Charge Fluctuations and transport coefficients near CEP

Use an effective chiral models and scaling theory to study:

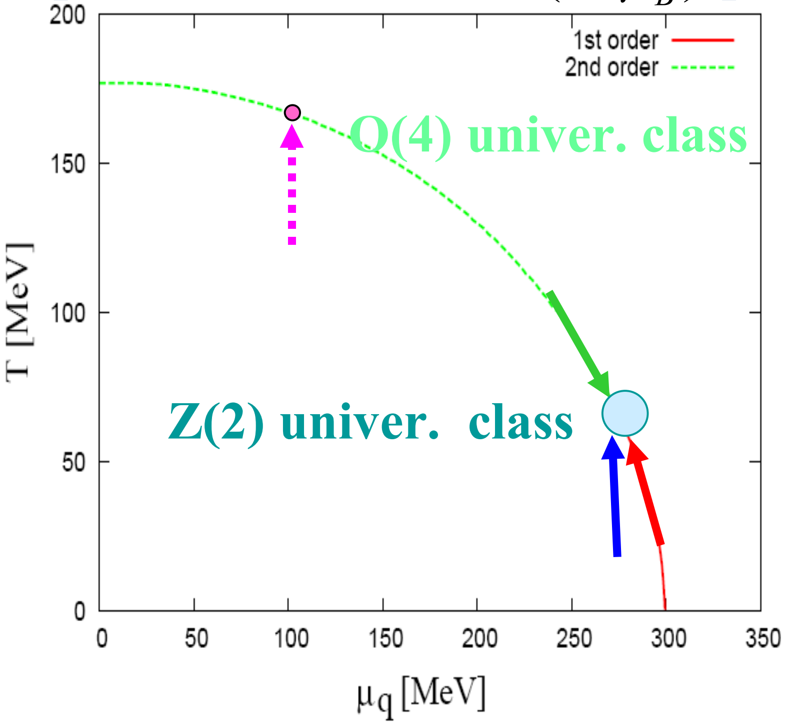
- Charge fluctuations in the presence of spinodal instabilities and their scaling
- Shear, and Bulk viscosities and its scaling near CEP

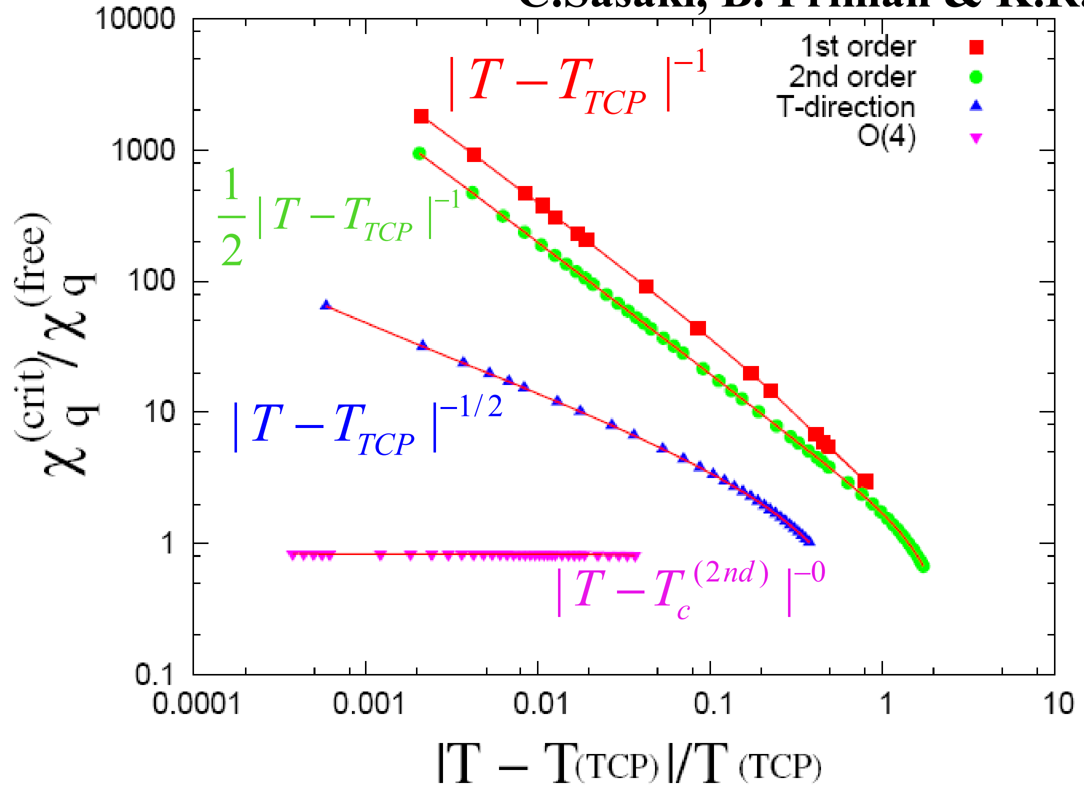
Krzysztof Redlich, Workshop on QCD-CP, INT 08

Scaling properties: $\chi_q = a + b\,|T - T_{TCP}|^{-\theta}$

C.Sasaki, B. Friman & K.R.

