum of Low Energy Baryons:

Multiplets with I = J; I,J = 1/2 -> I,J = N/2

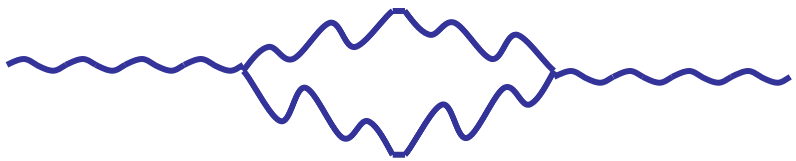
$$M_B(I,J) \sim M_N(1 + O(I^2/N_c^2, J^2/N_c^2, IJ/N_c^2))$$

$$M_\Delta - M_N \sim \Lambda_{QCD}^2/N_c$$

$$e^{(\mu_B - M_B)/T} = 0 \; if \; \mu_B < M_B$$

The confined world has no baryons!

Confinement at Finite Density:



$$g^2 N_c T^2 \sim \alpha_N T^2$$

Generates Debye Screening => Deconfinement at Tc

$$g^2 \mu_Q^2 \sim \alpha_N \mu_Q^2 / N_c$$

$$\mu_Q = \mu_B / N_c$$

Quark loops are always small by 1/N_c

For finite baryon fermi energy, confinement is never affected by the presence of quarks!

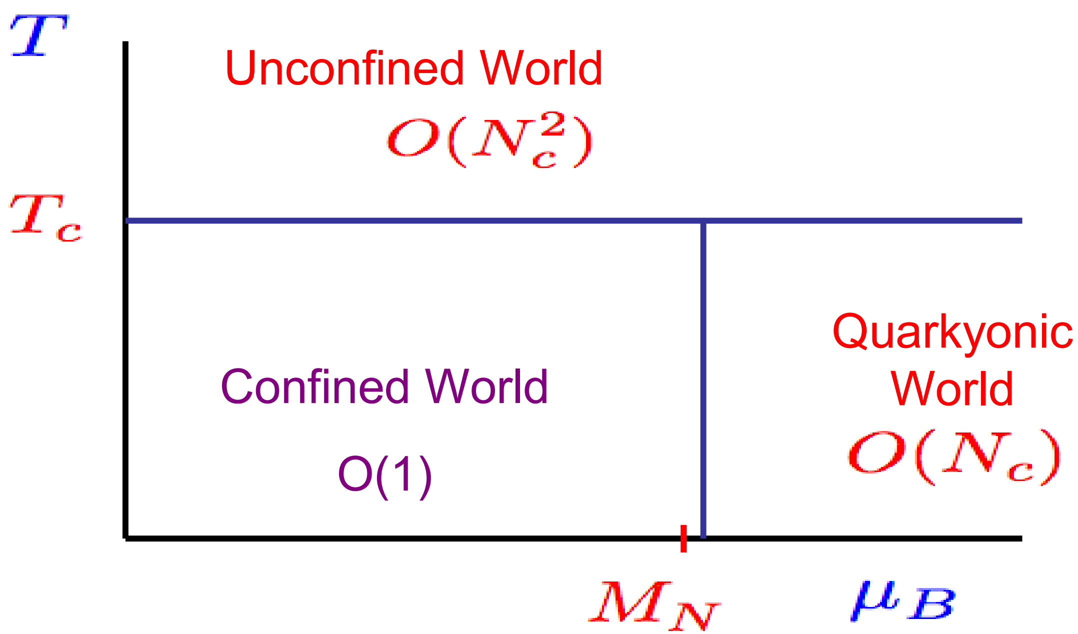
T_c does not depend upon baryon density!

Finite Baryon Density:

$$e^{(\mu_B - M_B)/T} = 0 \; if \;\; \mu_B < M_B$$

No baryons in the confined phase for $\mu_B < M_B$

For $\mu_B >> M_B \; (\mu_Q >> \Lambda_{QCD})$ weakly coupled gas of quarks.



Large N

If T < T_c, no free gluons, degrees of freedom are ~ Nc

Quarkyonic Matter:
Confined gas of perturbative quarks!

Confined: Mesons and Glueballs

Quarkyonic: Quarks and Glueballs

Unconfined: Quakrs and Gluons

Some Properties of Quarkyonic Matter

Quarks inside the Fermi Sea: Perturbative Interactions => At High Density can use perturbative quark Fermi gas for bulk properties

At Fermi Surface: Interactions sensitive to infrared => Confined baryons

Perturbative high density quark matter is chirally symmetric but confined => violates intuitive arguments that confinement => chiral symmetry

Quarkyonic matter appears when

$$\mu_B = M_B \ (\mu_Q = 330 MeV)$$

(Can be modified if quark matter is bound by interactions. Could be “strange quarkyonic matter”?

