A quasiparticle model for QCD thermodynamics

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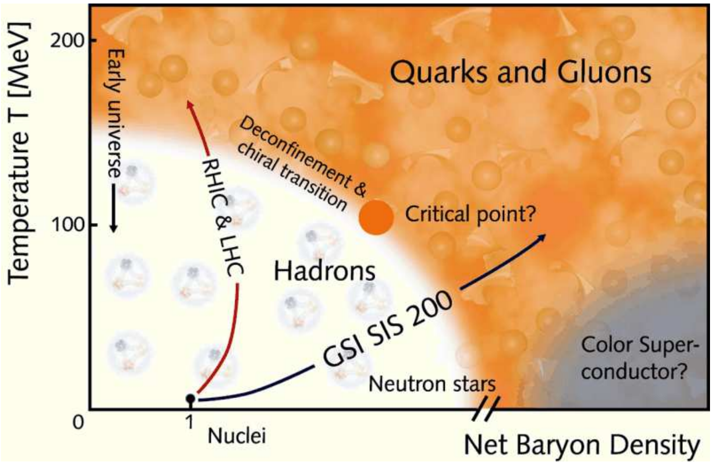
Modelling the CHIRAL
and DECONFINEMENT transitions
of QCD

Part I: In collaboration with Thomas Hell, Simon Rößner, Michael A. Thaler and Wolfram Weise, TU Munich

Part II: In collaboration with Kevin Dusling and Ismail Zahed, SUNY @ Stony Brook

The goal: QCD phase diagram



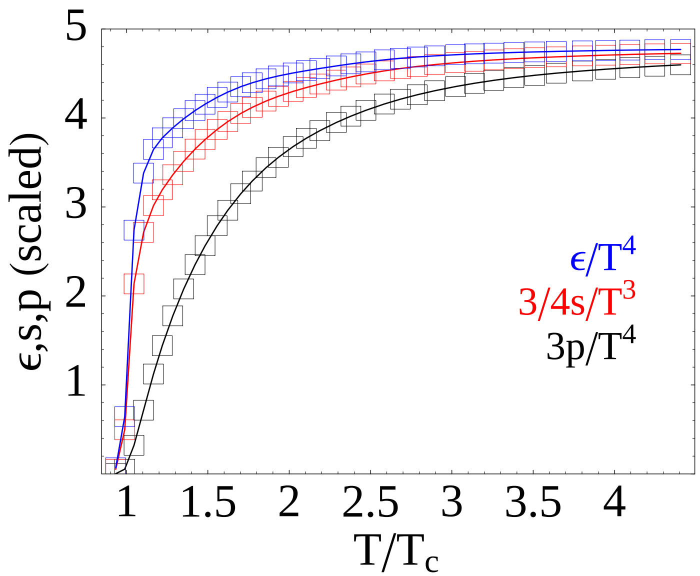
- QCD has a rich phase structure
- Many challenging items:
  - order of the phase transition
  - critical point
  - deconfinement and chiral symmetry
  - color superconductivity at high μ

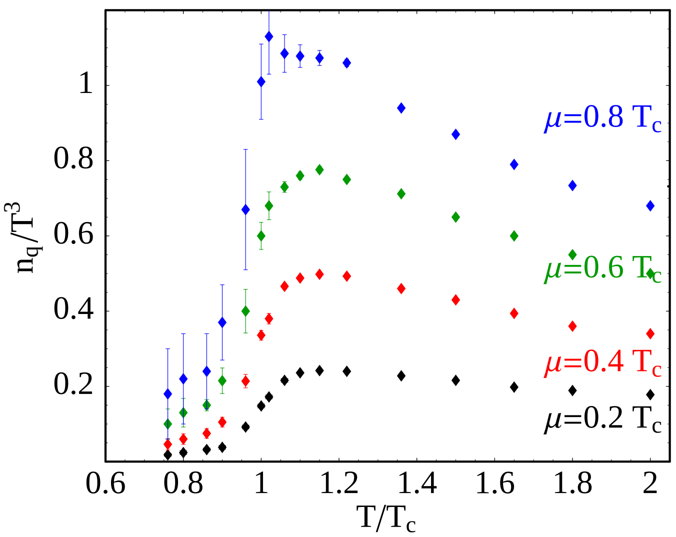
Possible approaches:

- Lattice QCD:
  - quarks easily introduced at $\mu = 0$
  - recent lattice data at small μ.

- Field theoretical models:
  - interpretation in terms of effective D.O.F.
  - regions not reachable on the lattice yet

“Data” base: QCD thermodynamics on the lattice





- **Equation of state of pure gauge QCD:**
  - pressure
  - energy density
  - entropy density

Boyd *et al.*, NPB 469 (1996).

- **Extrapolations to non-zero chemical potential**
  - different observables
  - Taylor expansion method

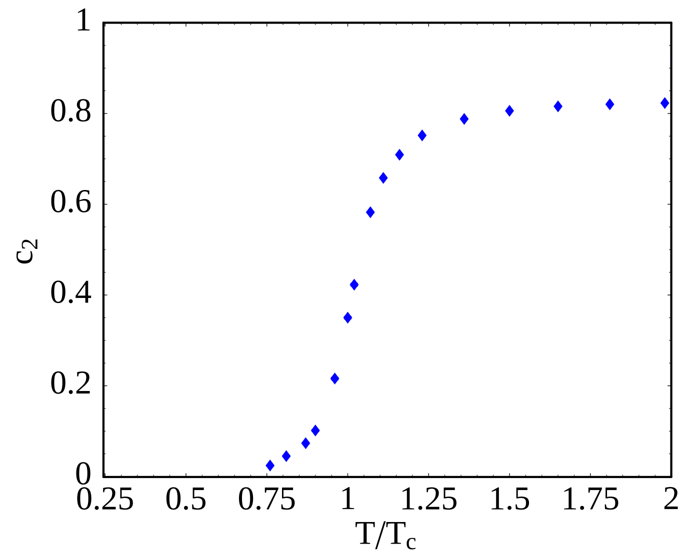
Allton *et al.*, (2005).

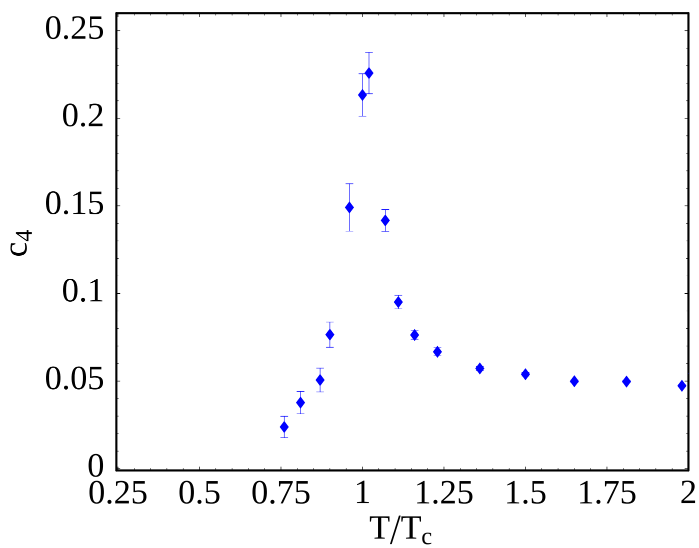
More details on the Taylor expansion method

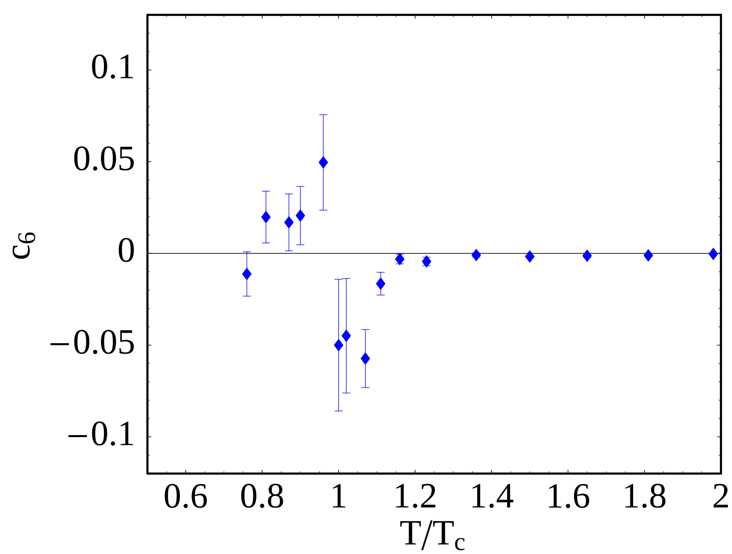
- The pressure can be written as a Taylor series in μ/T around $\mu = 0$:

$$\frac{p\left(T,\mu\right)}{T^4} = \sum_{n=0}^{\infty} c_n\left(T\right)\left(\frac{\mu}{T}\right)^n ;$$

$$c_n\left(T\right) = \frac{1}{n!}\frac{\partial^n\left(p\left(T,\mu\right)/T^4\right)}{\partial\left(\mu/T\right)^n}\Bigg|_{\mu=0}$$







Lattice data from Allton *et al.* (2005).

Purpose of our work

Part I: PNJL model in 3+1 dimensions

- Describe chiral and deconfinement phase transitions within a phenomenological model and study their interplay
- Interpret QCD thermodynamics in terms of effective degrees of freedom
- Study regions of the phase diagram NOT EXPLORABLE ON THE LATTICE

Part II: PNJL model in 0+1 dimensions

- Can the model catch some features of the thermodynamics in 3+1 dimensions?
- Build a RANDOM MATRIX model
- Study the Dirac spectrum across the phase transition

Our approach: Polyakov loop extended Nambu Jona-Lasinio (**PNJL**) model

Synthesis of

Polyakov loop dynamics

$\Rightarrow$ Aspects of **CONFINEMENT**

Nambu Jona-Lasinio model

$\Rightarrow$ **CHIRAL** phase transition

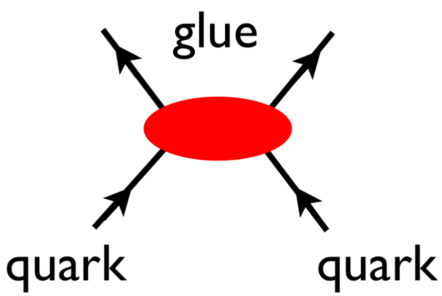
Identify order parameters as collective degrees of freedom
which drive dynamics and thermodynamics

Pisarski PRD62 (2000); Fukushima PLB591 (2004); C.R., Rößner, Thaler, Weise PRD73 (2006); PRD75 (2007).