The strength of the singularity at TCP depends on direction in (T, μ_B) plane





$\chi_q \propto |T - T_{TCP}|^{-1}$ **along 1st order line**

$\chi_q \propto |T - T_{TCP}|^{-1/2}$ **any direction not parallel**

$\chi_q \propto \frac{1}{2}|T - T_{TCP}|^{-1}$ **along 2nd order line**

See also Y. Hatta, T. Ikeda

Going beyond the mean field:
B.-J. Schaefer & J. Wambach

$\chi_q \propto |T - T_{TCP}|^{-0.53 (m \neq 0 => 0.78)}$

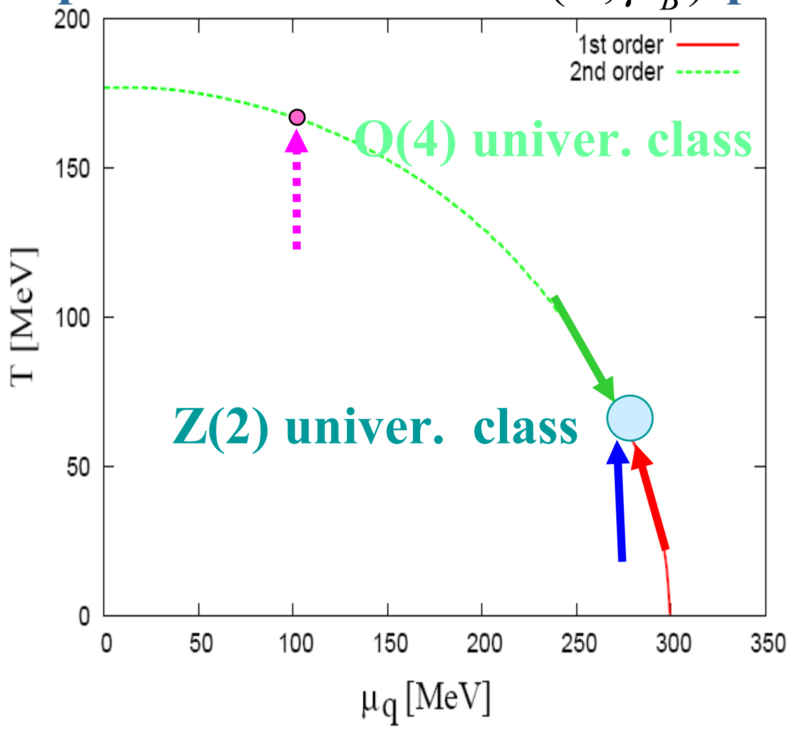
FRG: Stokic, Friman & K.R

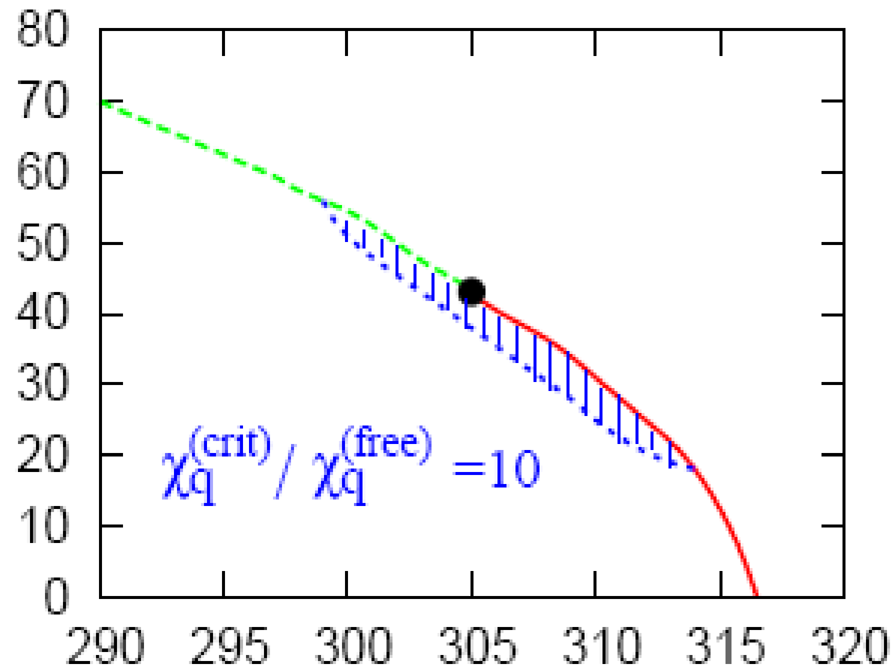
$\alpha = (0MF)\ (-0.25Z(2))(-0.3$

Scaling properties: $\chi_q = a + b\,|T - T_{TCP}|^{-\theta}$

C.Sasaki, B. Friman & K.R.

The strength of the singularity at TCP depends on direction in (T, μ_B) plane





$\chi_q \propto |T - T_{TCP}|^{-1}$ **along 1st order line**

$\chi_q \propto |T - T_{TCP}|^{-1/2}$ **any direction not parallel**

$\chi_q \propto \frac{1}{2}|T - T_{TCP}|^{-1}$ **along 2nd order line**

See also Y. Hatta, T. Ikeda

Going beyond the mean field:
B.-J. Schaefer & J. Wambach

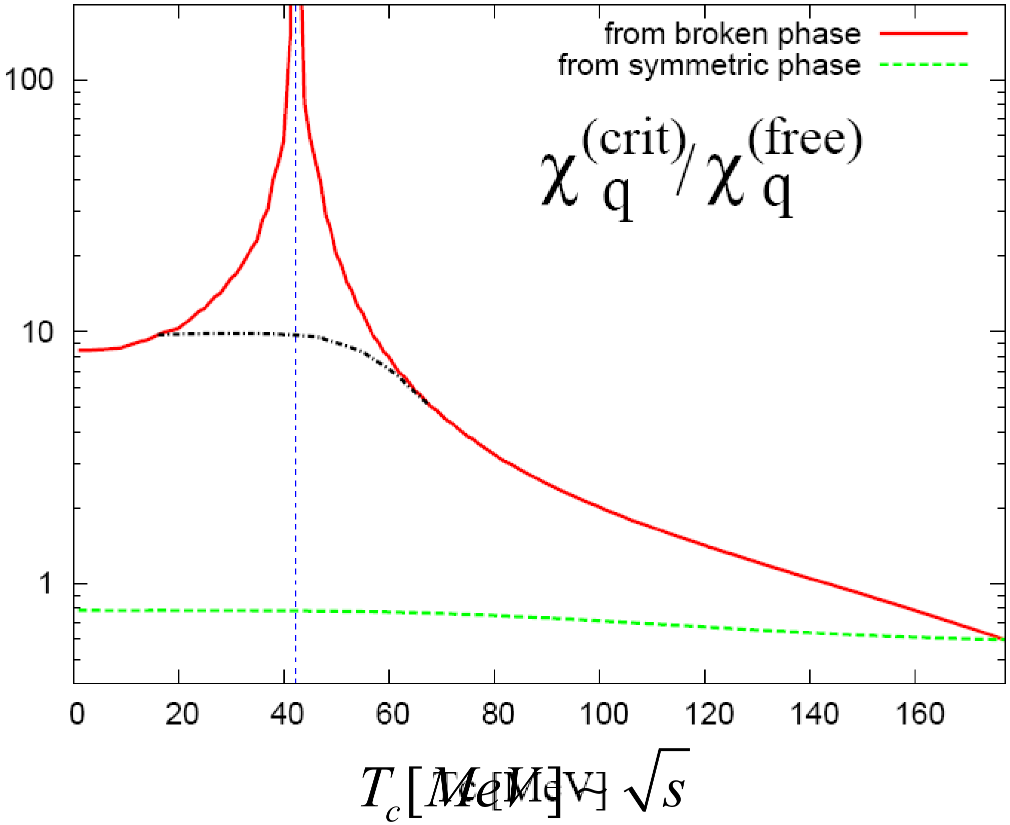
$\chi_q \propto |T - T_{TCP}|^{-0.53(m \neq 0 => 0.78)}$

