QCD Equation of State and Fluctuations on the Lattice

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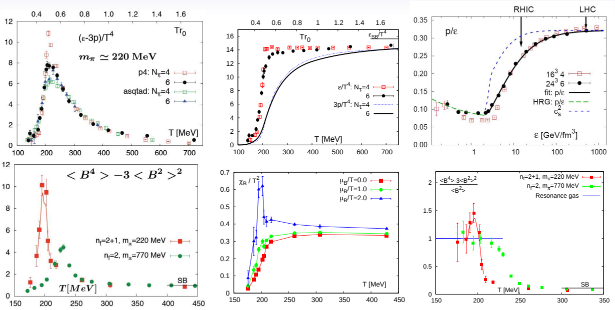
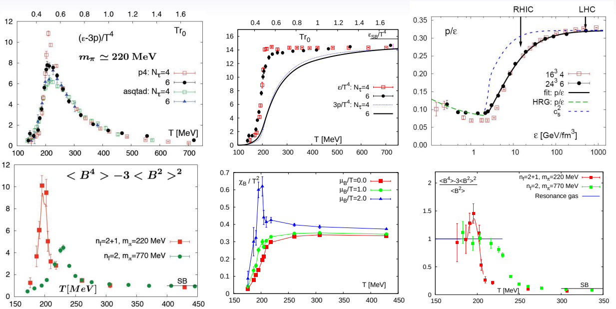
INT, 2008 Aug.

Outline



(ε-3p)/T^4
Tr0
mπ ≃ 220 MeV
p4: Nτ=4
6
asqtad: Nτ=4
6
T [MeV]
ε/T^4: Nτ=4
3p/T^4: Nτ=4
εSB/T^4
RHIC
LHC
p/ε
16^3 4
24^3 6
fit: p/ε
HRG: p/ε
c_s^2
ε [GeV/fm^3]
< B^4 > −3 < B^2 >^2
nf=2+1, mπ=220 MeV
nf=2, mπ=770 MeV
SB
T[MeV]
χB / T^2
μB/T=0.0
μB/T=1.0
μB/T=2.0
T [MeV]
<B^4>-3<B^2>^2
<B^2>
Resonance gas

Outline



- QCD equation of state at finite temperature and zero densities.
- QCD equation of state at finite temperature but small densities.
- Hadronic fluctuations.

Introduction

Goal QCD thermodynamics with realistic quark masses in (2+1)-f QCD and controlled extrapolation to the continuum limit

Status (i) Line of Constant Physics: physical m_K, $m_\pi \simeq 220$ MeV
(ii) $N_\tau = 4, 6, (8)$, $N_\sigma = 4N_\tau$
(iii) Wide temperature range scan: 140 MeV ~ 800 MeV
(iv) Exact algorithm (RHMC) and $\mathcal{O}(10^4)$ trajectories per ensemble

$$T = 1/N_\tau a(\beta)$$

Projects

RBC-Bielefeld: p4-action + 3-link smearing (p4fat3), $N_\tau = 4, 6$

[M. Cheng et al., PRD77, 014511 (2008)]

MILC: Naik-action + (3,5,7)-link smearing (asqtad), $N_\tau = 4, 6$

[C. Bernard et al., PRD75, 094505 (2007)]

HotQCD: Extend to $N_\tau = 8$ and compare p4fat3 and asqtad

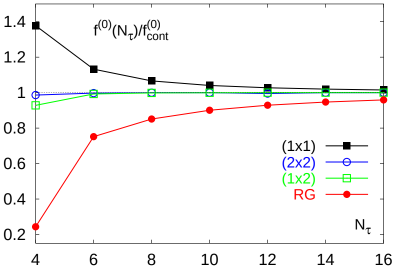
[prelimenary]

Cut off effects and improved action

- It is important to reduce cut-off effects for bulk thermodynamics.

$$f/T^4 \sim \text{Signal X } N_\tau^4 \Longrightarrow \text{small } N_\tau^4 \Longrightarrow \text{coarse } a \qquad a = 1/N_\tau T$$