Seems not true for N = 3)

Guess for Realistic Phase Diagram for N = 3

Will ignore “small effects” like Color Superconductivity

Number of degrees of freedom:

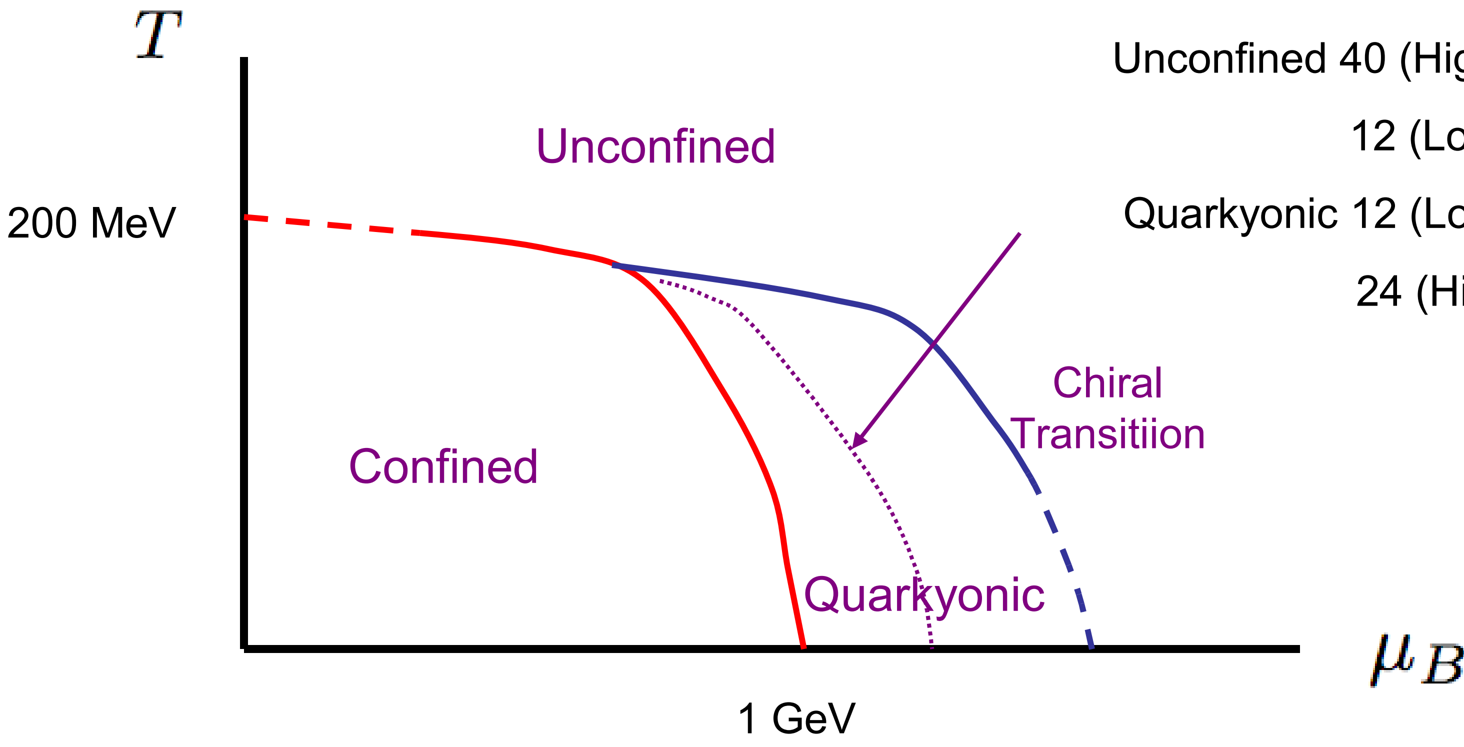
Confined 3

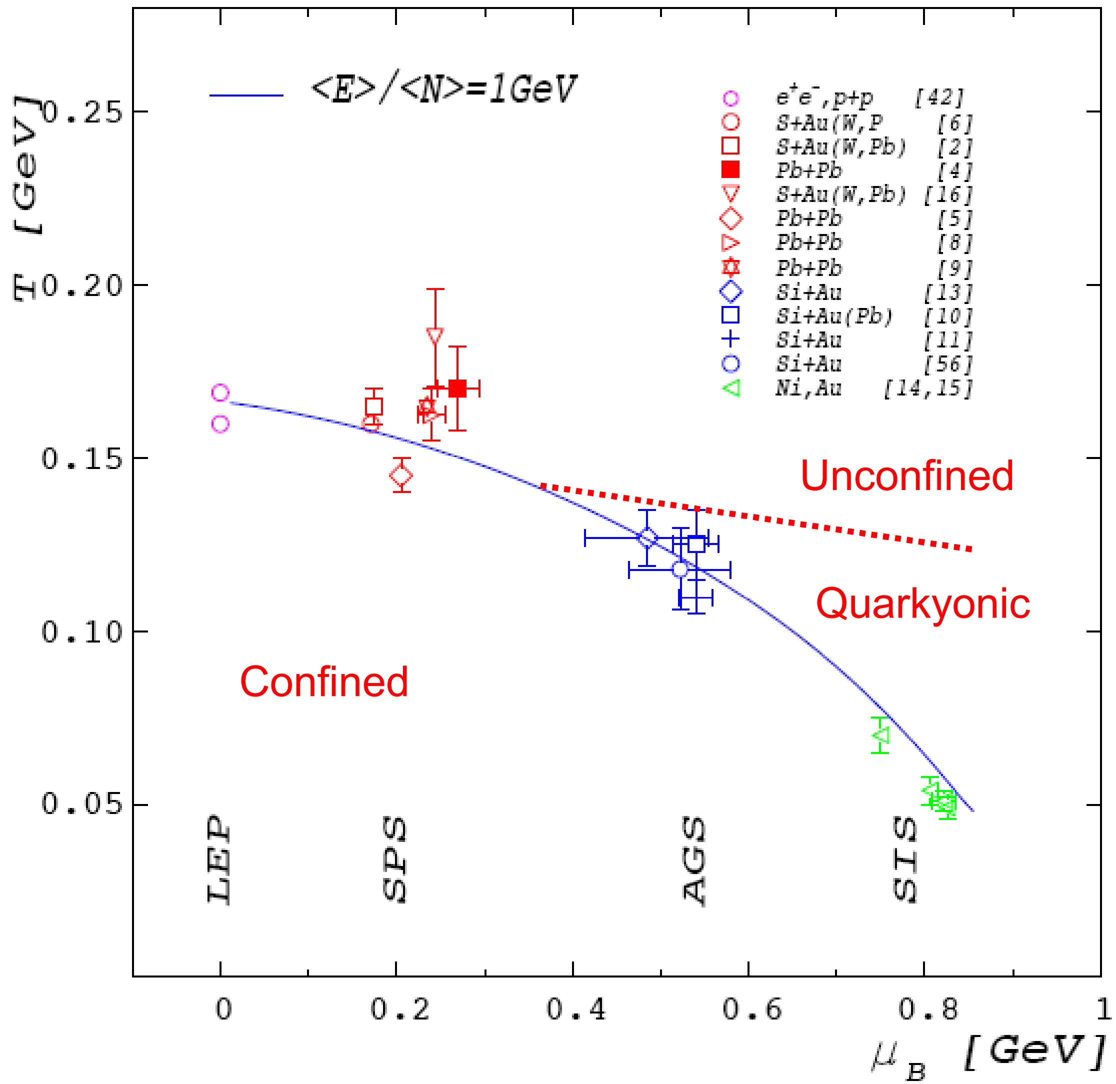
Unconfined 40 (High T)