The Nambu Jona-Lasinio model

$$J_\mu^a(x) = \bar{\psi}(x)\gamma_\mu \frac{\lambda^a}{2}\psi(x) \qquad \delta S \sim -g^2 \int d^4x \int d^4y J(x) D(x-y) J(y)$$

- ASSUMPTION : short correlation range for color transport between quarks



$$\mathcal{L}_{int} = -G_c J_\mu^a(x) J_a^\mu(x)$$

Local $SU(N_C)$ gauge symmetry of QCD $\Rightarrow$ Global $SU(N_C)$ gauge symmetry of NJL

- Gap equation:

$$M = m_0 - G\langle\bar{\psi}\psi\rangle$$

- Chiral condensate:

$$\langle\bar{\psi}\psi\rangle = -2iN_f N_c \int \frac{d^4p}{(2\pi)^4} \frac{M\theta(\Lambda^2 - \vec{p}^{\,2})}{p^2 - M^2 + i\epsilon}$$

➠ Spontaneous chiral symmetry breaking

...but no confinement

Part I

PNJL model in 3+1 dimensions

PNJL (Polyakov loop extended NJL) model

Starting point: NJL model in temporal background gauge field

$$\mathcal{S}_E(\psi,\psi^\dagger,\phi) = \int_0^{\beta=1/T} d\tau \int d^3x \left[\psi^\dagger\,\partial_\tau\,\psi + \mathcal{H}(\psi,\psi^\dagger,\phi)\right] - \frac{V}{T} V\left(\Phi, T\right).$$

where:

$$\mathcal{H} = -i\psi^\dagger\left(\vec{\alpha}\cdot\vec{\nabla} + \gamma_4\,m_0 - A_4\right)\psi - \frac{G}{2}\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}\,i\gamma_5\vec{\tau}\,\psi\right)^2\right].$$

Coupling between Polyakov loop and quarks uniquely determined by covariant derivative D_μ.

$$\Phi\left(x\right) = \frac{1}{N_c}\mathrm{Tr}\left[\mathcal{P}\exp\left(i\int_0^\beta A_4\left(x,\tau\right)d\tau\right)\right] = \frac{1}{3}\mathrm{Tr}\exp\left[\frac{i\mathcal{A}_4^a\lambda_a}{T}\right]$$

Parameters

Λ [GeV]	0.65
G [GeV^{-2}]	10.1
m_0 [MeV]	5.5

Physical quantities

f_π [MeV]	92.4
$\lvert\langle\bar{\psi}\psi\rangle\rvert^{1/3}$ [MeV]	247
m_π [MeV]	139.3

P.Meisinger and M.Ogilvie PLB379 (1996)- K. Fukushima PLB591 (2004)-C.R., M. Thaler, W. Weise PRD73 (2006)

Polyakov loop potential

The Polyakov loop is the order parameter related to the $Z(N_c)$ symmetry

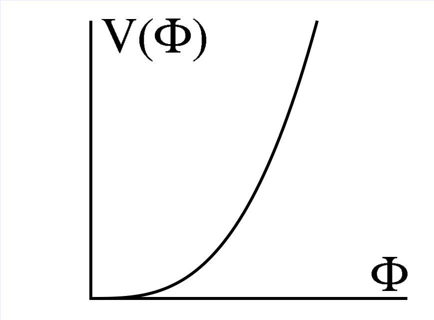
$$\frac{V(\Phi,T)}{T^4} = -\frac{b_2(T)}{2}\Phi^*\Phi - b_4\left(\frac{T_0}{T}\right)^3 \ln[1-6\Phi^*\Phi+4\left(\Phi^{*3}+\Phi^3\right)-3\left(\Phi^*\Phi\right)^2]$$

with

$$b_2(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2$$

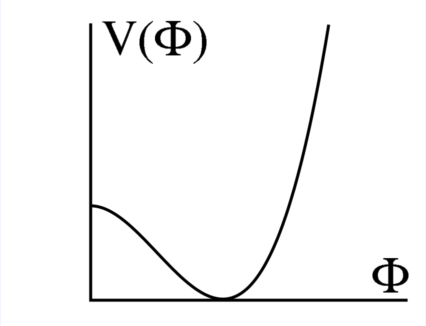
$T < T_0$

- color confinement
- $\langle\Phi\rangle = 0 \longrightarrow Z(3)$ unbroken



$T > T_0$

- color deconfinement
- $\langle\Phi\rangle \neq 0 \longrightarrow Z(3)$ broken

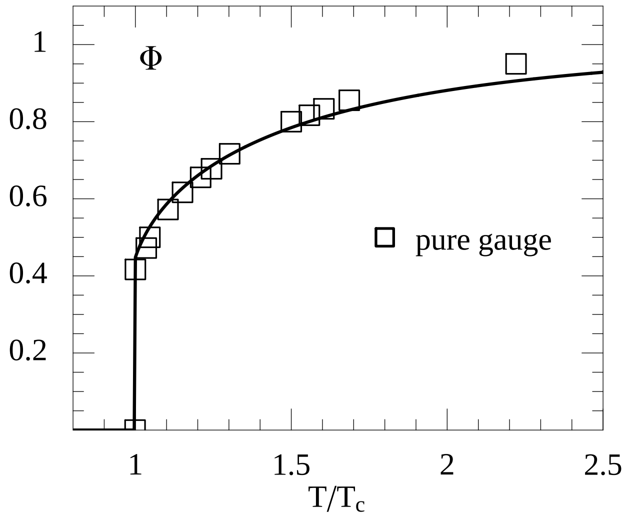


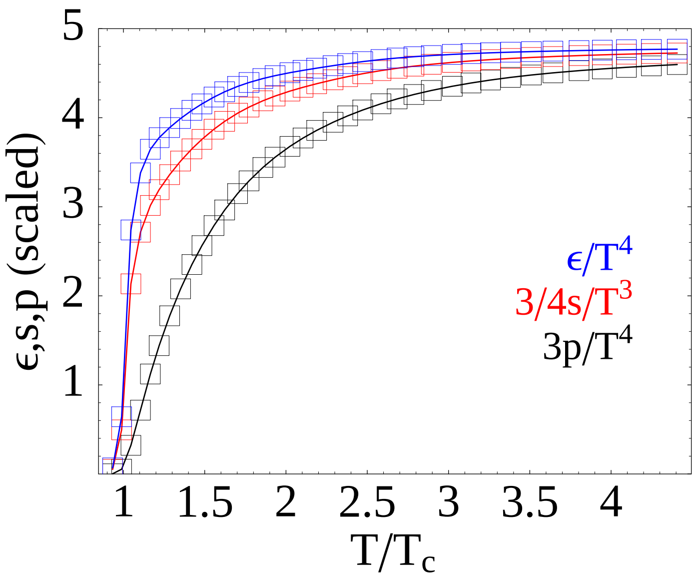
R. Pisarski, PRD62 (2000); K. Fukushima, PLB591 (2004); C.R., M. Thaler, W. Weise PRD73 (2006);

C.R., S. Rößner and W. Weise, PRD75 (2007).

Fit to Pure Gauge QCD lattice data

- Minimization of $V(\Phi, T)$: Polyakov loop behaviour as a function of T
- Comparison with lattice data from Kaczmarek *et al.* PLB 543 (2002)





- $p(T) = -V(\Phi(T), T)$
- $s(T) = \frac{dp}{dT} = -\frac{dV(\Phi(T),T)}{dT}$
- $\epsilon(T) = T\frac{dp}{dT} - p = Ts(T) - p(T)$

- Comparison with lattice data from Boyd *et al.* NPB 469 (1996)

PNJL model at finite temperature and quark chemical potential

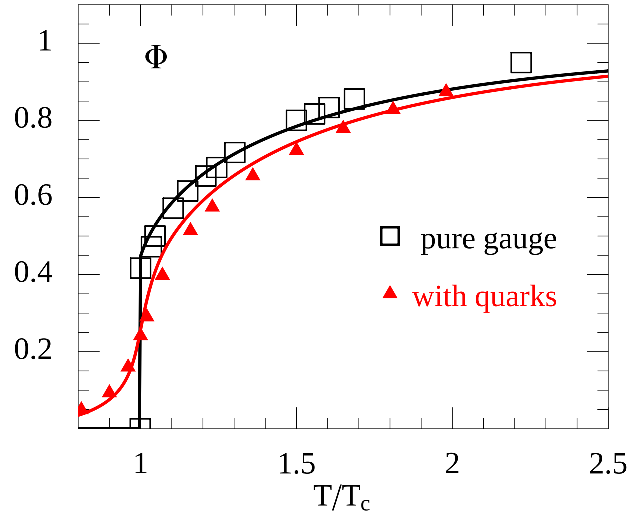
$$\Omega\left(T,\mu,\sigma,\Phi\right)=V\left(\Phi,T\right)+\frac{\sigma^2}{2G}$$
$$-2N_f\int\frac{\mathrm{d}^3p}{(2\pi)^3}\Big\{3E_p+T\Big[\ln\Big[1+3\Phi\mathrm{e}^{-(E_p-\mu)/T}+3\Phi^*\mathrm{e}^{-2(E_p-\mu)/T}+\mathrm{e}^{-3(E_p-\mu)/T}\Big]$$
$$+\ \ln\Big[1+3\Phi^*\mathrm{e}^{-(E_p+\mu)/T}+3\Phi\mathrm{e}^{-2(E_p+\mu)/T}+\mathrm{e}^{-3(E_p+\mu)/T}\Big]\Big]\Big\}$$

with $E_p=\sqrt{\vec{p}^{\,2}+m^2}$ and $m=m_0-\langle\sigma\rangle=m_0-2G\langle\bar{\psi}\psi\rangle$ is the constituent quark mass.

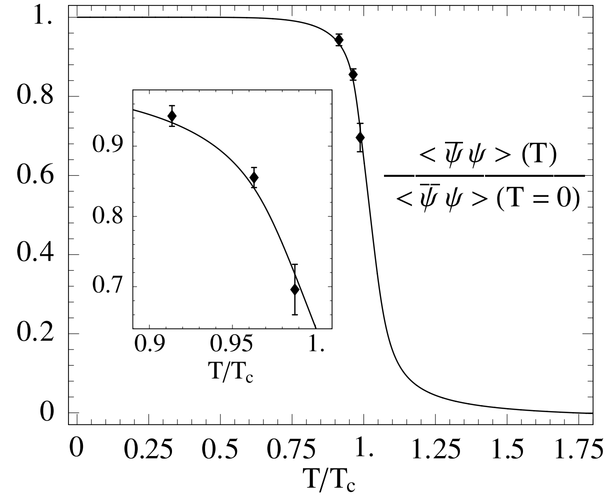
High temperature limit: $\Phi\longrightarrow 1$, $\Phi^*\longrightarrow 1$: we re-obtain the standard NJL formula:

$$\ln\left[1+3\Phi\mathrm{e}^{-(E_p-\mu)/T}+3\Phi^*\mathrm{e}^{-2(E_p-\mu)/T}+\mathrm{e}^{-3(E_p-\mu)/T}\right]$$
$$\downarrow\quad T\rightarrow\infty$$
$$\ln\left[1+\mathrm{e}^{-(E_p-\mu)/T}\right]^3=3\ln\left[1+\mathrm{e}^{-(E_p-\mu)/T}\right]$$

$\mu = 0$ PREDICTIONS



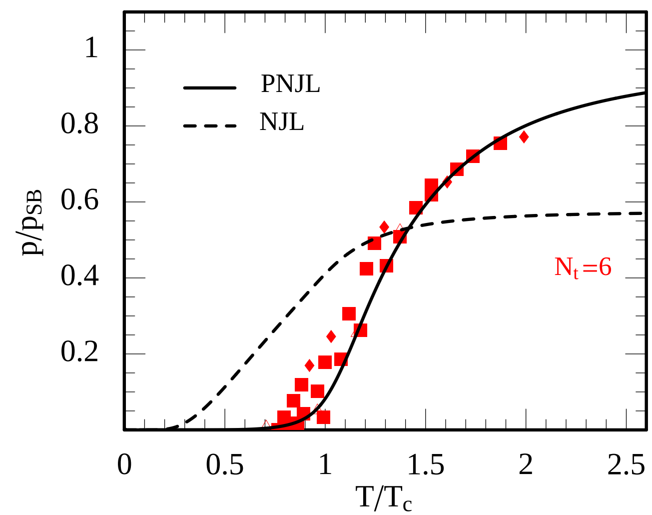
Lattice data from Kaczmarek *et al.* PLB 543 (2002) and Kaczmarek and Zantow PRD71 (2005).