- Gluonic sector: $\mathcal{O}(a^2)$ discretization error



$\Leftarrow$ the idea gas limit

free energy density on N_τ lattice normalized by its continuum limit

Cut-off effects for Staggered Fermions

similar as gluonic sector, but more severe

- p4 or Naik action remove $\mathcal{O}(a^2)$ errors in bulk thermodynamics

$$
\begin{aligned}
\frac{p}{p_{\mathrm{SB}}} &= 1+\frac{248}{147}\left(\frac{\pi}{N_\tau}\right)^2+\frac{635}{147}\left(\frac{\pi}{N_\tau}\right)^4+\cdots && (standard)\\
\frac{p}{p_{\mathrm{SB}}} &= 1+0 -\frac{1143}{980}\left(\frac{\pi}{N_\tau}\right)^4+\frac{73}{2079}\left(\frac{\pi}{N_\tau}\right)^6+\cdots && (p4)\\
\frac{p}{p_{\mathrm{SB}}} &= 1+0 -\frac{1143}{980}\left(\frac{\pi}{N_\tau}\right)^4-\frac{365}{77}\left(\frac{\pi}{N_\tau}\right)^6+\cdots && (Naik)
\end{aligned}
$$

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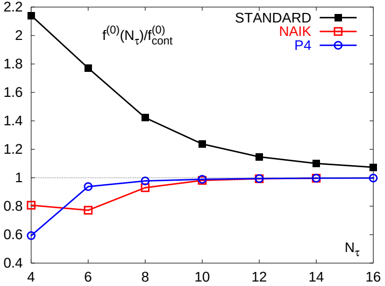

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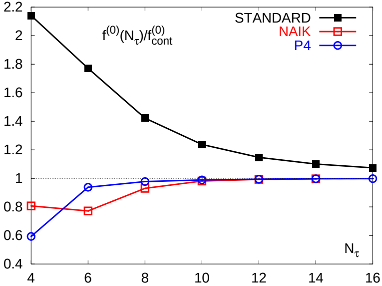


- smearing to improve flavor mixing:
  $\Rightarrow$ 3link-staple, 7link-staple (asqtad), stout, ...

RBC-Bielefeld choice of action: 2×1 tree level improved gauge action,
p4-fat3 improved staggered fermion

Bulk Thermodynamics at $\mu = 0$

Integration method

- Use integration method to calculate pressure

$$\frac{p(T)}{T^4} - \frac{p(T_0)}{T_0^4} = \int_{T_0}^{T} \mathrm{d}T' \frac{1}{T'} \frac{\varepsilon - 3p}{T'^4}$$

$T_0 = 100$ MeV $\Rightarrow$ integration constant $[p(T_0)/T_0^4]_{HRG} \simeq 0.265$

- the interaction measure

$$\frac{\varepsilon - 3p}{T^4} = T\frac{\mathrm{d}}{\mathrm{d}T}\left(\frac{p}{T^4}\right) = \left(a\frac{\mathrm{d}\beta}{\mathrm{d}a}\right)\frac{\mathrm{d}p/T^4}{\mathrm{d}\beta} \qquad \boxed{T = 1/N_\tau a(\beta)}$$

$$\begin{aligned} &= && R_\beta(\langle s_G\rangle_0 - \langle s_G\rangle_T)N_\tau^4 \\ & &-& R_\beta R_m(2\hat{m}_l\left(\langle\bar{\psi}\psi\rangle_{l,0} - \langle\bar{\psi}\psi\rangle_{l,T}\right) + \hat{m}_s\left(\langle\bar{\psi}\psi\rangle_{s,0} - \langle\bar{\psi}\psi\rangle_{s,T}\right))N_\tau^4 \\ & &-& R_\beta R_h\hat{m}_s\left(\langle\bar{\psi}\psi\rangle_{s,0} - \langle\bar{\psi}\psi\rangle_{s,T}\right)N_\tau^4 \end{aligned}$$

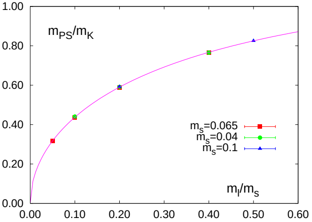
$$\boxed{R_\beta = -a\frac{\mathrm{d}\beta}{\mathrm{d}a} \quad R_m = \frac{1}{\hat{m}_l}\frac{\mathrm{d}\hat{m}_l}{\mathrm{d}\beta} \quad R_h = \frac{\hat{m}_l}{\hat{m}_s}\frac{\mathrm{d}\hat{m}_s/\hat{m}_l}{\mathrm{d}\beta}}$$

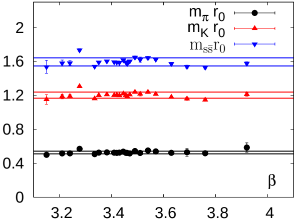
$\Longrightarrow$ UV divergence subtracted from $T = 0$ counter parts.