12 (Low T)

Quarkyonic 12 (LowT)

24 (High T)

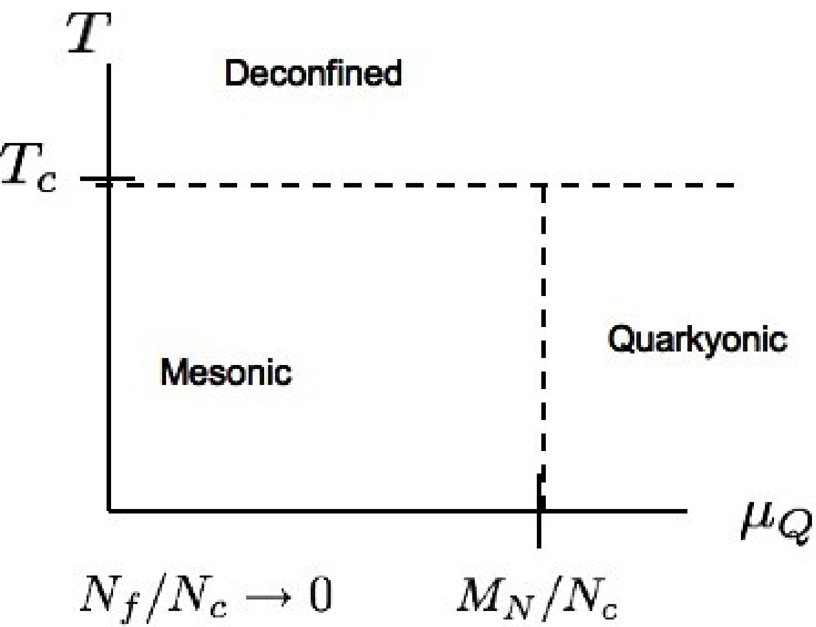


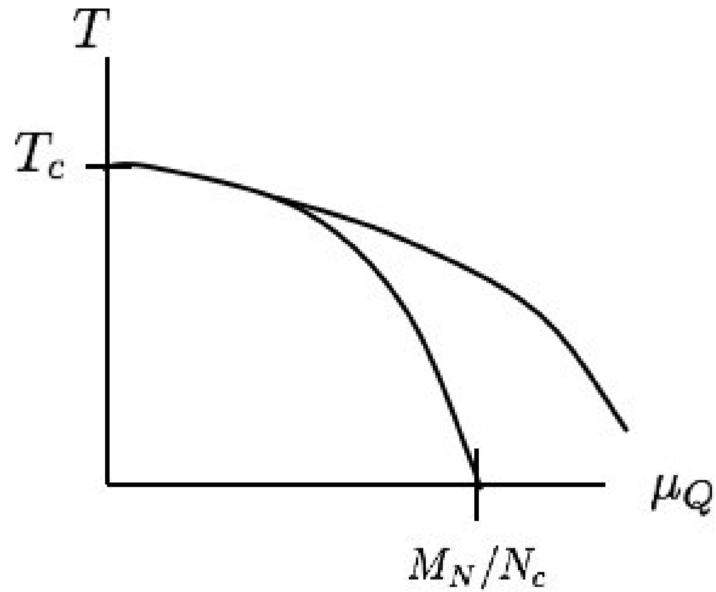


Maybe it looks a little like this?

Maybe somewhere around the AGS there is a tricritical point where these worlds merge?

Decoupling probably occurs along at low T probably occurs between confined and quarkyonic worlds. Consistent with Cleymans-Redlich-Stachel-Braun-Munzinger observations!





Nf/Nc fixed, large Nc

Confinement not an order parameter

Baryon number is

Large density of states:

Lowest mass baryons

QuickTime™ and a decompressor are needed to see this picture.