Lattice data from Boyd *et al.* PLB349 (1995).

❖ Scaled pressure as a function of T/T_c

$$\frac{p\,(T,\mu=0)}{T^4} = -\frac{\Omega\,(T,\mu=0)}{T^4}$$



Lattice data from CP-PACS collaboration (2001)

PNJL model results from:

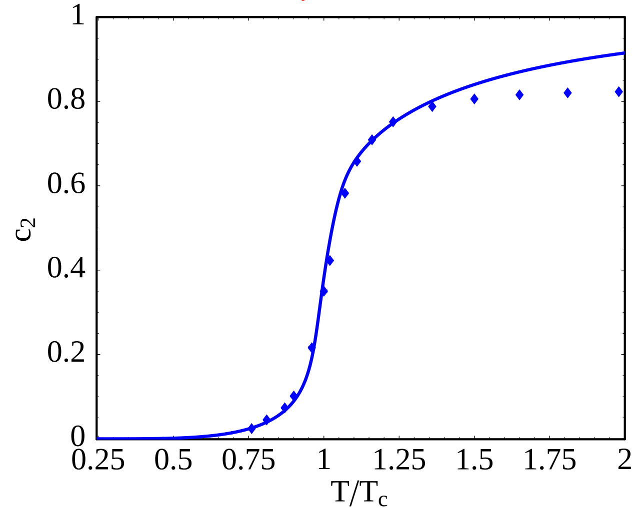
C.R., M. A. Thaler and W. Weise, PRD 73 (2006).

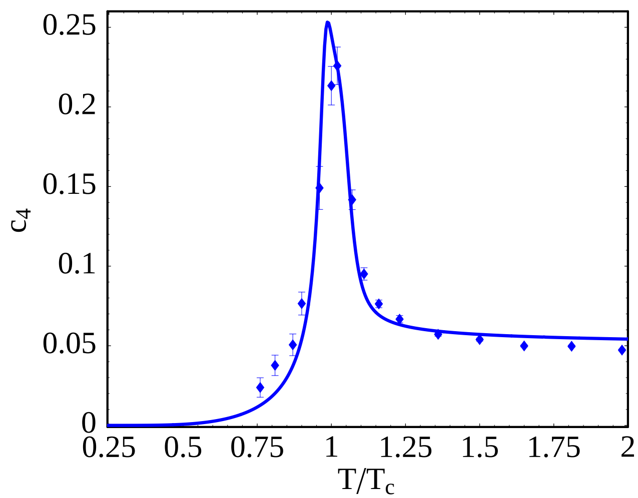
C.R., S. Rößner and W. Weise, PRD75 (2007).

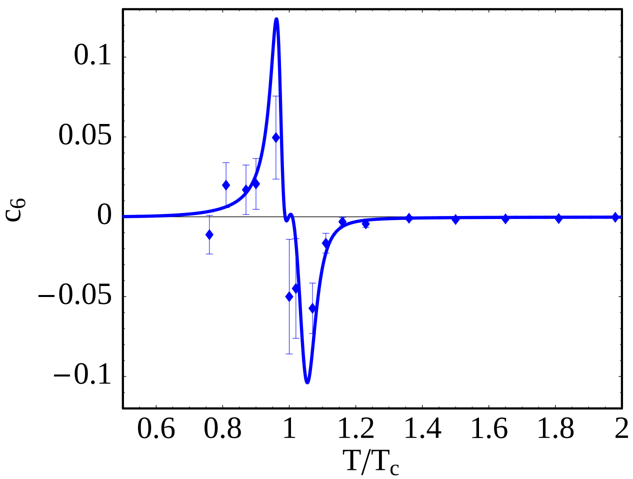
Taylor expansion of pressure around $\mu = 0$

$$\frac{p(T,\mu)}{T^4} = \sum_{n=0}^{\infty} c_n(T)\left(\frac{\mu}{T}\right)^n ;$$

$$c_n(T) = \frac{1}{n!}\frac{\partial^n\left(p(T,\mu)/T^4\right)}{\partial(\mu/T)^n}\bigg|_{\mu=0}$$



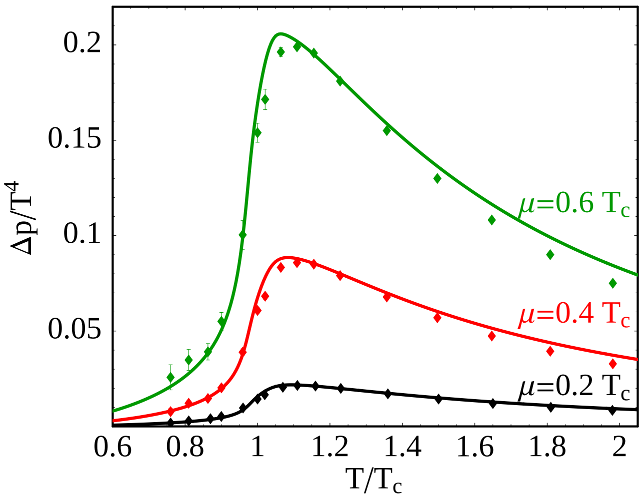


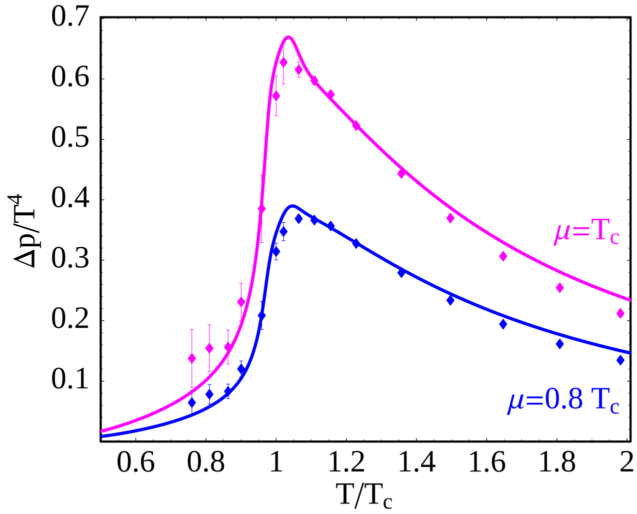


C.R., S. Rößner, M. A. Thaler and W. Weise, EPJC49 (2006).

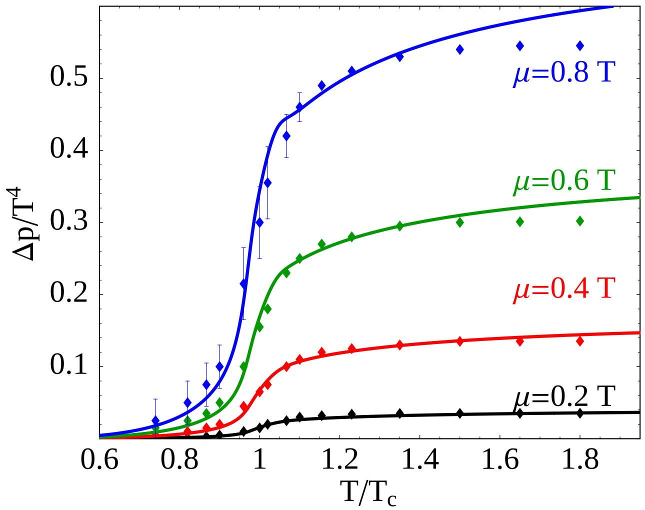
C.R., S. Rößner and W. Weise, PRD75 (2007). Lattice data from Allton *et al.* (2005).

Finite μ PREDICTIONS: pressure





$$\frac{\Delta p\,(T,\mu)}{T^4} = c_2\,(T)\left(\frac{\mu}{T}\right)^2$$
$$+c_4\,(T)\left(\frac{\mu}{T}\right)^4 + c_6\,(T)\left(\frac{\mu}{T}\right)^6$$

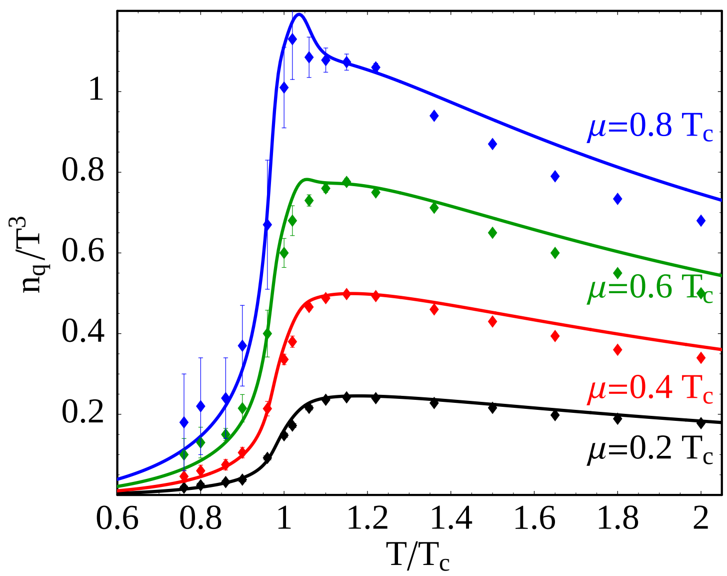


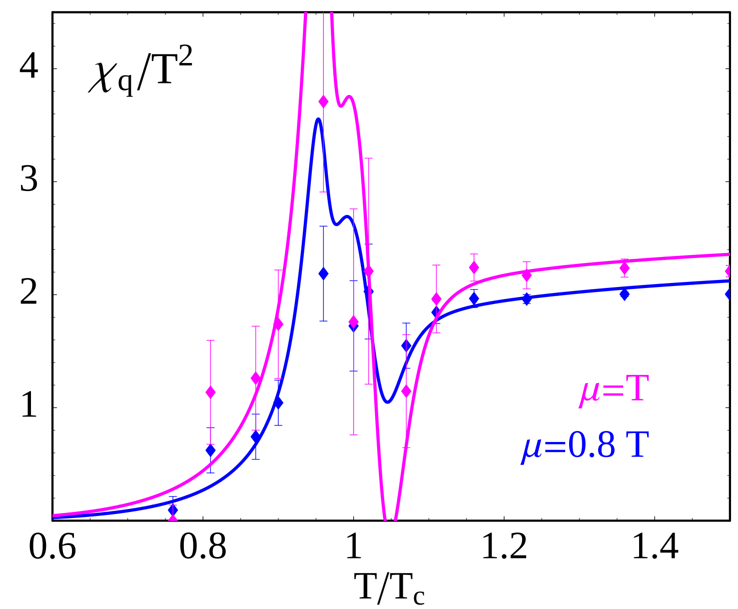
C.R., S. Rößner, M. A. Thaler and W. Weise, EPJC49 (2006). Lattice: Allton *et al.* (2005).