$\Longrightarrow$ Line of Constant Physics ($m_\pi \sim 220$ MeV):
mass parameters ($\hat{m}_l,\ h = \frac{\hat{m}_s}{\hat{m}_l}$) temperature scale ($T = \frac{1}{N_\tau a(\beta)}$)

Line of Constant Physics

Bare quark masses tuned so that m_K physical and $m_\pi \simeq 220MeV$.





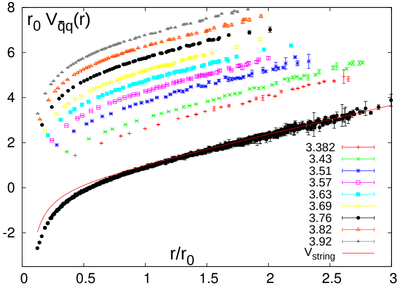
m_π/m_K
depends only on
$\hat{m}_s/\hat{m}_l$

fix $\frac{\hat{m}_s}{\hat{m}_l} = 10$
and fine tune $\hat{m}_s$

$\implies R_h = 0$ and R_m

Scale set by heavy quark potential slope parameter $(T = 0)$

$$V_{\bar{q}q}(r) = -\frac{\alpha}{r} + \sigma r + c, \qquad r^2 \frac{\mathrm{d}V}{dr}\bigg|_{r=r_0} = 1.65$$



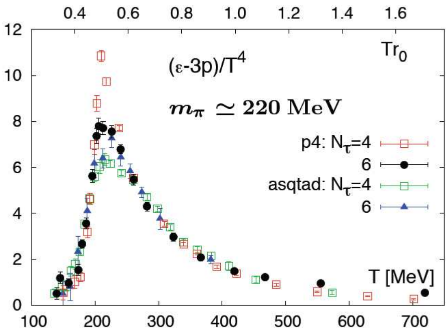
r_0 has
no significant cut-off dependence,
while a changes by factor 6!

$r_0 = 0.469(7)$ fm from lattice calculations of quarkonium spectroscopy

[A. Gray et al, Phys. Rev. D72 (2005) 094507]

$\Longrightarrow a(\beta)$ and $R_\beta(\beta)$

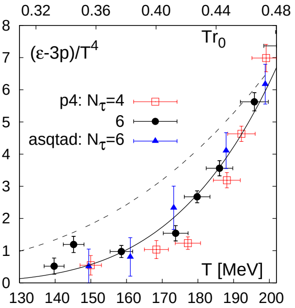
Interaction measure on LCP (I)



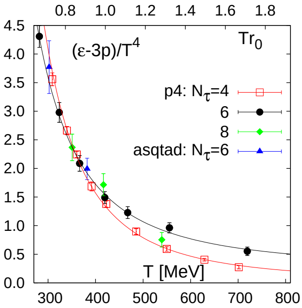
- p4 vs. asqtad: overall good agreement
- $N_\tau = 4$ vs. 6: Peak height reduces, but softest point $\simeq 200$ MeV

Integration measure on LCP (II)

low T:



high T:



LGT v.s. hadron resonance gas

$\Longrightarrow$ physical quark masses

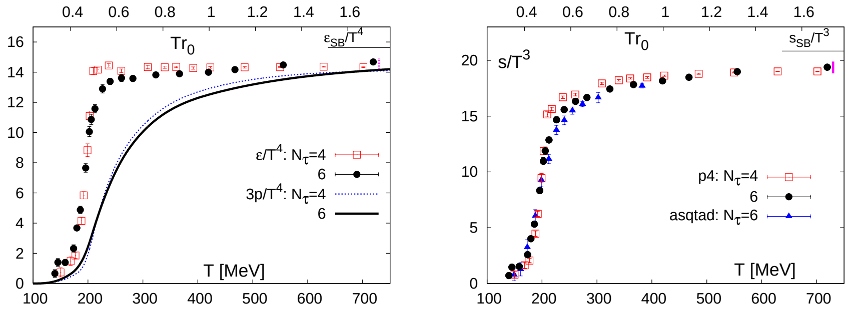
deviation from ideal gas

$\Longrightarrow$ cut-off effects

$\Longrightarrow$ continuum extrapolation

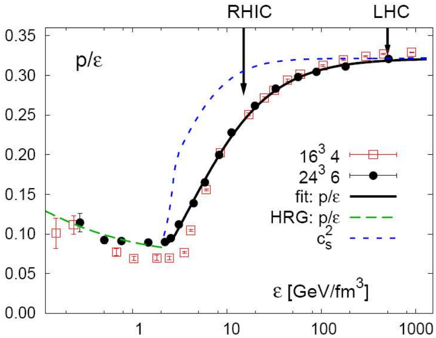
EoS at $\mu = 0$

10% differences from the Stephan-Boltzmann limit



EoS and velocity of sound

$$c_s^2 = \frac{\mathrm{d}p}{\mathrm{d}\varepsilon} = \varepsilon \frac{\mathrm{d}p/\varepsilon}{\mathrm{d}\varepsilon} + \frac{p}{\varepsilon}$$