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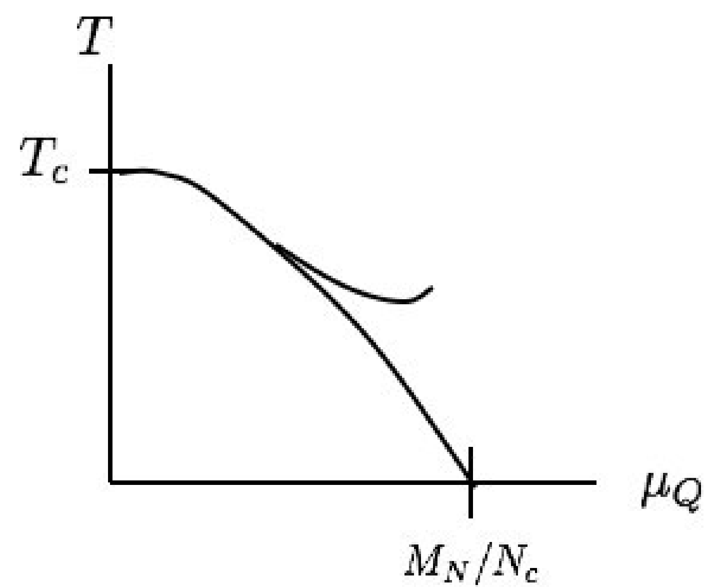
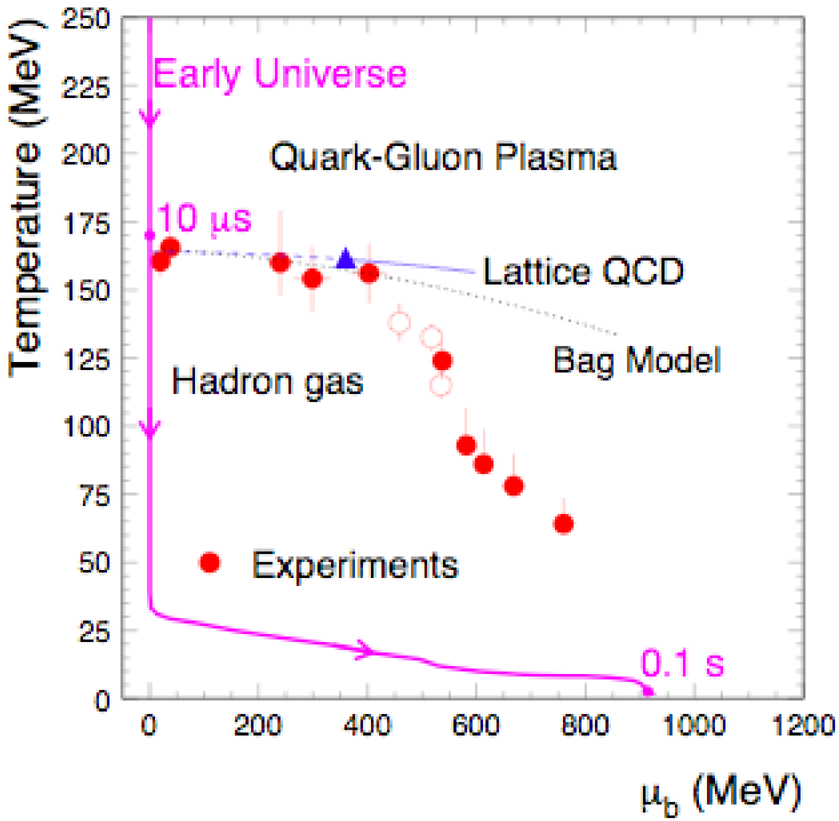




Conclusions:

There are three phases of QCD at large N:

Confined

Unconfined

Quarkyonic

They have very different bulk properties

There may be a tri-critical point somewhere near AGS energies

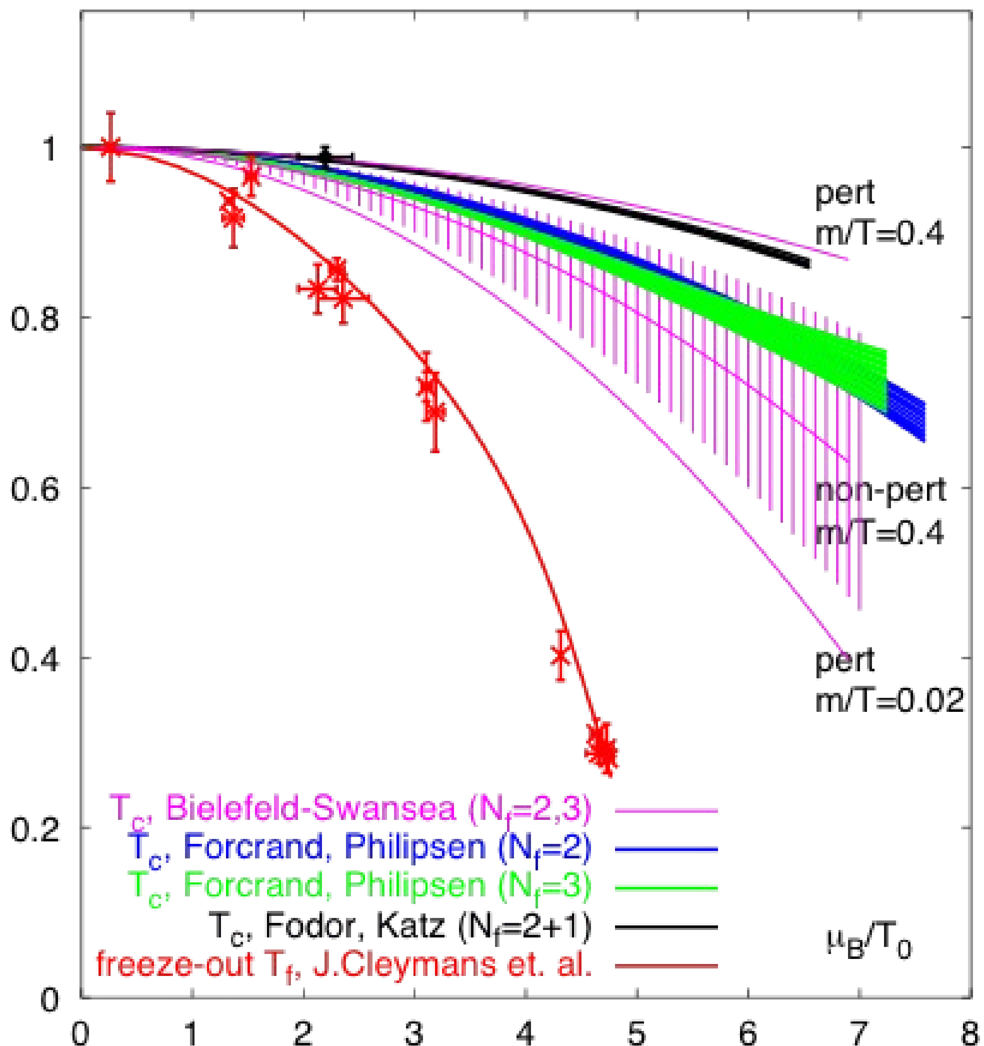
The early observations of Cleymans, Redlich,Braun-Munzinger and Stachel strongly support that this picture reflects N = 3.

Experiment vs. Lattice

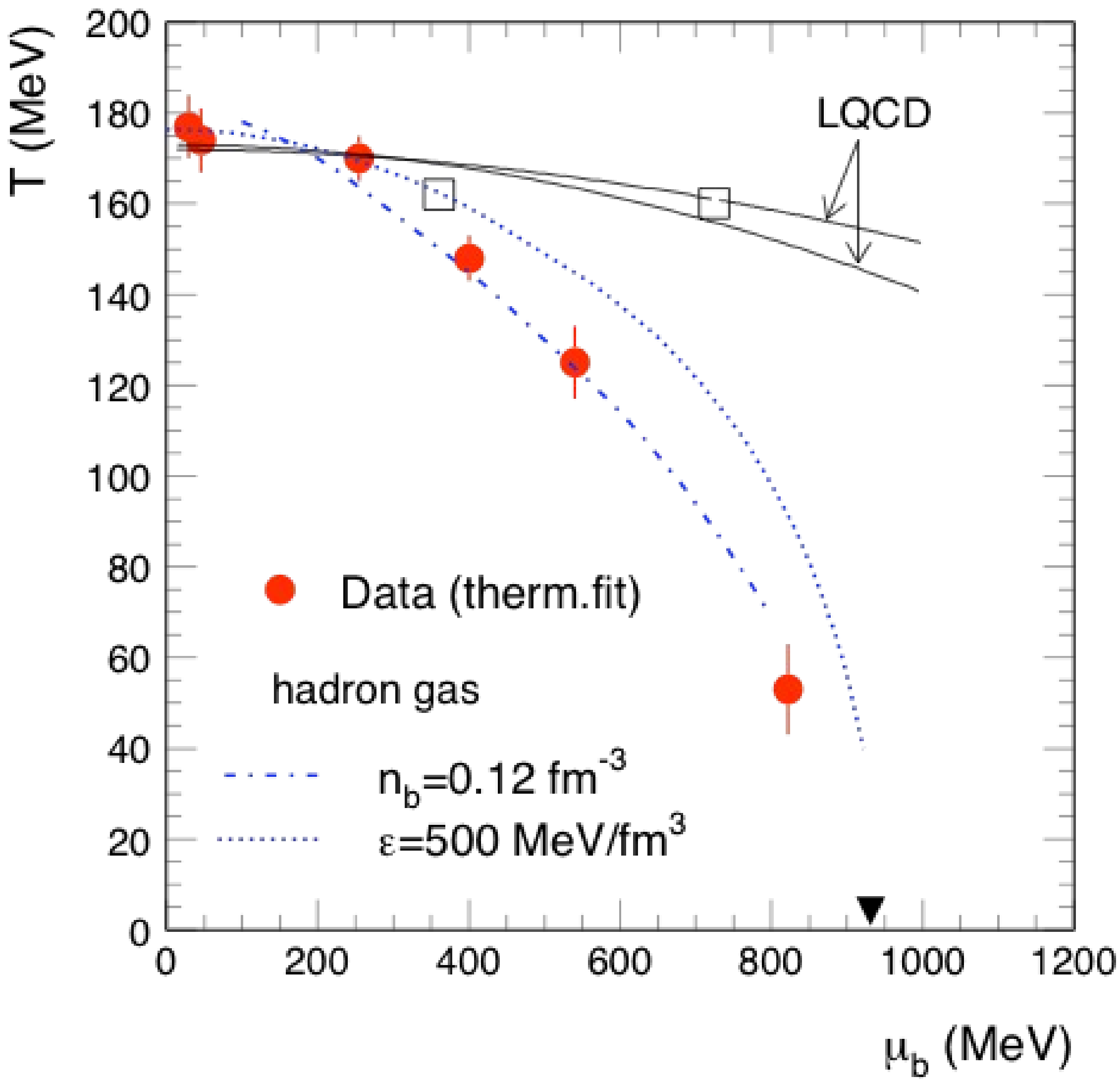
Lattice "transition" appears *above* freezeout line? Schmidt '07

N.B.: small change in T_c with μ?