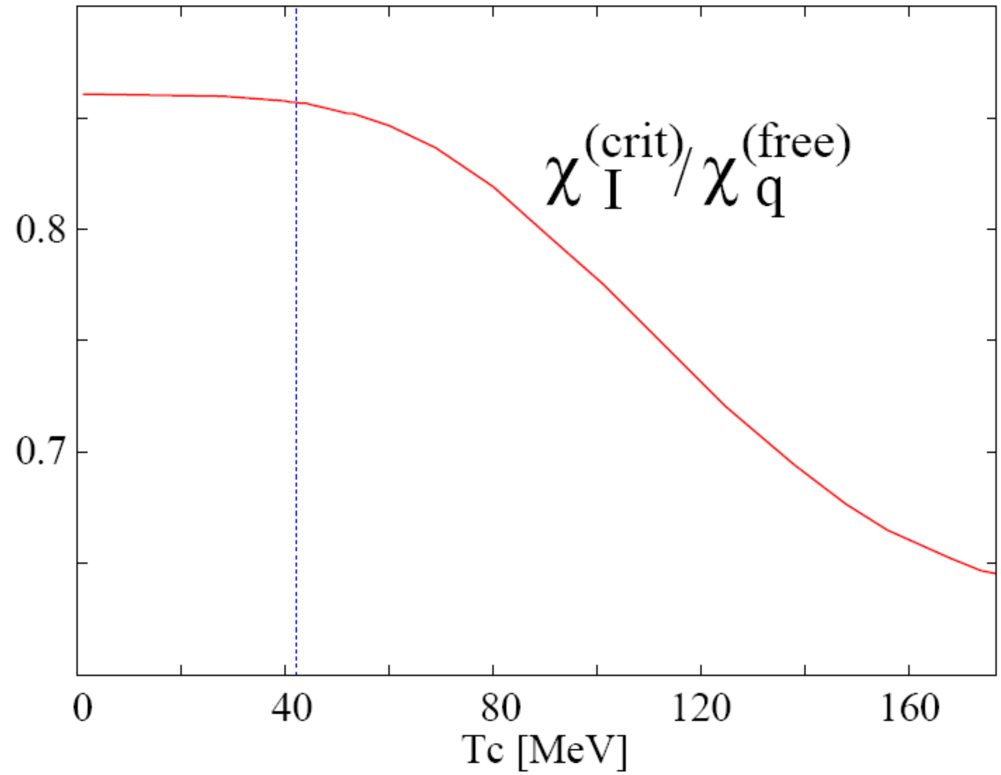
FRG: Stokic, Friman & K.R

$\alpha = (0MF)\ (-0.25Z(2))(-0.3$

Quark and isovector fluctuations along the critical line

NJL-model results: C. Sasaki, B. Friman, K.R.



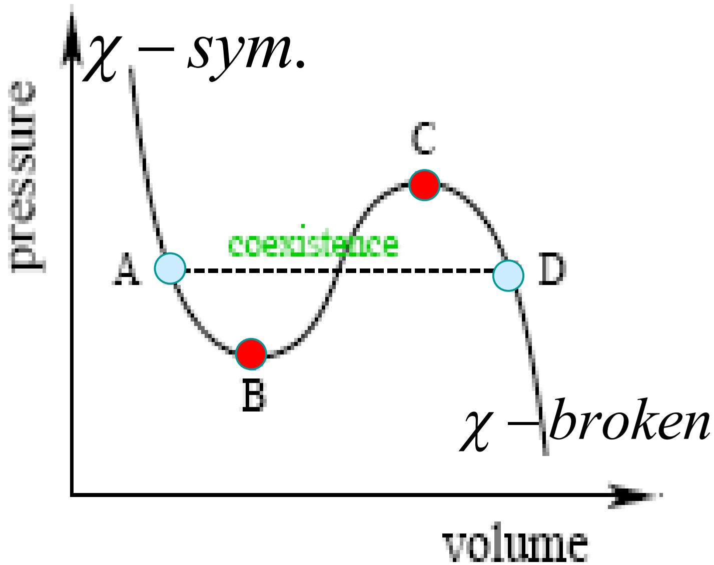


Non-monotonic behavior of the net quark susceptibility as function of (T_c, μ_c) in LGT or $\sqrt{s}$ in HIC

sensitive probes of TCP/CEP

The nature of the 1st order chiral phase transition

instability of a system:



$\partial P / \partial V < 0$: stable

$\partial P / \partial V > 0$: unstable

$\partial P / \partial V = 0$: spinodal

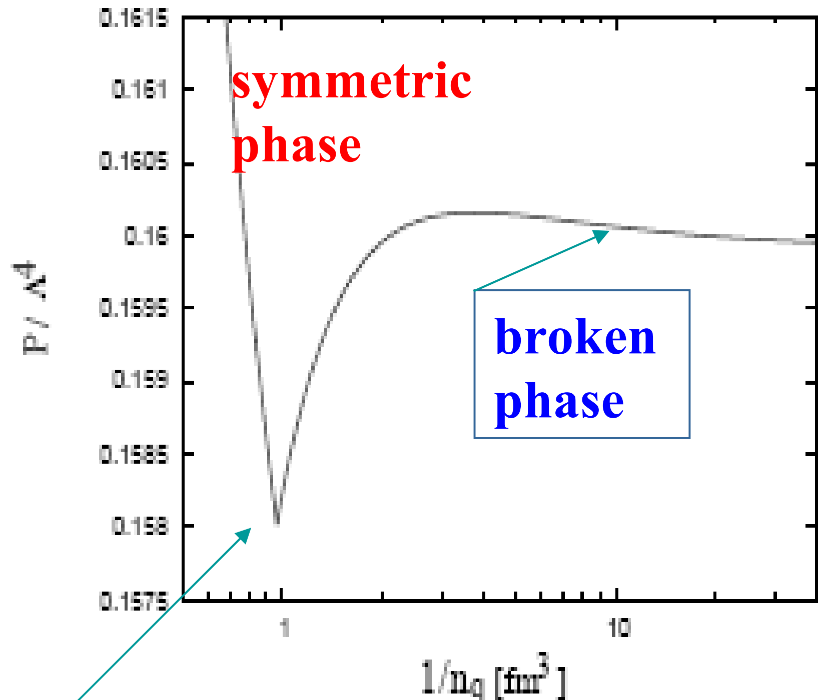
A-B: supercooling (symmetric phase)

B-C: non-equilibrium state

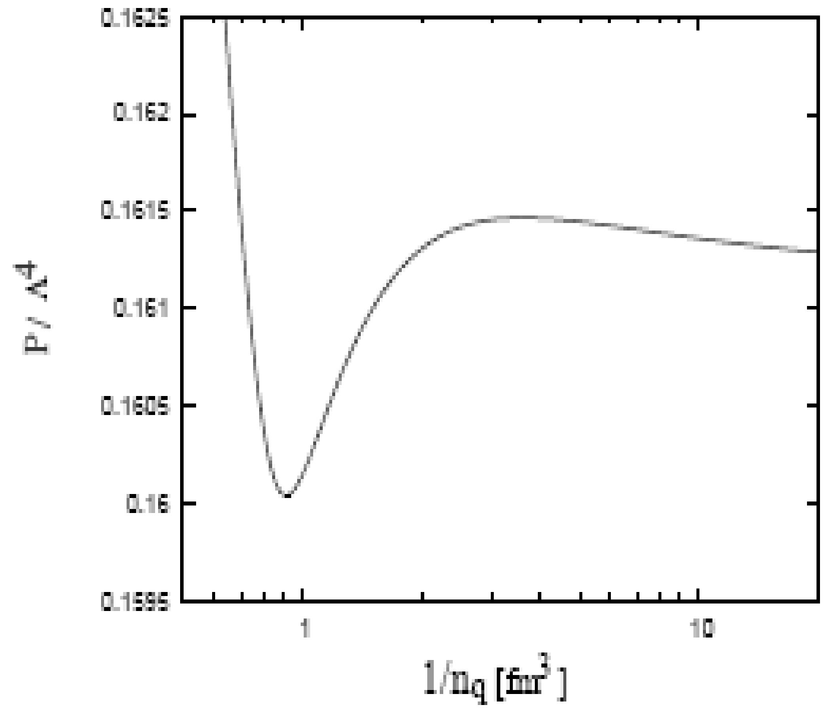
C-D: superheating (broken phase)