Finite μ PREDICTIONS: quark number density and susceptibilities

$$\frac{n_q(T,\mu)}{T^3} = \frac{\partial\left(p/T^4\right)}{\partial\mu/T} = 2c_2\frac{\mu}{T} + 4c_4\left(\frac{\mu}{T}\right)^3 + 6c_6\left(\frac{\mu}{T}\right)^5$$

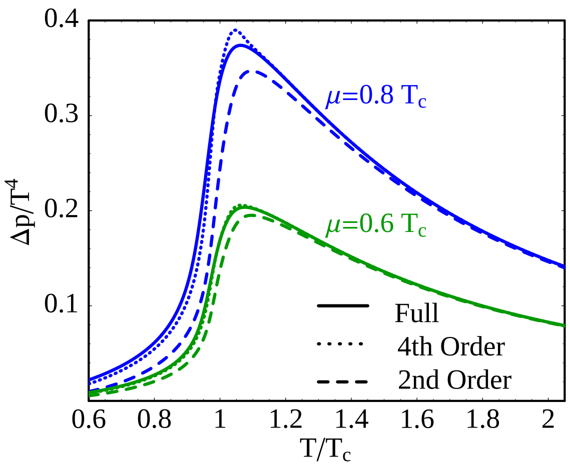
$$\frac{\chi_q(T,\mu)}{T^2} = \frac{\partial\left(n_q/T^3\right)}{\partial\mu/T} = 2c_2 + 12c_4\left(\frac{\mu}{T}\right)^2 + 30c_6\left(\frac{\mu}{T}\right)^4$$

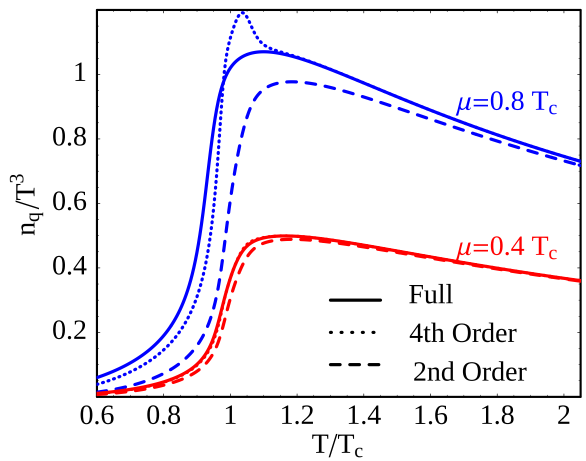


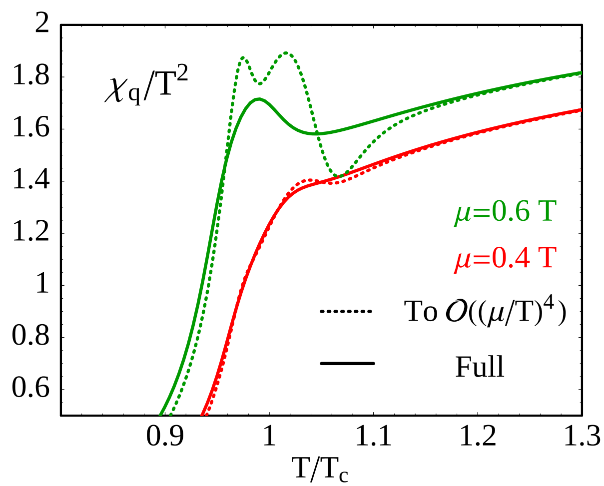


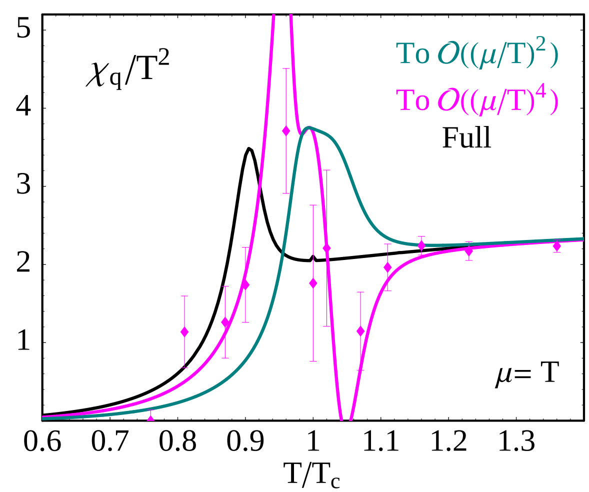
C.R., S. Rößner, M. A. Thaler and W. Weise, EPJC49 (2006). Lattice: Allton *et al.* (2005).

Comparison between Taylor-expanded and full results





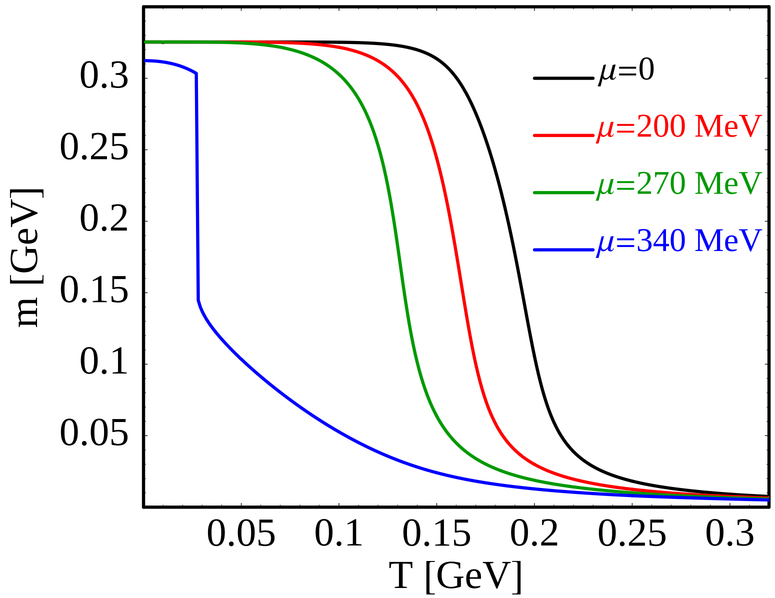


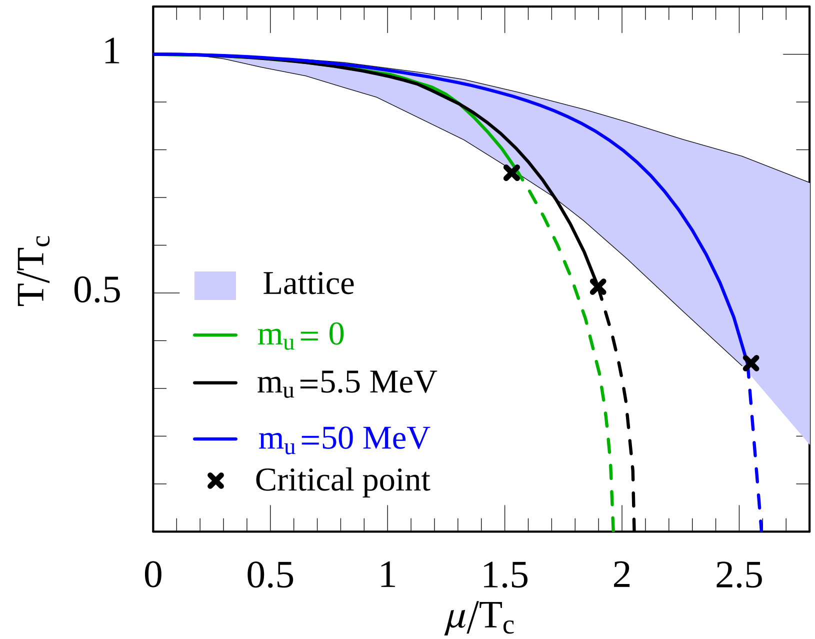


Some problems around T_c in the Taylor series convergence at high μ

C.R., S. Rößner, M. A. Thaler and W. Weise, EPJC49 (2006); C.R., S. Rößner, W. Weise, PLB649 (2007).

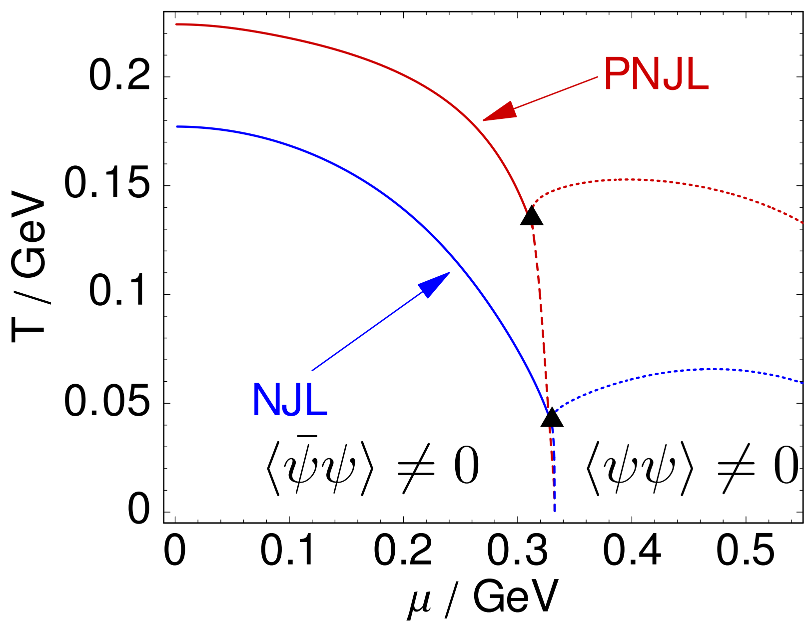
Phase diagram

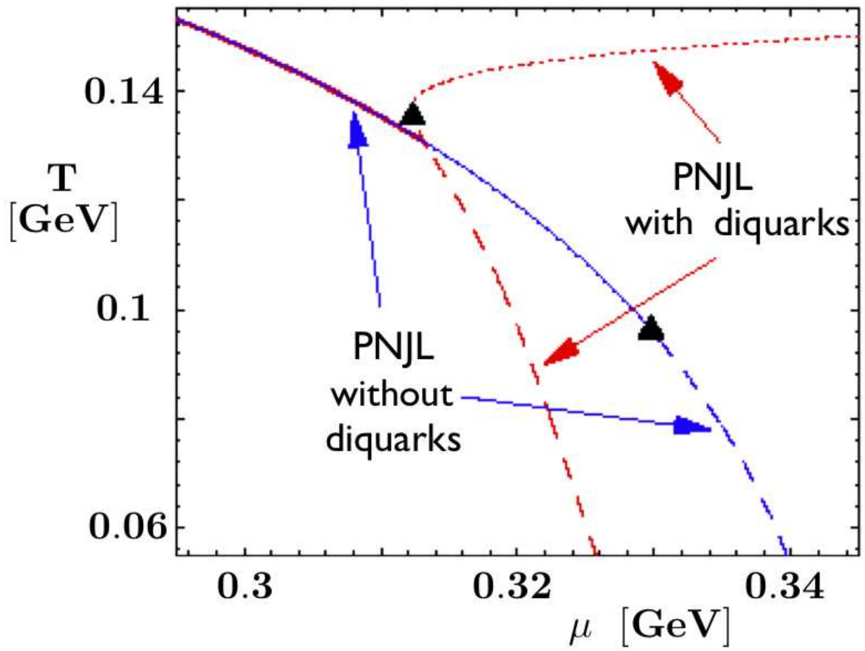




- Critical point: the phase transition becomes first order at high μ
- Location of critical point and phase diagram curvature depend sensitively on the value of the bare quark mass

Phase diagram at high μ: color superconductivity





- Nonvanishing diquark condensate at high μ
- Location of critical point depends sensitively on active degrees of freedom involved

C.R., S. Rößner and W. Weise, PRD75 (2007).