T ↑



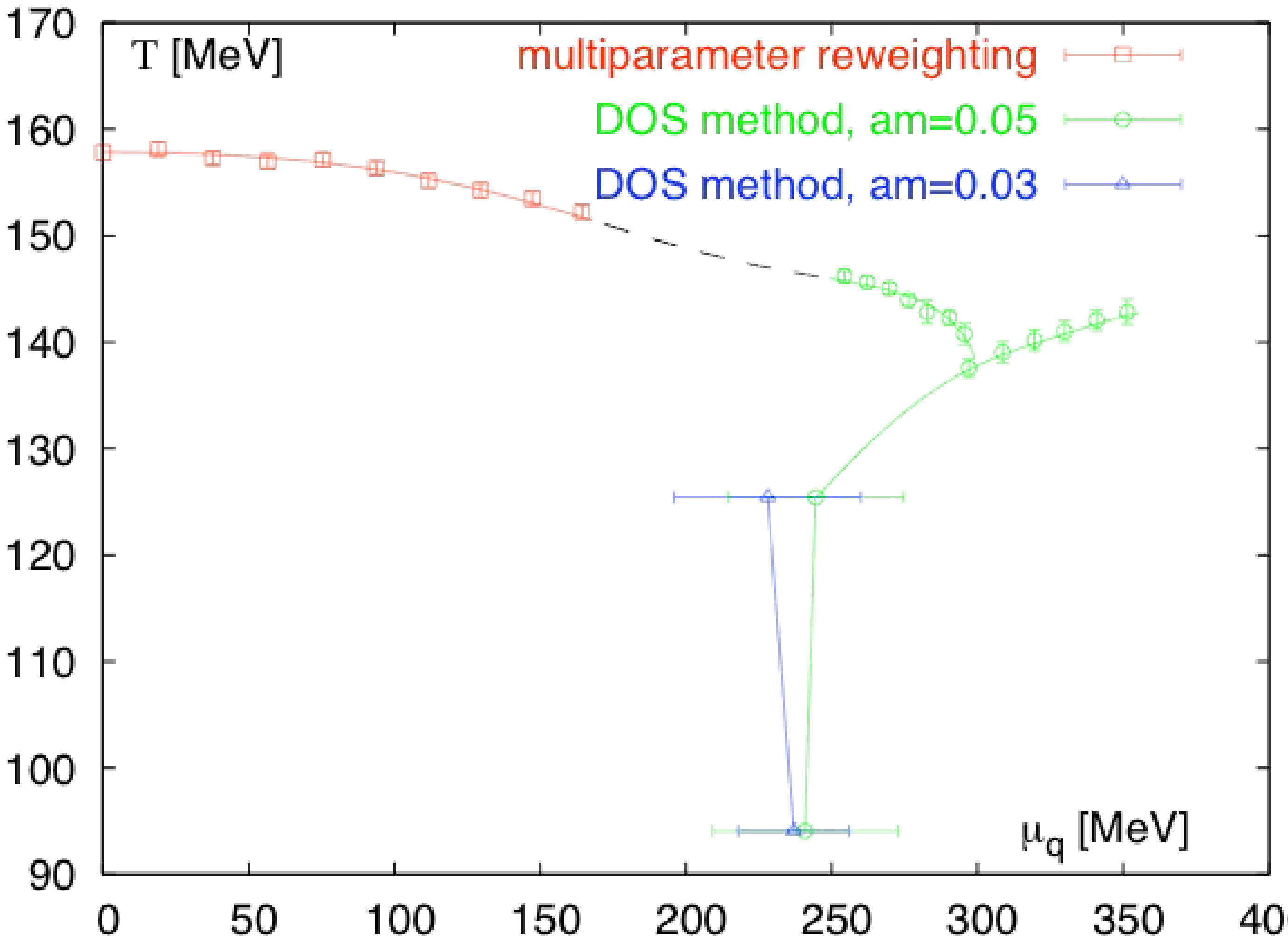
μ_{quark} →


T (MeV)
200
180
160
140
120
100
80
60
40
20
0
LQCD
Data (therm.fit)
hadron gas
n_b=0.12 fm^{-3}
ε=500 MeV/fm^3
0
200
400
600
800
1000
1200
μ_b (MeV)

Lattice T_c , vs μ

Rather small change in T_c vs μ? Depends where μ_c is at T = 0. Fodor & Katz '06

T ↑



μ_{quark} →

How Does Transition Occur?

Kinetic Energies ~ $\frac{k_F^4}{N_c}\left(\frac{k_F}{\Lambda_{QCD}}\right)$

Resonance Sum ~ $\frac{k_F^4}{N_c}\left(\frac{k_F}{\Lambda_{QCD}}\right)^{\delta}$

Interactions ~ $N_c\, k_F^4\left(\frac{k_F}{\Lambda_{QCD}}\right)^{\gamma}$

For a dilute gas, interactions give $\gamma = 3$

Interactions dominate kinetic energies when $k_F \sim \Lambda_{QCD}/N_c$

Liquid-Gas Phase Transition?