Convex anomaly in thermodynamic pressure
NJL model results

$m_q = 0$



$m_q \neq 0$

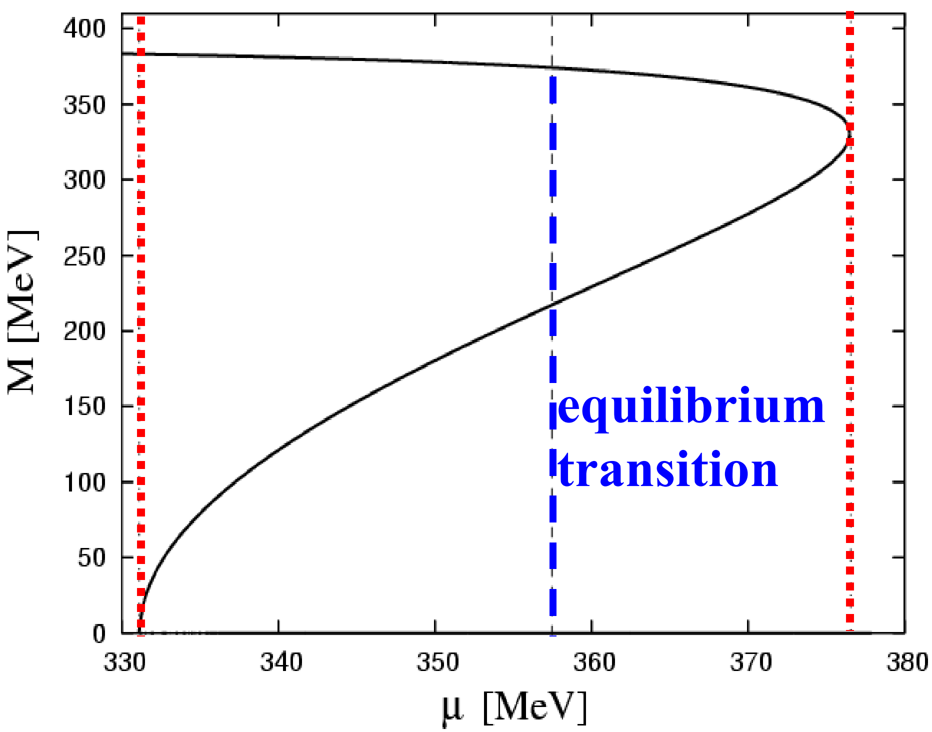


P- has a cusp at the point where the dynamical quark mass vanishes

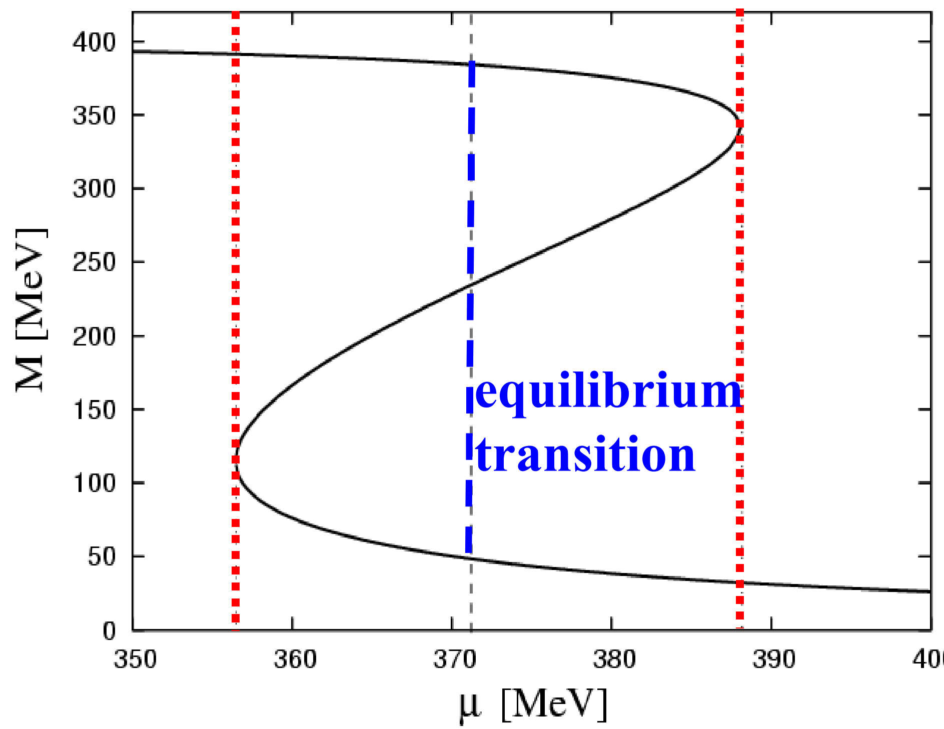
P- is differentiable at all values of μ

Dynamical Quark Mass $M(\mu, T = fixed)$ and Instabilities

$m_q = 0$



$m_q \neq 0$

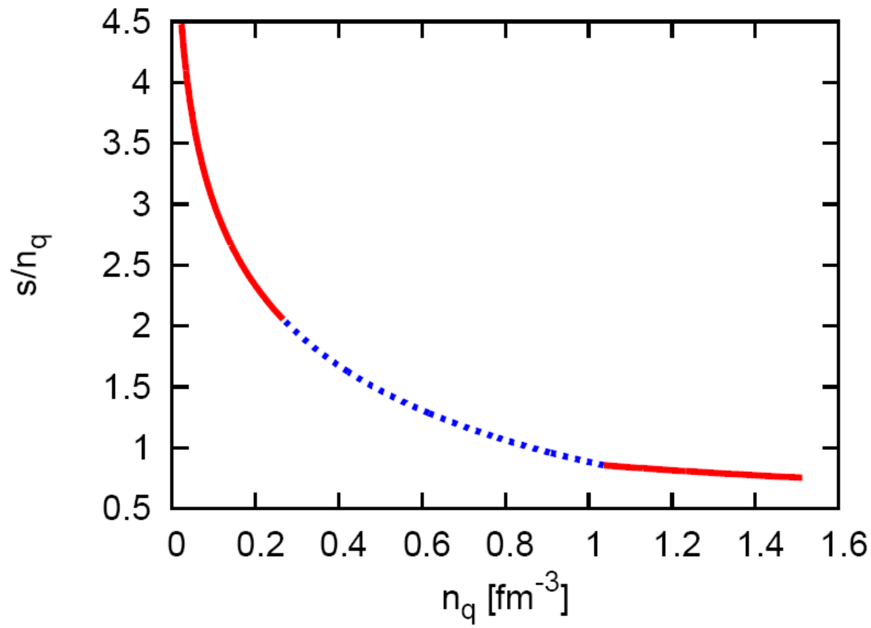


Spinodal Lines : **no-unique solution of the gap equation for μ between spinodals**