Part II

PNJL model in 0+1 dimensions

Thermodynamic potential

$$\Omega = V(\phi, \bar{\phi}, T) + \frac{N_c \sigma^2}{G} - T \sum_{n=-\infty}^{\infty} Tr_c \ln[\beta^2 \omega^2 + \beta^2 (\omega_n + i(\mu + iA_4))^2]$$

Thermodynamic potential

$$\Omega = V(\phi,\bar{\phi},T) + \frac{N_c\sigma^2}{G} - T\sum_{n=-\infty}^{\infty} Tr_c \ln[\beta^2\omega^2 + \beta^2(\omega_n + i(\mu + iA_4))^2]$$

Sum over ALL Matsubara frequencies

$$\begin{aligned}\Omega &= V(\phi,\bar{\phi},T) + \frac{N_c\sigma^2}{G} - N_c\omega \\ &-T\ln[1 + 3\phi e^{-\beta(\omega-\mu)} + 3\bar{\phi}e^{-2\beta(\omega-\mu)} + e^{-3\beta(\omega-\mu)}] \\ &-T\ln[1 + 3\bar{\phi}e^{-\beta(\omega+\mu)} + 3\phi e^{-2\beta(\omega+\mu)} + e^{-3\beta(\omega+\mu)}]\end{aligned}$$

Thermodynamic potential

$$\Omega = V(\phi,\bar{\phi},T) + \frac{N_c\sigma^2}{G} - T\sum_{n=-\infty}^{\infty} Tr_c \ln[\beta^2\omega^2 + \beta^2(\omega_n + i(\mu + iA_4))^2]$$

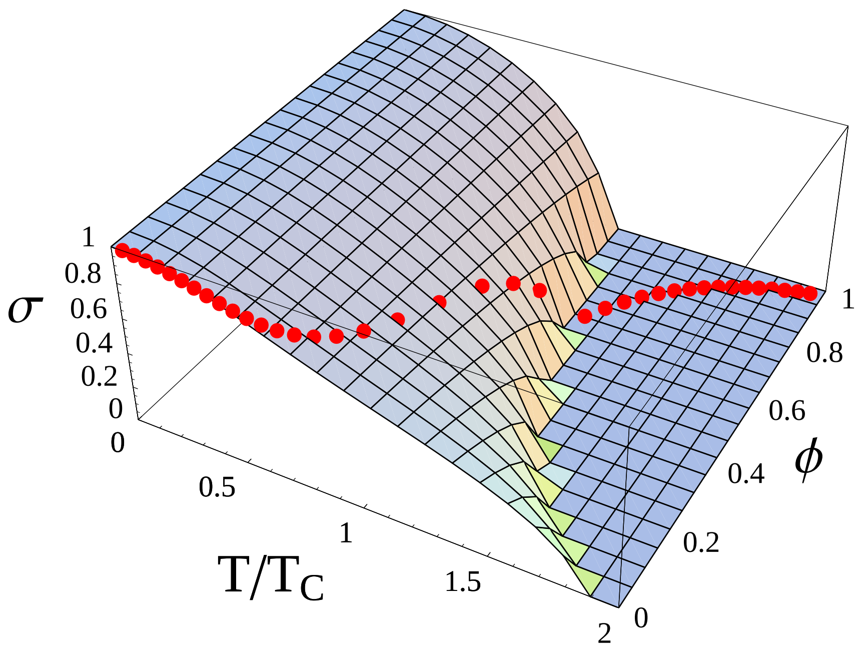
Sum over **ALL** Matsubara frequencies

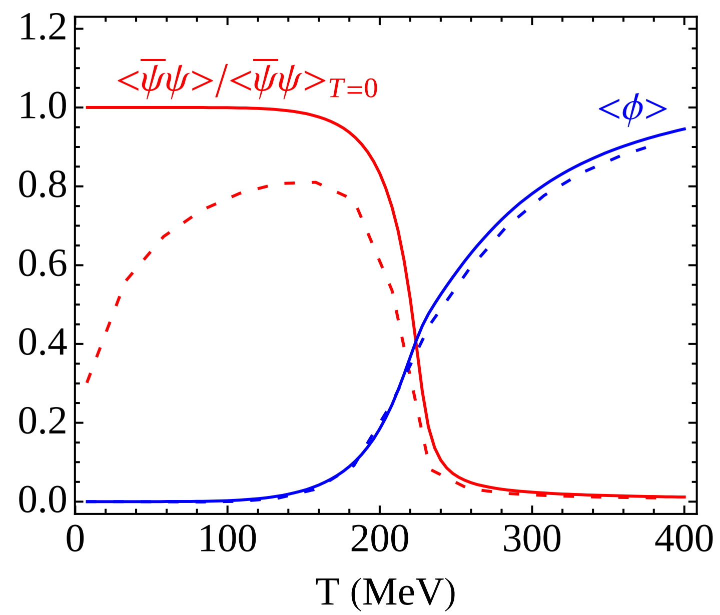
$$\Omega = V(\phi,\bar{\phi},T) + \frac{N_c\sigma^2}{G} - N_c\omega$$
$$-T\ln[1 + 3\phi e^{-\beta(\omega-\mu)} + 3\bar{\phi}e^{-2\beta(\omega-\mu)} + e^{-3\beta(\omega-\mu)}]$$
$$-T\ln[1 + 3\bar{\phi} e^{-\beta(\omega+\mu)} + 3\phi e^{-2\beta(\omega+\mu)} + e^{-3\beta(\omega+\mu)}]$$

Sum over **TWO lower** Matsubara frequencies

$$\Omega = V(\phi,\bar{\phi},T) + \frac{N_c\sigma^2}{G}$$
$$-T\sum_{n=-1}^{0} Tr_c \ln[\beta^2\omega^2 + \beta^2(\omega_n + i(\mu + iA_4))^2]$$

Results

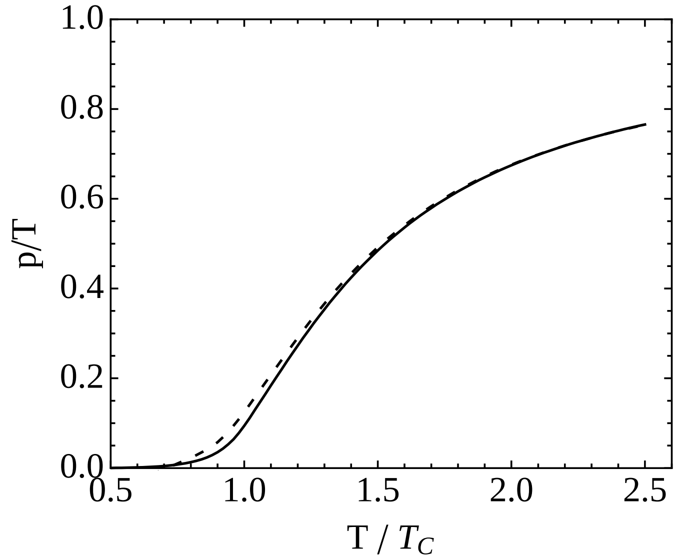


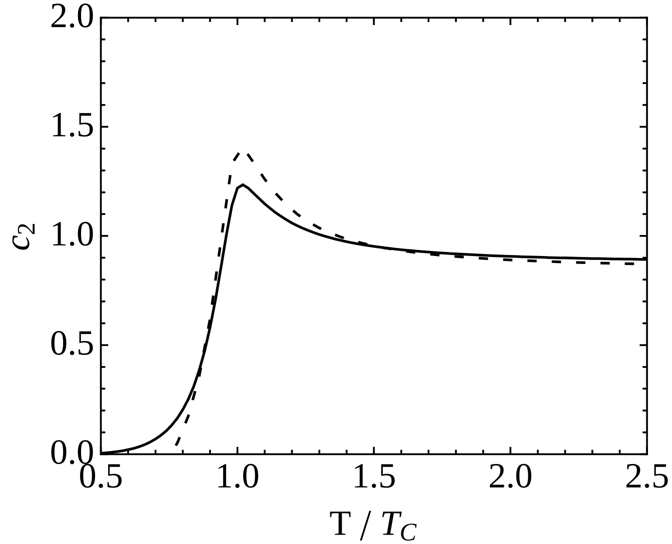


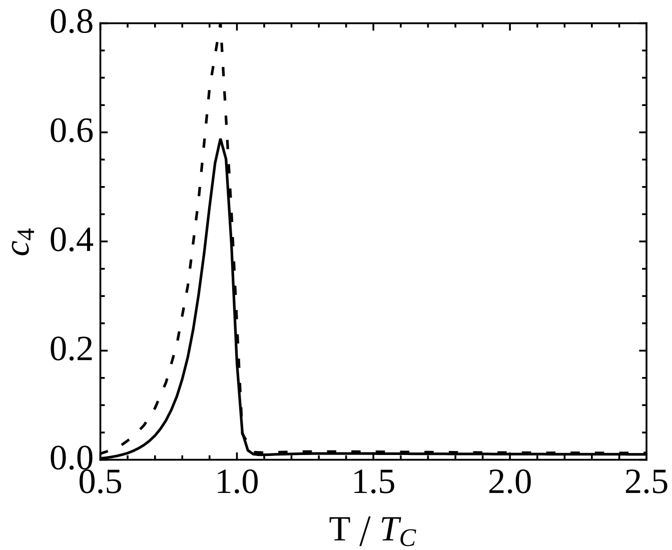
- Good agreement with results in 3+1 dimensions
- Reduction to two lower Matsubaras preserves the main features

K. Dusling, C. R., I. Zahed, 0807.2879.

Thermodynamics







- Main features of 3+1 dim. thermodynamics are nicely reproduced

K. Dusling, C. R., I. Zahed, 0807.2879.

Quark spectrum

- Left and right quarks are put on a discretized grid

$$\int_0^\beta d\tau((\psi^\dagger\psi)^2+(\psi^\dagger i\gamma_5\psi)^2)=4\sum_{n,m,k,l}\delta_{n,m}\delta_{k,l}\psi^\dagger_{Rn}\psi_{Rm}\psi^\dagger_{Lk}\psi_{Ll}$$

- We bosonize quark pairs of opposite chirality using the auxiliary matrix

$$W^{x,y}_{n,m}=\psi^x_{Rm}\psi^{\dagger y}_{Ln}$$

- Resulting partition function

$$Z=\int\mathcal{D}[W]e^{-\frac{N\beta\Sigma^2}{2}TrWW^\dagger}\prod_{f=1}^{N_f}det\begin{pmatrix} m & iW+i\omega_n+\mu-iA^4 \\ iW^\dagger+i\omega_n+\mu-iA^4 & m\end{pmatrix}$$

- We can now study the quark spectrum in the presence of the background gauge field

Random matrix model: details

- The matrix model is composed of a random part $\mathbf{R}$ and a deterministic part $\mathbf{D}$

$$Z = \int \mathcal{D}[R] e^{-\frac{N\beta\Sigma^2}{2} Tr_{\mathbf{x},n,N} RR^\dagger} det_{\mathbf{x},n,N} \mathbf{Q}$$

where

$$\mathbf{Q} = \begin{pmatrix} 0 & \mathbf{D} \\ \mathbf{D} & 0 \end{pmatrix} + \begin{pmatrix} 0 & \mathbf{R} \\ \mathbf{R}^\dagger & 0 \end{pmatrix}$$

and $\mathbf{D} = \mathbf{1}_x \otimes diag(\pi T + \nu, \pi T, \pi T - \nu, -\pi T + \nu, -\pi T, -\pi T - \nu)$, with

$$\nu = T arccos\left(\frac{3\Phi - 1}{2}\right)$$

- The spectral function is

$$\rho(\lambda) = -\frac{1}{\pi} \lim_{\epsilon \to 0} \mathrm{Im} G(\lambda + i\epsilon)$$