Skyrmionic Solid?

Expect transition when $k_F \sim \Lambda_{QCD}$

Width of the Transition Region:

$$k_F \sim \Lambda_{QCD}$$

Baryons are non-relativistic: $k_F/M_N \sim v \sim 1/N_c$

$$\mu_B \sim M_N + k_F^2/2M_N \sim N_c\Lambda_{QCD}(1 + O(1/N_c^2))$$

$$\mu_Q \sim \Lambda_{QCD}(1 + O(1/N_c^2))$$

Nuclear physics is in a width of order
$1/N_c^2$ around the baryon mass!

Large Nc world looks like our world:
Nuclear matter is non-relativistic, and
there is a narrow window between
confined and quarkyonic world

Virtues of the Skyrmion Treatment of Nuclear Matter

$$S = \int d^4x \; \left(f_\pi^2 \; trV^\mu V_\mu^\dagger + \kappa tr[V^\mu, V^\nu]^2\right)$$

$$\kappa, f_\pi^2 \sim N_c$$

Nuclear matter would like to have energy density
and pressure of order N

At low density, except for the rest mass contribution to
energy density, ~ 1/N

Baryons are very massive, and in the Skyrme model, the
energy density arises from translational zero modes.

Interactions are small because nucleons are far separated.

When energy density is of order N, however, higher order
terms in Skyrme model are important, but correct
parametric dependence is obtained

Skyrmions and $N_c = \infty$ baryons

Witten '83; Adkins, Nappi, Witten '83: Skyrme model for baryons

$$\mathcal{L} = f_\pi^2 \, \mathrm{tr}|V_\mu|^2 + \kappa \, \mathrm{tr}[V_\mu, V_\nu]^2 \; , \; V_\mu = U^\dagger \partial_\mu U \; , \; U = e^{i\pi/f_\pi}$$

Baryon soliton of pion Lagrangian: $f_\pi \sim N_c^{1/2}$, $\kappa \sim N_c$, mass $\sim f_\pi^2 \sim \kappa \sim N_c$.

Single baryon: at $r = \infty$, $\pi^a = 0$, $U = 1$. At $r = 0$, $\pi^a = \pi \, r^a/r$.

Baryon number topological: Wess & Zumino '71; Witten '83.

Huge degeneracy of baryons: multiplets of isospin and spin, $I = J$: $1/2 \ldots N_c/2$.

Obvious as collective coordinates of soliton, coupling spin & isospin

Dashen & Manohar '93, Dashen, Jenkins, & Manohar '94:

Baryon-meson coupling $\sim N_c^{1/2}$,

Cancellations from extended $SU(2\,N_f)$ symmetry.

Towards the phase diagram at $N_c = \infty$

As example, consider gluon polarization tensor at zero momentum.
(at leading order, ~ Debye mass^2 , gauge invariant)

$$\Pi^{\mu\mu}(0) = g^2 \left(\left(N_c + \frac{N_f}{2} \right) \frac{T^2}{3} + \frac{N_f \mu^2}{2\pi^2} \right) = \lambda \, \frac{T^2}{3} \; , \; N_c = \infty$$

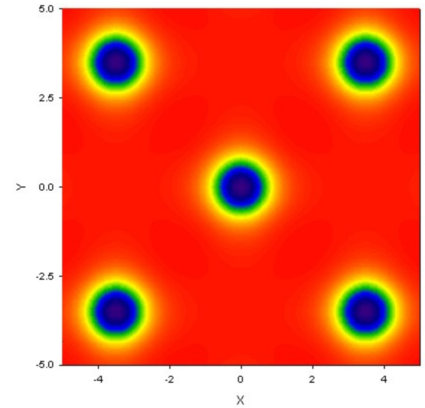
For $\mu \sim N_c^0 \sim 1$, at $N_c = \infty$ the gluons are blind to quarks.

When $\mu \sim 1$, deconfining transition temperature $T_d(\mu) = T_d(0)$

Chemical potential only matters when larger than mass:
$\mu_{Baryon} > M_{Baryon}$. Define $m_{quark} = M_{Baryon}/N_c$; so $\mu > m_{quark}$.

"Box" for $T < T_c$; $\mu < m_{quark}$: confined phase baryon free, since their mass ~ N_c
Thermal excitation ~ $\exp(-m_B/T) \sim \exp(-N_c) = 0$ at large N_c.
So hadronic phase in "box" = mesons & glueballs only, *no* baryons.

Skyrmion crystals

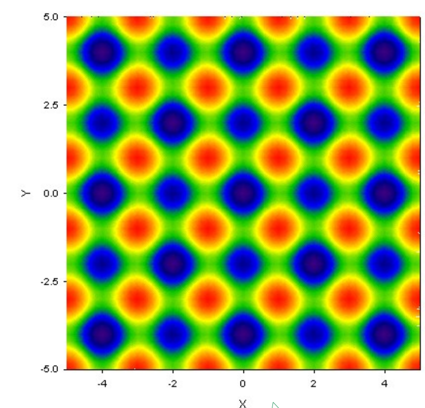


Skyrmion crystal: soliton periodic in space.
Kutschera, Pethick & Ravenhall (KPR) ’84; Klebanov ’85 + ...
Lee, Park, Min, Rho & Vento, hep-ph/0302019 =>

At low density, chiral symmetry brokenby Skyrme crystal, as in vacuum.