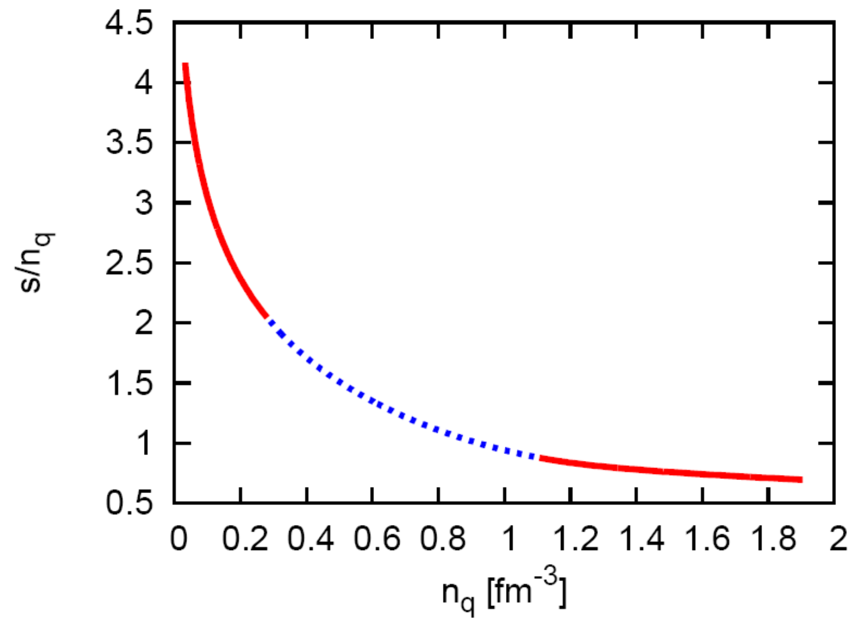
Maxwell constration : **two degenerate minima in the thermodynamic potential**

Entropy per baryon and 1st order phase transition

$m_q = 0$



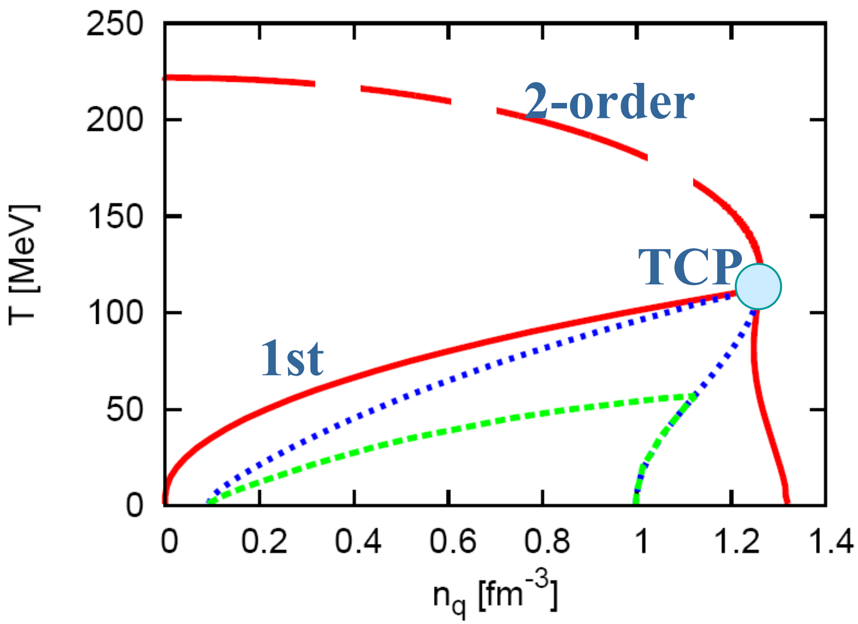
$m_q \neq 0$



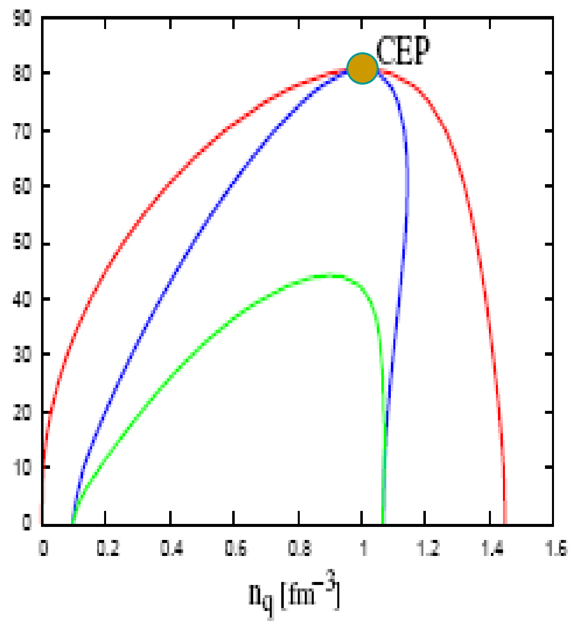
Smooth evolution of entropy/baryon across the 1st order transition

Phase diagram and spinodals

$m_q = 0$



$m_q \neq 0$



critical end point (CEP) :

$T = 81$ MeV, $\mu = 330$ MeV

spinodal lines :

$$\left(\frac{\partial P}{\partial V}\right)_T = 0 \quad : \text{isothermal}$$

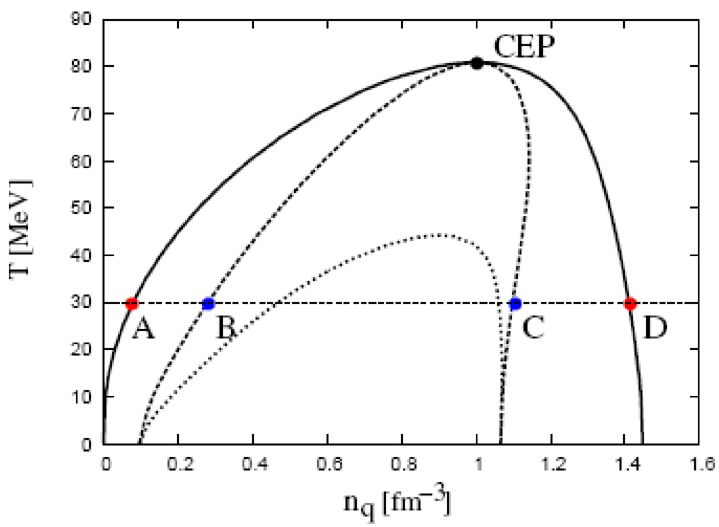
$$\left(\frac{\partial P}{\partial V}\right)_S = 0 \quad : \text{isentropic}$$