- The resolvent for the RMM is

$$G(z)+\frac{1}{6}\sum_{n=1}^{6}\frac{1}{G(z)-\mathbf{D_n}}=z$$

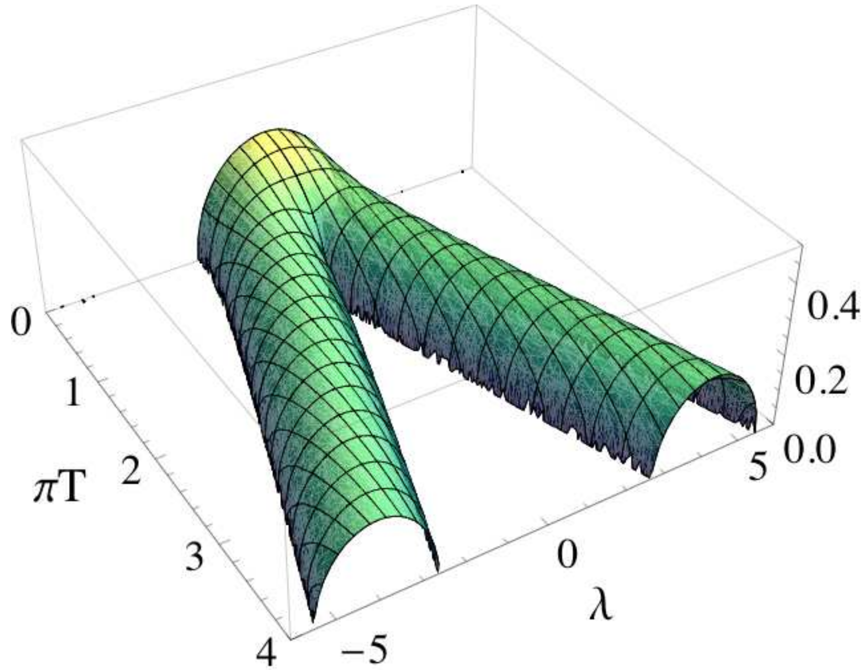
which is a seventh order algebraic equation for $G(z)$.

$$G^7+a_6G^6+a_5G^5+a_4G^4+a_3G^3+a_2G^2+a_1G+a_0=0$$

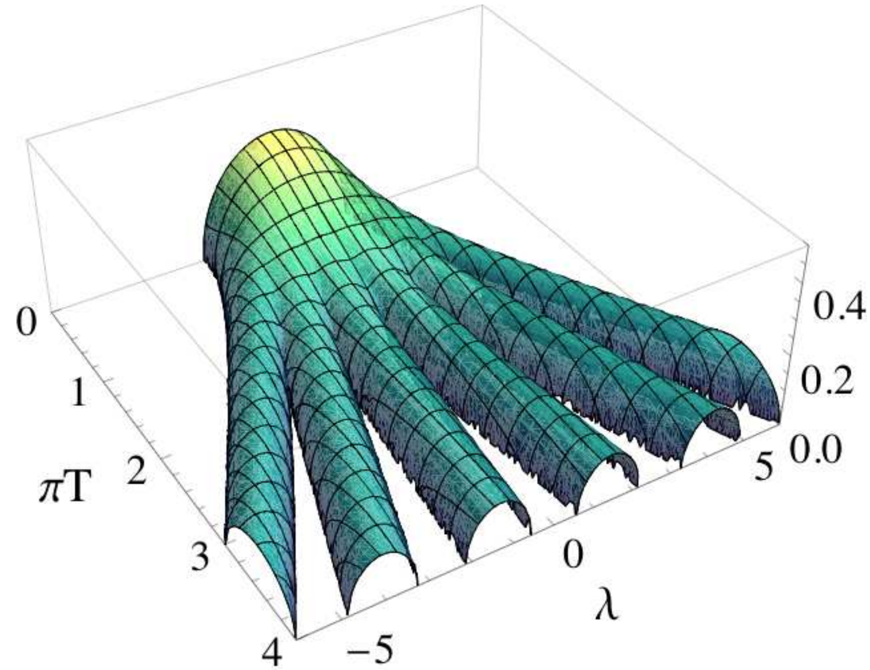
with

$$\begin{aligned}
a_6 &= -6z\\
a_5 &= 1-3\pi^2T^2-2\nu^2+15z^2\\
a_4 &= z(-5+12\pi^2T^2+8\nu^2-20z^2)\\
a_3 &= 3\pi^4T^4+\nu^4+5z^2(2+3z^2)-2\pi^2T^2(1+9z^2)-4/3\nu^2(1+9z^2)\\
a_2 &= -2z(3\pi^4T^4+\nu^4-3\pi^2T^2(1+2z^2)-2\nu^2(1+2z^2)+z^2(5+3z^2))\\
a_1 &= -\pi^6T^6+\nu^4(1/3+z^2)-2\nu^2z^2(2+z^2)+z^4(5+z^2)\\
&\quad +\pi^4T^4(1+2\nu^2+3z^2)-\pi^2T^2(\nu^4+3z^2(2+z^2))\\
a_0 &= -z/3(3\pi^4T^4+\nu^4-6\pi^2T^2z^2-4\nu^2z^2+3z^4)
\end{aligned}$$

Quark spectrum: maximal Polyakov loop effect



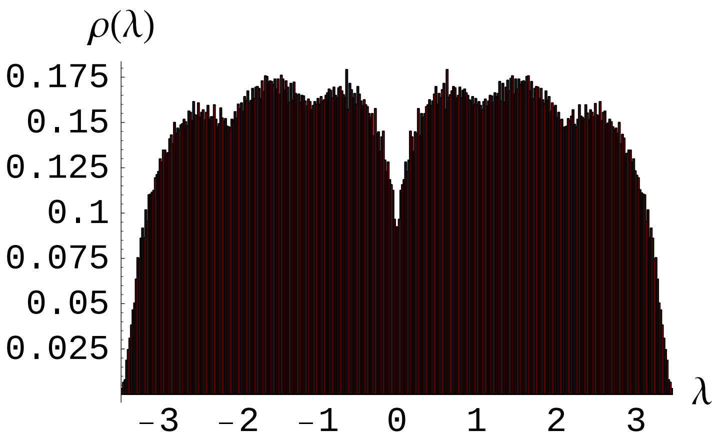
$\Phi = 1$



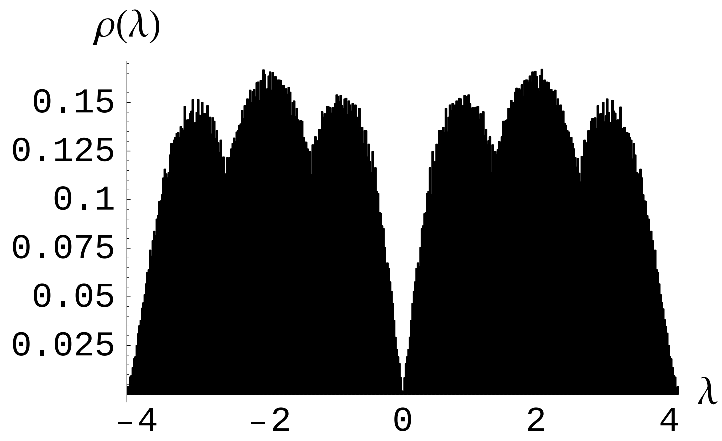
$\Phi = 0$

- Inclusion of Polyakov loop causes the spectra to split into separate domains

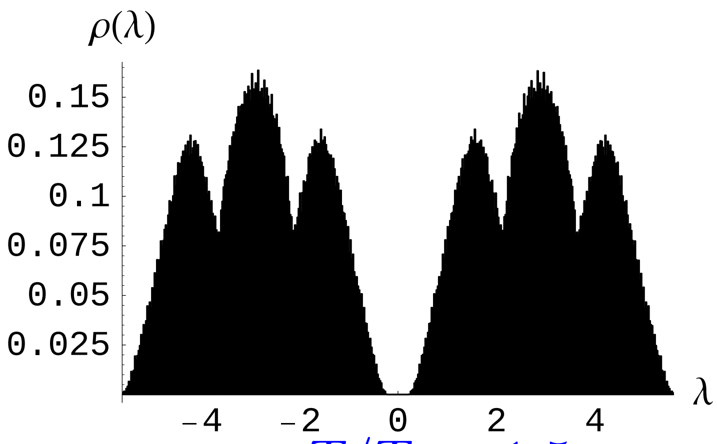
Quark spectrum: averaged Polyakov loop



$T/T_c = 0.75$



$T/T_c = 1$



$T/T_c = 1.5$

- Value of Φ sampled from the distribution given by $V(\Phi, T)$
- Oscillations increase with temperature

K. Dusling, C. R., I. Zahed, 0807.2879.

Conclusions

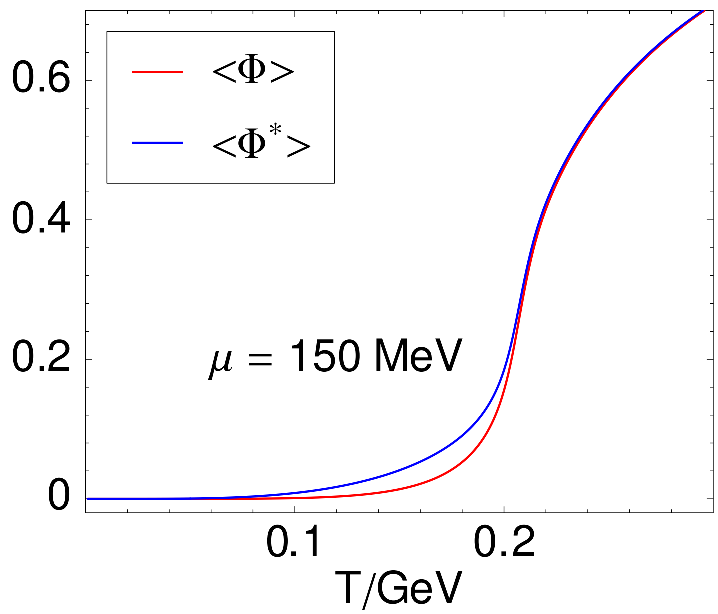
- PNJL model: minimal synthesis of features of confinement and χ-symmetry breaking
- A description of QCD thermodynamics with our simple model works very well
- Phase diagram and critical point
- 0+1 dimensional PNJL model: study of quark spectrum

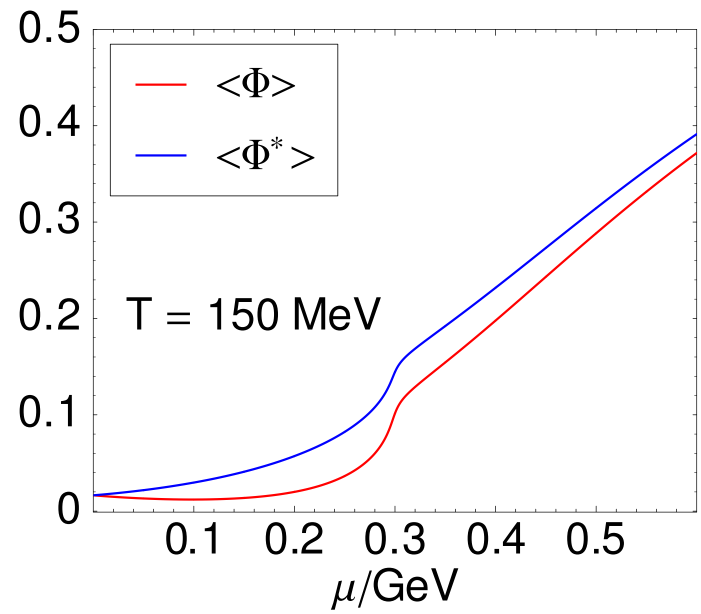
Outlook

- Study of the quark spectrum at finite chemical potential
- Inclusion of strange quarks