[RBC-Bielefeld, M. Cheng et al., PRD 77, 014511 (2008)]

Bulk Thermodynamics at $\mu \neq 0$

Taylor expansion method at small chemical potential (I)

- Pressure are Taylor expanded in terms of $\mu_{u,d,s}$ or $\mu_{B,Q,S}$:

$$\frac{p}{T^4} = \frac{\ln Z(\mu_u,\mu_d,\mu_s)}{VT^3} = \sum_{i,j,k} c^{uds}_{i\,j\,k} \left(\frac{\mu_u}{T}\right)^i \left(\frac{\mu_d}{T}\right)^j \left(\frac{\mu_s}{T}\right)^k$$

$$= \sum_{B,Q,S} c^{BQS}_{i\,j\,k} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

$$\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q \quad \mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q \quad \mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q + \mu_S\ ,$$

- $c^{uds}_{i,j,k}$ are directly measured, since u,d,s quarks are microscopic dof.
  $\Longrightarrow$ Use unbiased, noisy estimator to compute $c^{uds}_{i,j,k}$.
- $c^{BQS}_{i,j,k}$ are linear combinations of $c^{uds}_{i,j,k}$.

Taylor expansion method (II)

- We then can construct thermal quantities at finite chemical potential.

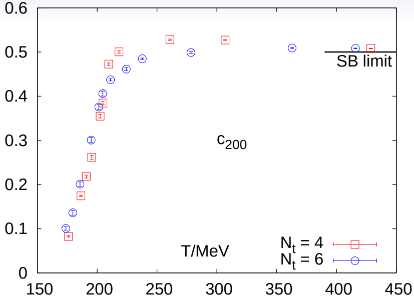
$$\frac{n_B}{T^3} \equiv \frac{\partial\left(p/T^4\right)}{\partial\left(\mu_B/T\right)} = \sum_{i,j,k}(i+1)c^{B,Q,S}_{i+1,j,k}\left(\frac{\mu_B}{T}\right)^i\left(\frac{\mu_Q}{T}\right)^j\left(\frac{\mu_S}{T}\right)^k$$

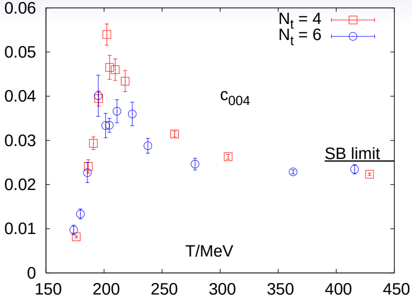
$$\frac{\chi_B}{T^2} \equiv \frac{\partial^2\left(p/T^4\right)}{\partial\left(\mu_B/T\right)^2} = 2c^{BQS}_{200} + 12c^{BQS}_{400}\left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}\left[\left(\frac{\mu_B}{T}\right)^4\right]$$

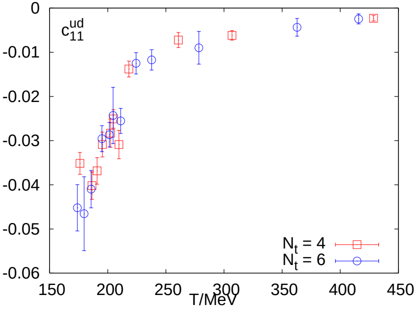
- Also, the coefficients are fluctuations (correlations) at zero chemical potential.

$$\begin{aligned}
c_2^x &\equiv \frac{1}{2VT^3}\frac{\partial^2 \ln Z}{\partial(\hat{\mu}_x)^2}\bigg|_{\mu=0} = \frac{1}{2VT^3}\langle(\delta N_x)^2\rangle_{\mu=0} = \frac{1}{2VT^3}\langle N_x^2\rangle_{\mu=0}, \\
c_4^x &\equiv \frac{1}{24VT^3}\frac{\partial^4 \ln Z}{\partial(\hat{\mu}_x)^4}\bigg|_{\mu=0} = \frac{1}{24VT^3}\left(\left\langle(N_x)^4\right\rangle - 3\left\langle(N_x)^2\right\rangle^2\right)_{\mu=0}, \\
c_{11}^{xy} &\equiv \frac{1}{VT^3}\frac{\partial^2 \ln Z}{\partial(\hat{\mu}_x)\partial(\hat{\mu}_y)}\bigg|_{\mu=0} = \frac{1}{VT^3}\left(\langle N_x N_y\rangle - \langle N_x\rangle\langle N_y\rangle\right)_{\mu=0}.
\end{aligned}$$