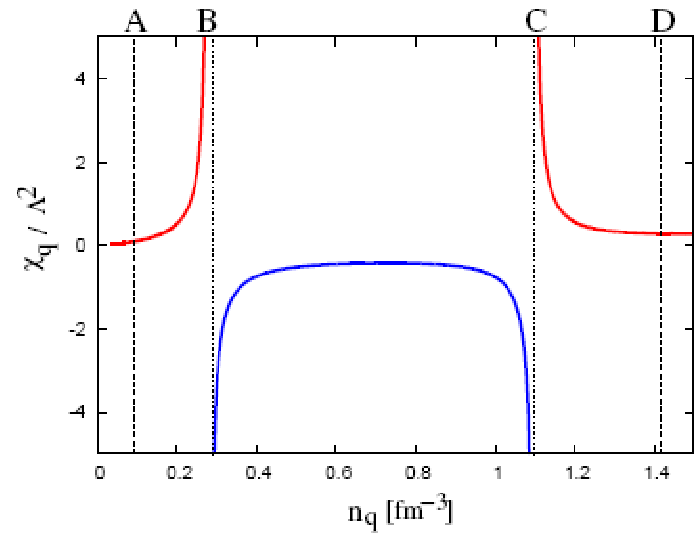
$$\left(\frac{\partial P}{\partial V}\right)_T = \left(\frac{\partial P}{\partial V}\right)_S + \frac{T}{C_V}\left[\left(\frac{\partial P}{\partial T}\right)_V\right]^2$$

mixed phase - Maxwell construction

Quark number susceptibility

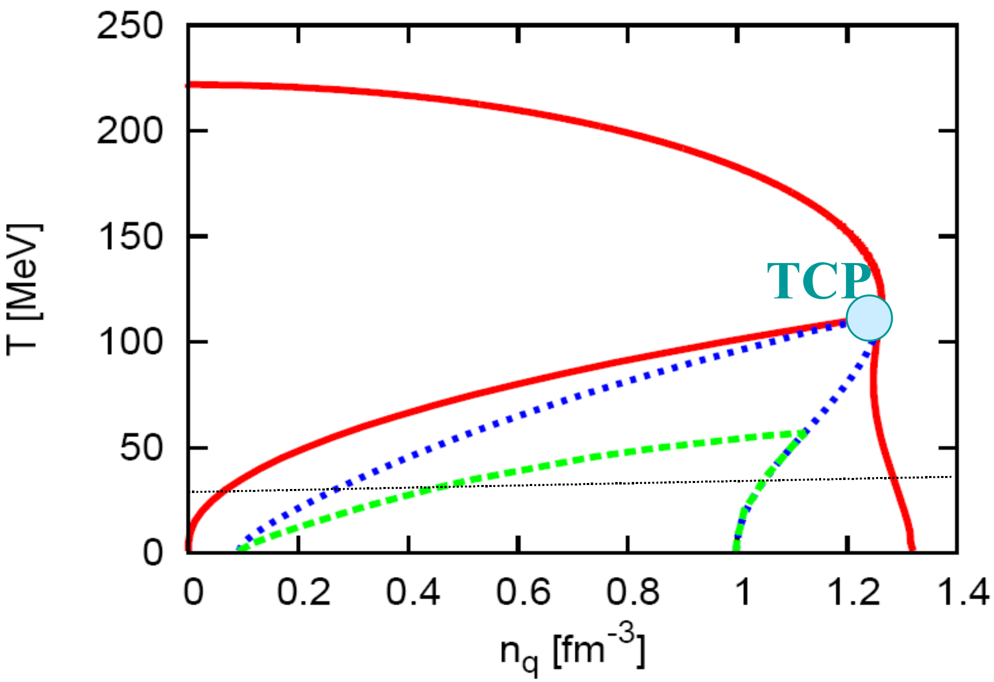
- deviation from equilibrium, large fluctuations induced by instabilities

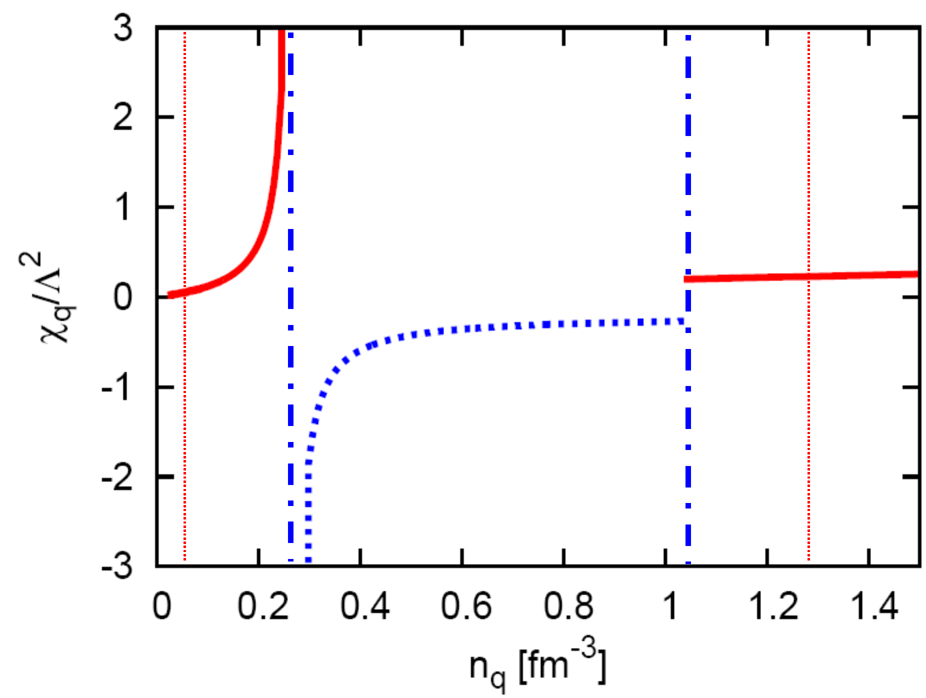




- at 1st order transition point (A, D) : χ_q is finite
- at isothermal spinodal point (B, C) : χ_q diverges and changes its sign
  $\frac{\partial P}{\partial V} < 0$ for stable/meta-stable state $\Rightarrow$ $\frac{\partial P}{\partial V} > 0$ for unstable state
- in unstable region (B-C) : χ_q is finite and negative