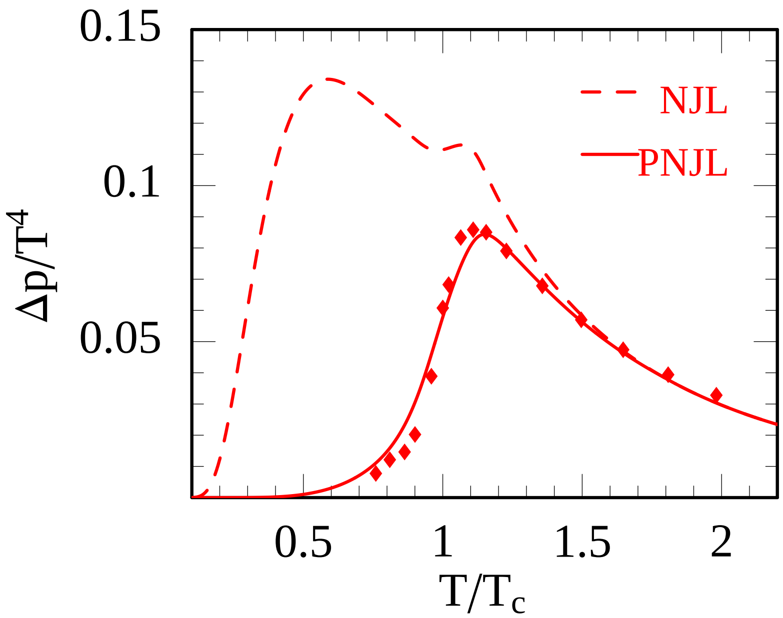
Backup slides

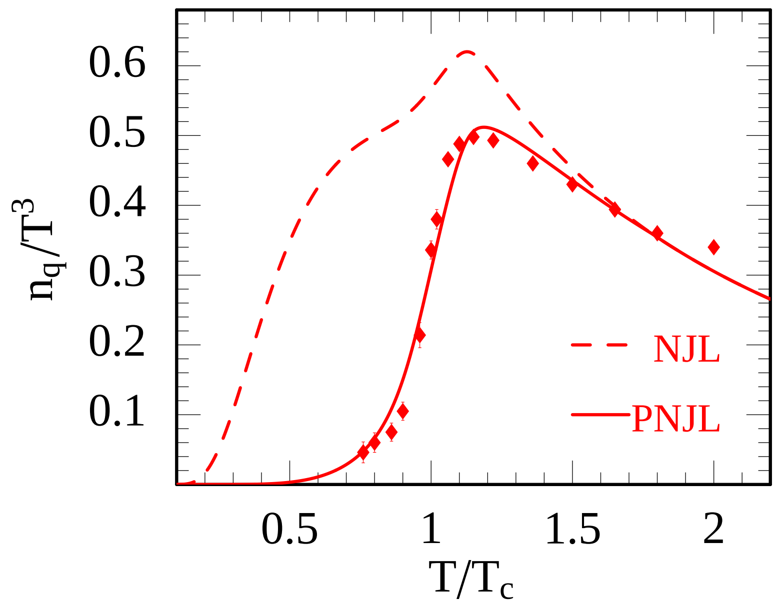
Finite μ results





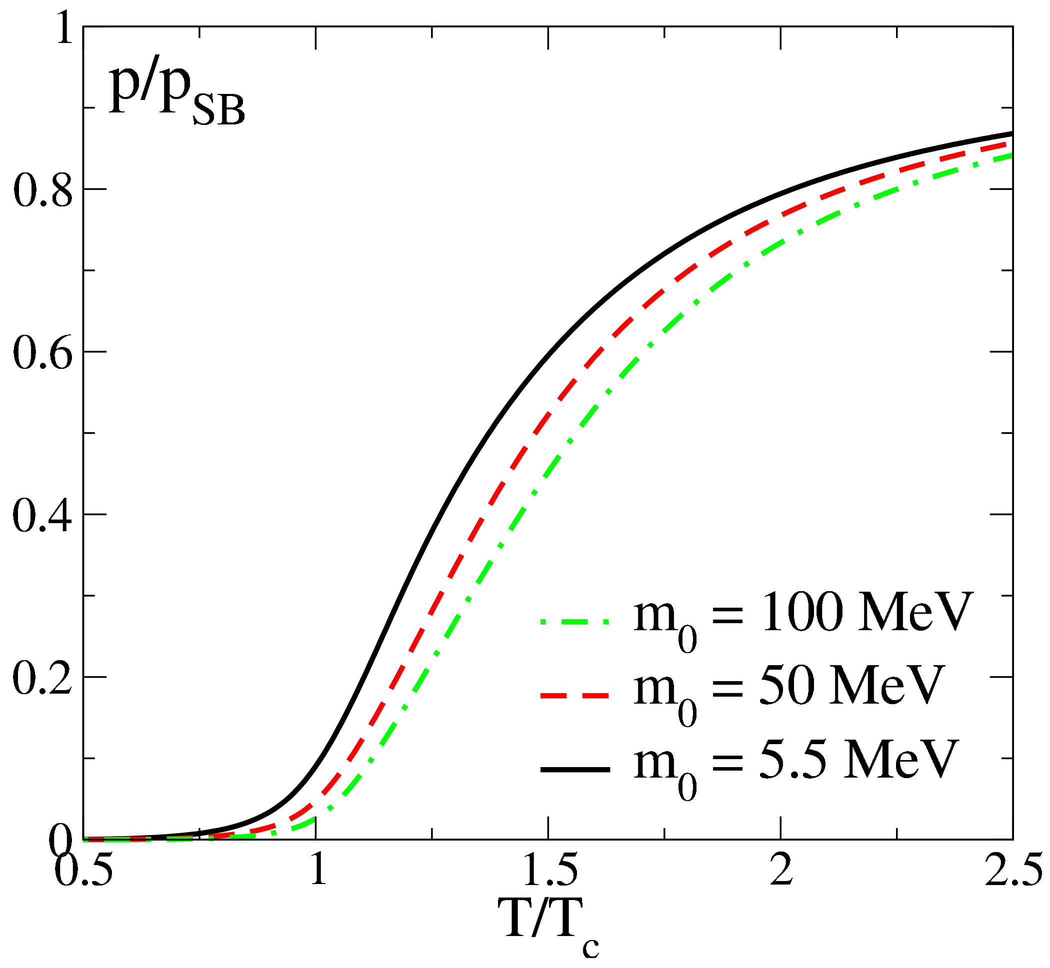
Finite μ results in the standard NJL model





- Comparison between PNJL and standard NJL model results at finite μ
- NJL model alone fails in reproducing lattice data
- Incorporation of confinement crucial to reproduce lattice results

Quark mass dependence



Parameter fixing

We have three free parameters in the model: $m_0,\ \Lambda,\ G$. They are fixed by:

- The pion decay constant f_π is evaluated in the NJL model through the following relation:

$$f_\pi^2 = 4m^2 I_\Lambda^{(1)}(m) \quad \text{where} \quad I_\Lambda^{(1)}(m) = -iN_c \int \frac{d^4p}{(2\pi)^4} \frac{\theta\left(\Lambda^2 - \vec{p}^{\,2}\right)}{(p^2 - m^2 + i\epsilon)^2}.$$

  The empirical value is $f_\pi = 92.4$ MeV.

- The quark condensate becomes

$$\left\langle \bar{\psi}_u \psi_u \right\rangle = -4m I_\Lambda^{(0)}(m) \qquad \text{with} \qquad I_\Lambda^{(0)}(m) = iN_c \int \frac{d^4p}{(2\pi)^4} \frac{\theta\left(\Lambda^2 - \vec{p}^{\,2}\right)}{p^2 - m^2 + i\epsilon}.$$

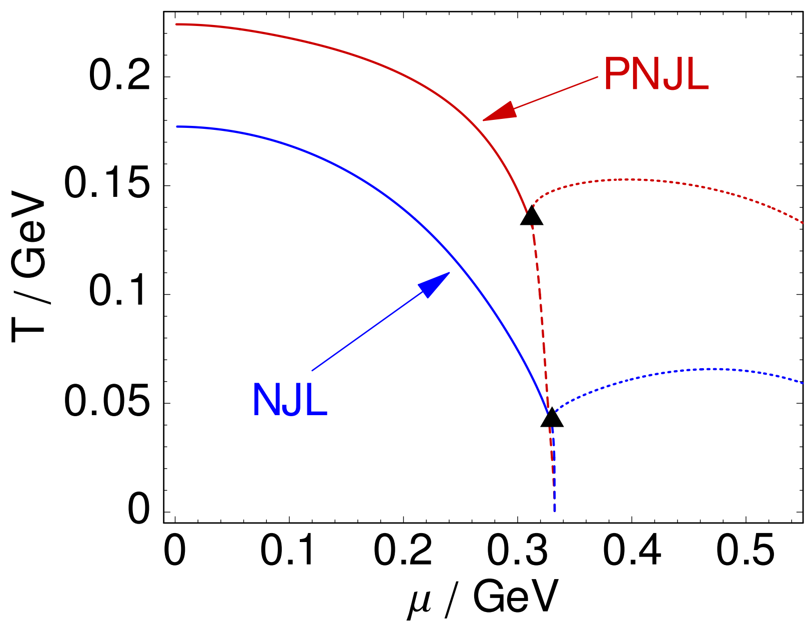
  Its “empirical” value derived from QCD sum rules is

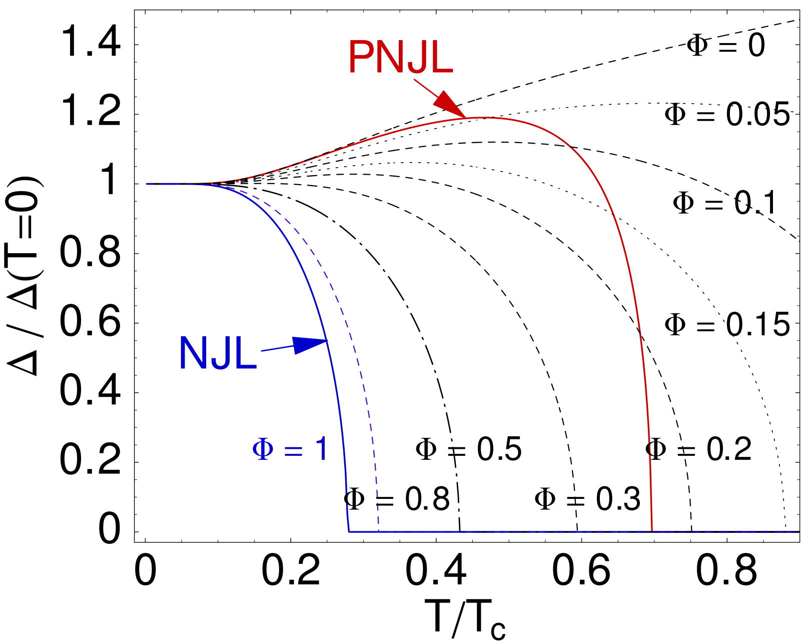
$$\langle \bar{\psi}_u \psi_u \rangle^{1/3} \simeq \langle \bar{\psi}_d \psi_d \rangle^{1/3} = -\,(240 \pm 20)\ \text{MeV}.$$