Chiral symmetry *restored* at nonzero density: $< U > = 0$ in each cell.

Goldhaber & Manton ’87: due to “half” Skyrmion symmetry in each cell.
Forkel, Jackson et al, ’89: excitations *are* chirally symmetric.



Easiest to understand with “spherical” crystal, KPR ’84, Manton ’87.
Take same boundary conditions as a single baryon, but for sphere of radius R:

At $r = R$: $\pi^a = 0$. At $r = 0$, $\pi^a = \pi\, r^a/r$. Density one baryon/$(4 \pi R^3/3)$.

At high density, term ~ κ dominates, so energy density ~ baryon density$^{4/3}$.
Like perturbative QCD! Accident of simplest Skyrme Lagrangian.

Chirally symmetric baryons

B. Lee, ‘72; DeTar & Kunihiro ’89; Jido, Oka & Hosaka, hep-ph/0110005; Zschiesche et al nucl-th/0608044. Consider *two* baryon multiplets. One usual nucleon, other parity partner, transforming *opposite* under chiral transformations:

$$\psi_{L,R} \rightarrow U_{L,R}\,\psi_{L,R} \;\; ; \;\; \chi_{L,R} \rightarrow U_{R,L}\,\chi_{L,R}$$

With two multiplets, can form chirally symmetric (parity even) mass term:

$$\psi_L\,\chi_R \,-\, \psi_R\,\chi_L \,+\, \chi_R\,\psi_L \,-\, \chi_L\,\psi_R$$

Also: usual sigma field, $\Phi \rightarrow U_L\,\Phi\,U_R^\dagger$, couplings for linear sigma model:

$$g_1\,\psi_L\,\Phi\,\psi_R \,+\, g_2\,\chi_R\,\Phi\,\chi_L$$

Generalized model at μ ≠ 0: D. Fernandez-Fraile & RDP ’07...

Anomalies?

't Hooft, '80: anomalies rule *out* massive, parity doubled baryons in vacuum:

No massless modes to saturate anomaly condition

Itoyama & Mueller'83; RDP, Trueman & Tytgat '97:

At $T \neq 0$, $\mu \neq 0$, anomaly constraints *far* less restrictive (many more amplitudes)

E.g.: anomaly unchanged at $T \neq 0$, $\mu \neq 0$, but Sutherland-Veltman theorem *fails*

Must do: show parity doubled baryons consistent with anomalies at $\mu \neq 0$.

At $T \neq 0$, $\mu = 0$, no massless modes. Anomalies probably rule out model(s).

But at $\mu \neq 0$, *always* have massless modes near the Fermi surface.

Casher '79: heuristically, confinement => chiral sym. breaking in vacuum

Especially at large N_c, carries over to $T \neq 0$, $\mu = 0$.

Does *not* apply at $\mu \neq 0$: baryons strongly interacting at large N_c.

Banks & Casher '80: chiral sym. breaking from eigenvalue density at origin.

Splittorff & Verbaarschot '07: at $\mu \neq 0$, eigenvalues spread in complex plane.

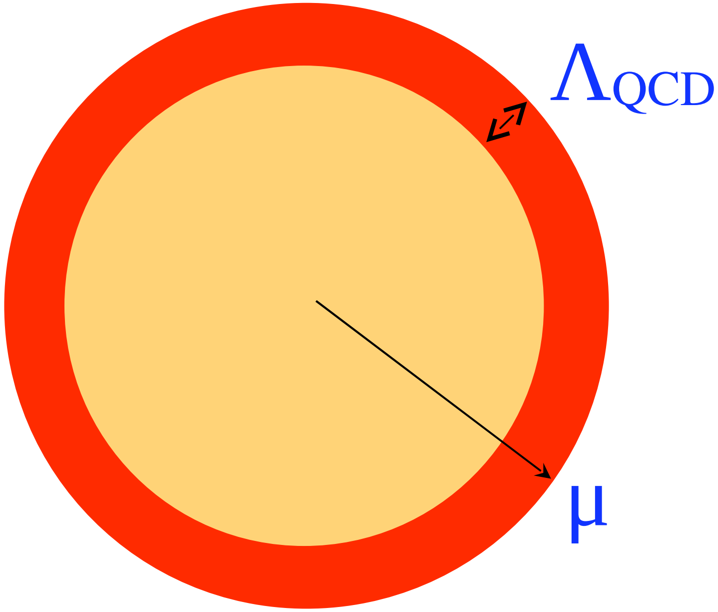
(Another) heuristic argument for chiral sym. restoration in quarkyonic phase.

"Quarkyonic" phase at large N_c

As gluons blind to quarks at large N_c, for $\mu \sim N_c^0 \sim 1$, *confined* phase for $T < T_d$

This includes $\mu >> \Lambda_{QCD}$! Central puzzle. We suggest:

To left: Fermi sea.



Deep in the Fermi sea, $k << \mu$,
looks like quarks.

But: within ~ Λ_{QCD} of the Fermi surface,
confinement => *baryons*

We term combination "quark-yonic"

OK for $\mu >> \Lambda_{QCD}$. When $\mu \sim \Lambda_{QCD}$, baryonic "skin" entire Fermi sea.

But what about chiral symmetry breaking?