Fluctuations in the chiral limit across spinodals

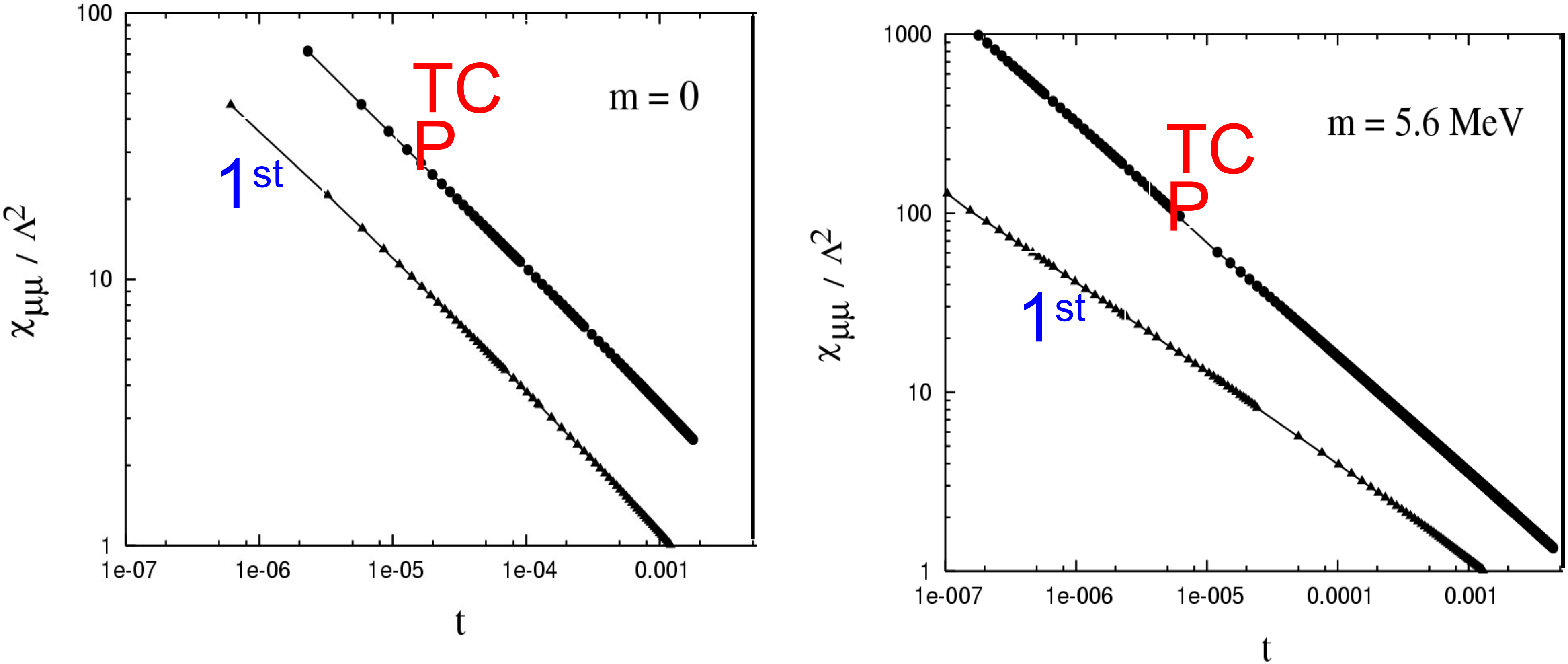




- Non-singular behavior of fluctuations when crossing spinodal line from the side of symmetric phase:

=> directly related with a cusp structure of the pressure

Critical exponents at 1st order line and CEP

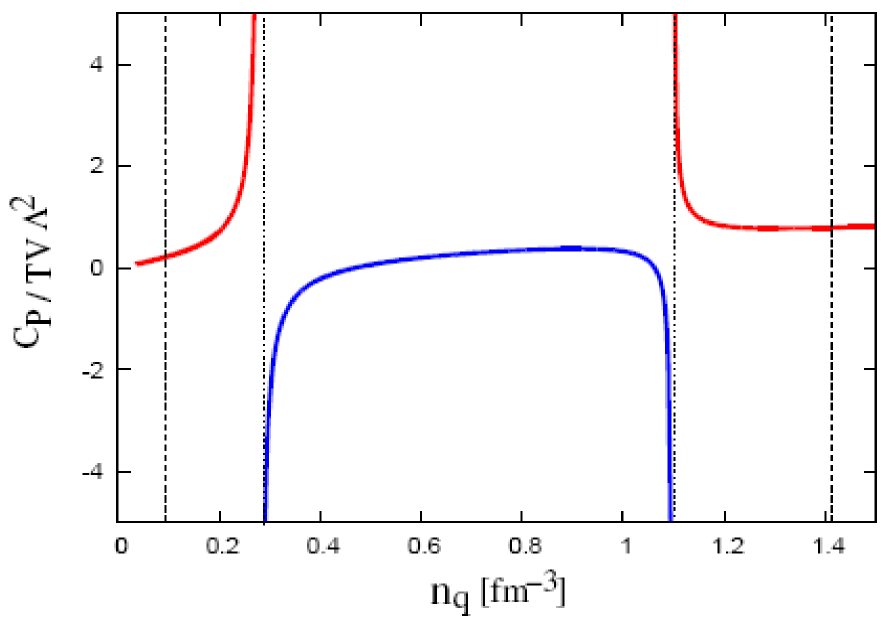


$$\chi_q \sim \left| \frac{\mu - \mu_c}{\mu_c} \right|^{-\gamma} \quad \textbf{with} \quad \gamma_{m_q=0} = \begin{cases} 1/2 \ \ (0.53) & TCP \\ 1/2 & 1st \end{cases}, \gamma_{m_q \neq 0} = \begin{cases} 2/3 \ \ (0.78) & CEP \\ 1/2 & 1st \end{cases}$$

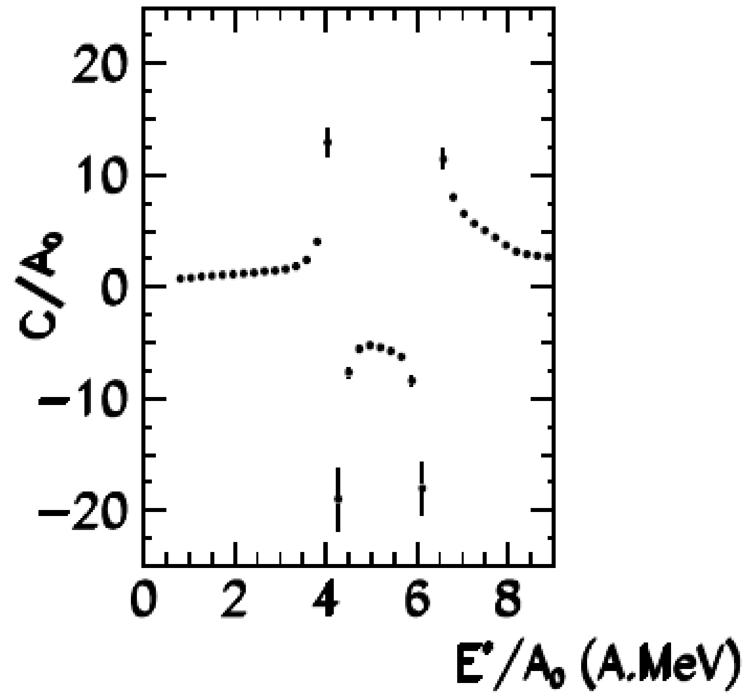
B. Friman, C. Sasaki & K.R. , Phys.Rev.D77:034024,2008.