Results: quark coefficients

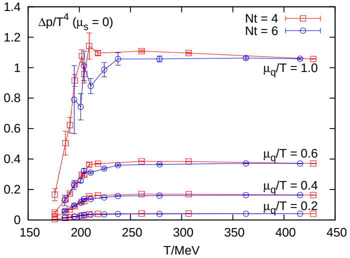


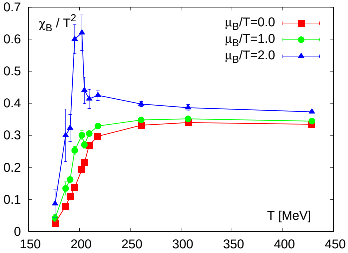




small cut off effects
SB limits well matched

Pressure and baryon susceptibility at finite μ





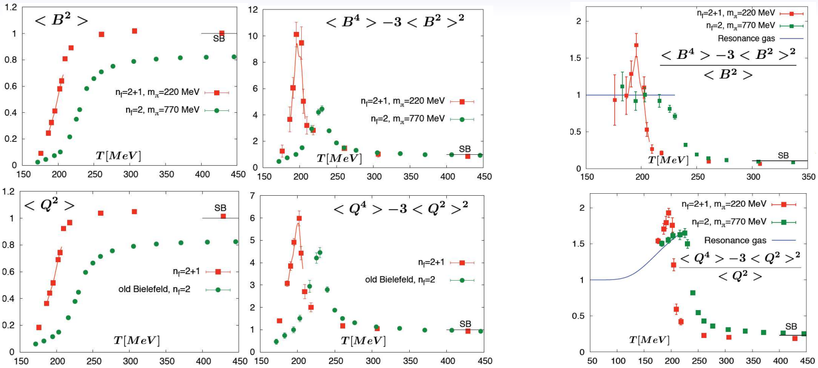
$$\Delta p = p - p(\mu = 0)$$

for light (u,d) quarks

$$\chi_B = 2c_{200}^{BQS} + 12c_{400}^{BQS}\left(\frac{\mu_B}{T}\right)^2$$

large baryon number fluctuation

Hadronic fluctuation

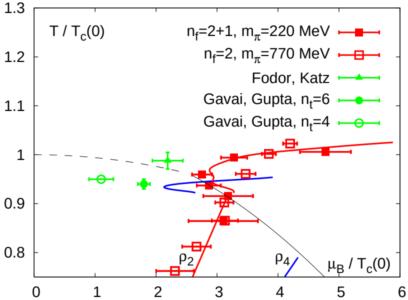


$$\frac{\langle X^4\rangle - 3\langle X^2\rangle^2}{\langle X^2\rangle} = \frac{1}{12}\frac{c_4^X}{c_2^X}$$

- Fluctuations increase with decreasing mass. $\Longrightarrow$ physical pion mass
- Peak shown for baryon at on $N_\tau = 4$ lattices. $\Longrightarrow$ $N_\tau = 6$ lattices.

Convergence radius for Taylor expansion of the pressure

$$\rho_n = \sqrt{\frac{c_n^x}{c_{n+2}^x}}$$



Hint of the phase boundary?

Conclusion

- 2+1 flavor QCD on a line of constant physics:
  physical Kaon mass and pseudo scalar mass as low as $220\ \mathrm{MeV}$
- a wide temperature extent $(140\ \mathrm{MeV} \sim 800\ \mathrm{MeV})$,
  corresponding to lattice cut-off a changing by factor of 6
- p4fat3 improved staggered action:
  reduced flavor mixing and improved bulk thermodynamics to $\mathcal{O}(a^4)$
- $N_\tau = 4, 6, (8)$ lattices: cut-off effects are under good control
- EoS at finite temperature: integration method
  pressure, energy and entropy density, 10% differences from the SB limit at $T \sim 700\ \mathrm{MeV}$ showing that QGP at this temperature is strongly interacting matter
- EoS at finite density: Taylor expansion of the pressure
  hadronic fluctuations, convergence radius