- The current quark mass m_0 is fixed from the Gell-Mann, Oakes, Renner (GMOR) relation:

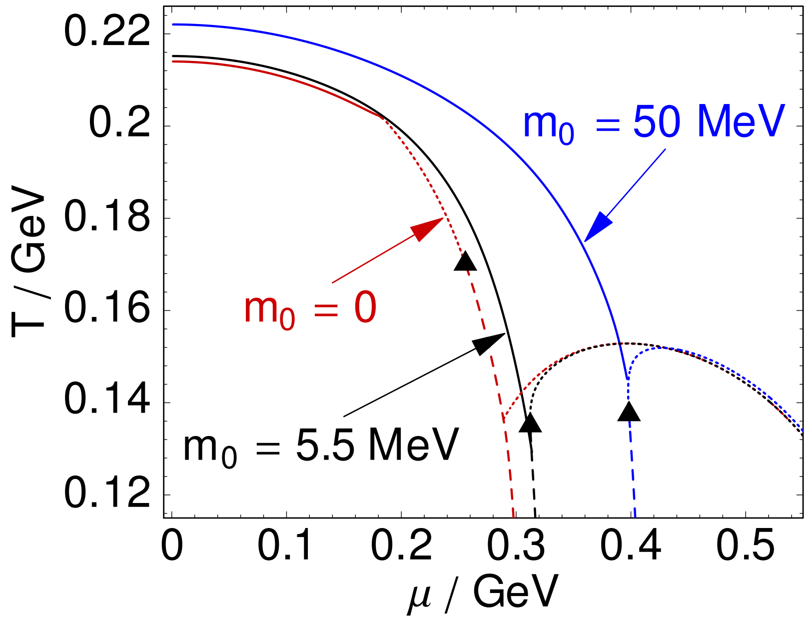
$$m_\pi^2 = \frac{-m_0 \left\langle \bar{\psi}\psi \right\rangle}{f_\pi^2}.$$

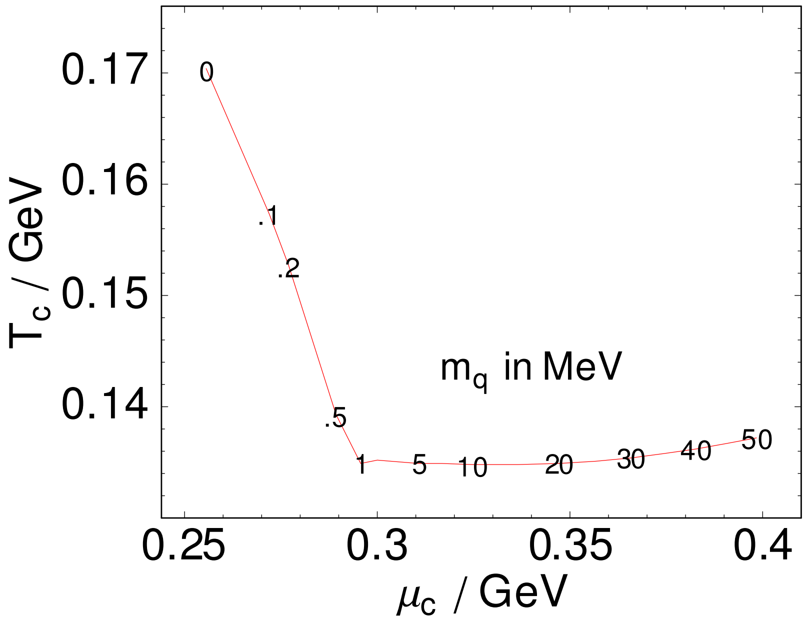
Phase diagram at high μ: NJL vs PNJL model





Quark mass dependence of phase diagram and critical endpoint





Thermodynamic potential: intermediate steps

The thermodynamic potential of the system is:

$$\Omega\left(T,\mu\right)=V\left(\Phi,T\right)-T\sum_n\int\frac{d^3p}{(2\pi)^3}\frac{1}{2}\,\mathrm{Tr}\,\ln\left(\frac{1}{T}\tilde{S}^{-1}\left(i\omega_n,\vec{p}\right)\right)+\frac{\sigma^2}{2G},$$

where $\omega_n=(2n+1)\,\pi\,T$ are the Matsubara frequencies for fermions and

$$\tilde{S}^{-1}\left(p^0,\vec{p}\right)=\begin{pmatrix} \not{p}-\hat{m}-\mu\gamma_0+gA^0\gamma_0 & 0 \\ 0 & \not{p}-\hat{m}+\mu\gamma_0-gA^0\gamma_0 \end{pmatrix}.$$

The constituent quark mass is defined as

$$m=m_0-\langle\sigma\rangle=m_0-G\langle\bar{\psi}\psi\rangle.$$

Final form of Ω:

$$\begin{aligned}\Omega\left(T,\mu\right) \quad &= \quad V\left(\Phi,T\right)+\frac{\sigma^2}{2G}-2N_f\,T\int\frac{\mathrm{d}^3p}{(2\pi)^3}\left\{\mathrm{Tr}_c\ln\left[1+L\,\mathrm{e}^{-(E_p-\mu)/T}\right]\right.\\ &+ \quad \left.\mathrm{Tr}_c\ln\left[1+L^\dagger\,\mathrm{e}^{-(E_p+\mu)/T}\right]+\frac{E_p}{T}\theta\left(\Lambda^2-\vec{p}^{\;2}\right)\right\}\end{aligned}$$

The Z_3 center symmetry of pure gauge QCD (I)

We consider the partition function of pure gauge QCD

$$Z = \int DA \exp\left(-S\left[A\right]\right)$$

with

$$S\left[A\right] = \frac{1}{2g^2}\int_0^\beta dx_4 \int d^3x \mathrm{Tr} F_{\mu\nu}F_{\mu\nu}$$

and

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \left[A_\mu, A_\nu\right], \qquad A_\mu = i A_\mu^a \lambda^a$$

The gauge fields obey periodic boundary conditions:

$$A_\mu\left(\vec{x}, x_4 + \beta\right) = A_\mu\left(\vec{x}, x_4\right)$$

The above action is invariant under gauge transformations of the form

$${}^g A_\mu = g\left(A_\mu + \partial_\mu\right) g^\dagger$$

under which the field strength transforms as

$${}^g F_{\mu\nu} = g F_{\mu\nu} g^\dagger$$

The Z_3 center symmetry of pure gauge QCD (II)

The periodic boundary conditions of A_μ must be maintained. In addition to the strictly periodic gauge transformations

$$g\left(\vec{x}, x_4+\beta\right)=g\left(\vec{x}, x_4\right)$$

there are nontrivial transformations

$$g\left(\vec{x}, x_4+\beta\right)=h g\left(\vec{x}, x_4\right)$$

When such a transformation is applied to a strictly periodic vector potential A_μ, it turns into

$${}^g A_\mu\left(\vec{x}, x_4+\beta\right)=h\,{}^g A_\mu\left(\vec{x}, x_4\right) h^{\dagger}$$

The gauge transformed vector potential ${}^g A_\mu$ obeys periodic boundary conditions only if

$$h\,{}^g A_\mu\left(\vec{x}, x_4+\right) h^{\dagger}={}^g A_\mu\left(\vec{x}, x_4\right)$$

which is satisfied only if h commutes with ${}^g A_\mu$.

h must be an element in the center Z_3 of the $SU(3)$ gauge group

$$h=z 1, z=\exp (2 \pi i n / 3)$$

The Z_3 center symmetry of pure gauge QCD (III)

The Z_3 symmetry gets explicitly broken in the presence of dynamical fields that transform in the fundamental representation.

Polyakov loop: order parameter for Z_3 symmetry in pure gauge QCD. It is defined as

$$\Phi(x) = \frac{1}{N_c}\mathrm{Tr}\left[\mathcal{P}\exp\left(i\int_0^\beta A_4(x,\tau)\,d\tau\right)\right]$$

It transforms nontrivially under Z_3 transformations:

$$^g\Phi(\vec{x}) = z\Phi(\vec{x})$$

The expectation value of the Polyakov loop measures the free energy of an external static quark

$$\langle\Phi\rangle = \exp\left[-\beta F\right]$$

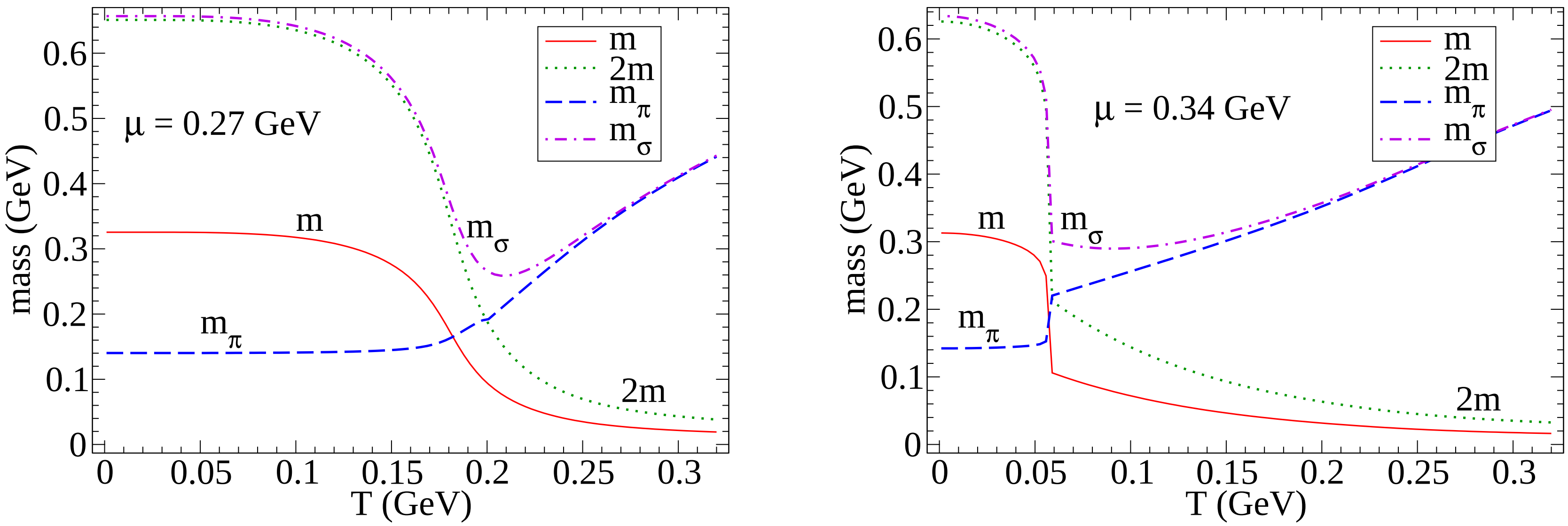
$T < T_0$

- color confinement
- $\langle\Phi\rangle = 0 \longrightarrow Z(3)$ unbroken
- $F = \infty$

$T > T_0$

- color deconfinement
- $\langle\Phi\rangle \neq 0 \longrightarrow Z(3)$ broken
- F finite

Mesonic properties in the PNJL model



H. Hansen, W. M. Alberico, A. Beraudo, A. Molinari, M. Nardi, C. R., hep-ph/0609116, PRD (2007)