Experimental Evidence for 1st order transition

Specific heat for constant pressure:



Low energy nuclear collisions



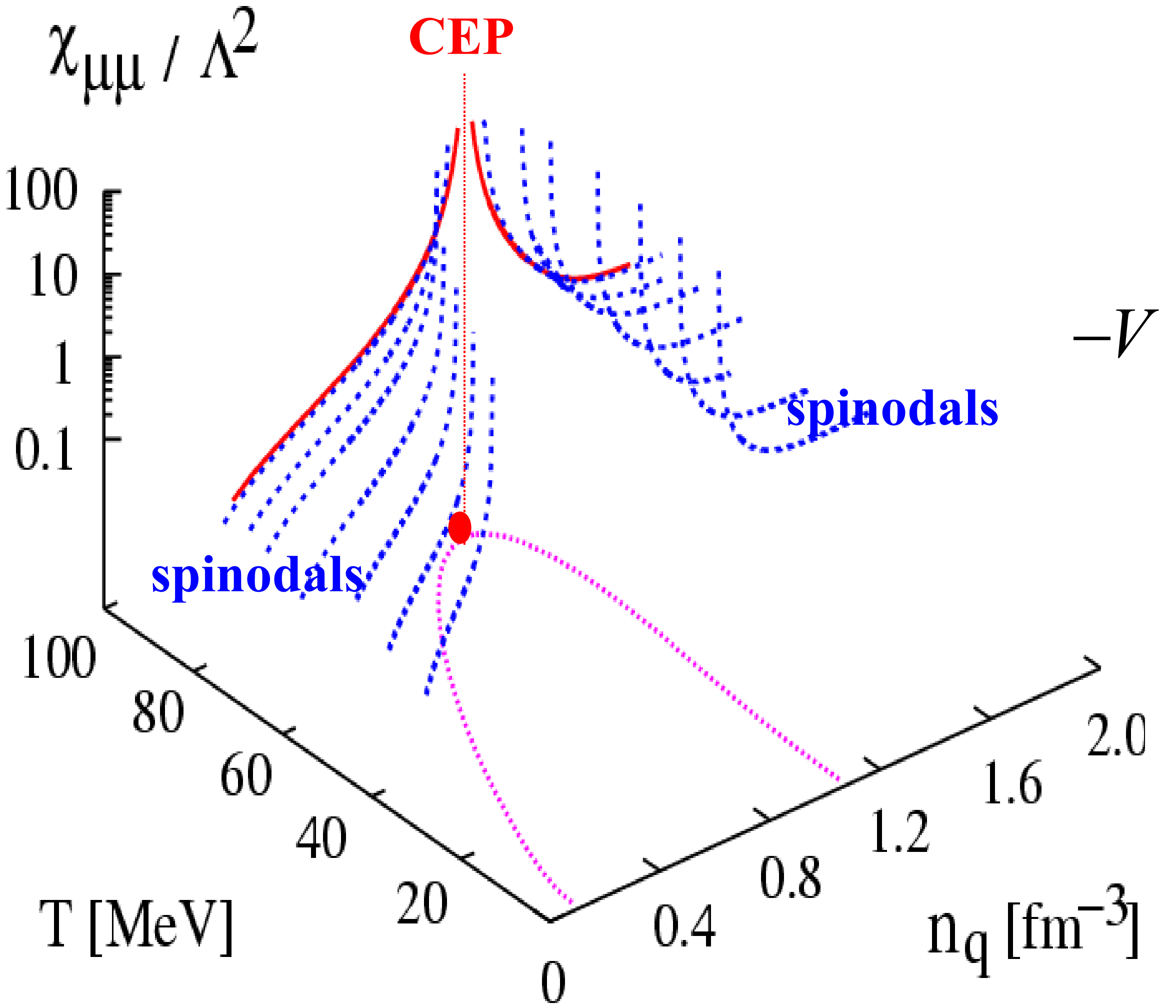
$$C_P = T\left(\frac{\partial S}{\partial T}\right)_P = TV\left[\chi_{TT} - \frac{2s}{n_q}\chi_{\mu T} + \frac{s^2}{n_q{}^2}\chi_q\right]$$

M. D'Agostino *et al.*, Phys. Lett. B 473, 219 (2000)

negative heat capacity : anomalously large fluctuations

$\Rightarrow$ an evidence of the 1st order liquid-gas phase transition

Net-quark fluctuations on spinodals



at any spinodal points:

$$-V \frac{\partial P}{\partial V}\Big|_T = \frac{n_q^{\ 2}}{\chi_q} = \frac{1}{compres.}$$

Singularity at CEP are the remnant of that along the spinodals

C. Sasaki, B. Friman & K.R., Phys.Rev.Lett.99:232301,2007.

Transport coefficients near CEP

- Minimum of shear viscosity may indicate the location of T_c : L. Csernai, J. Kapusta & L. McLerran (06)
- Divergent bulk viscosity at CEP from QCD trace anomaly argument: D. Kharzeev & Tuchin 07

  F. Karsch, D. Kharzeev & Tuchin 08

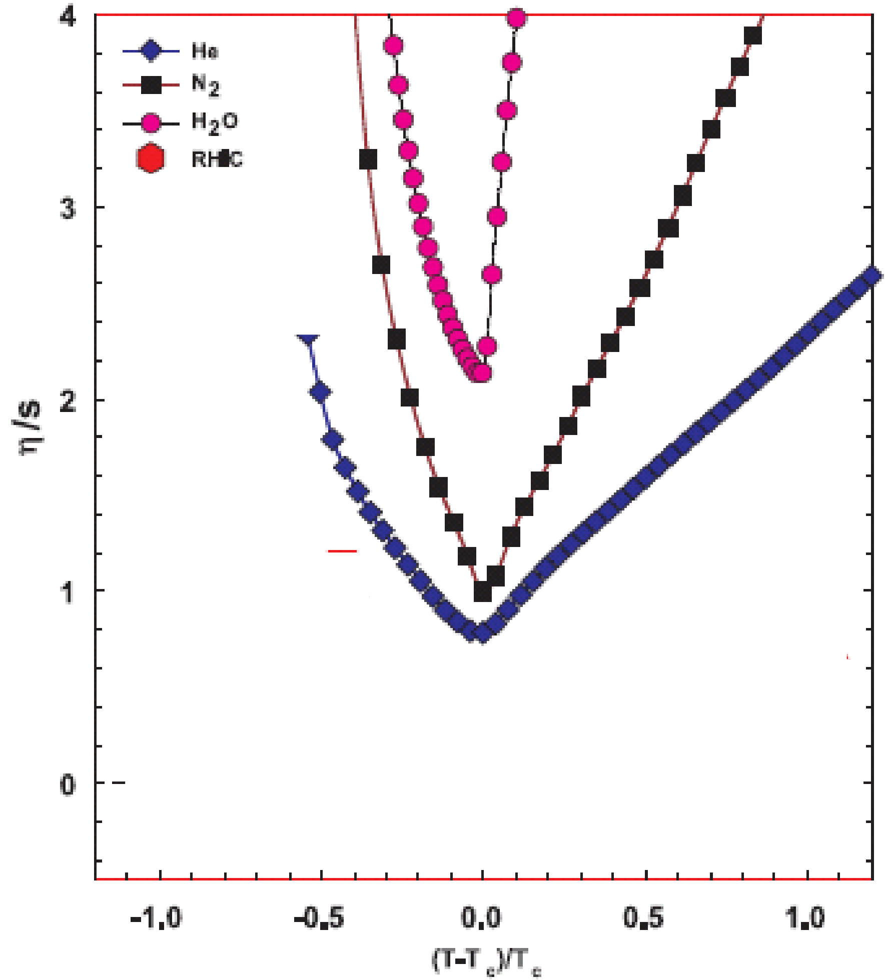
Our goal: C. Sasaki & K.R. **hep-ph 0806.4745**

- Find viscosities in quasi particle models with dynamically generated mass $M(T,\mu) = m_{bare} + f(T,\mu)$, under relaxation time approximation
- Derive scaling behavior of viscosities in O(4) and Z(2) universality class using scaling theory

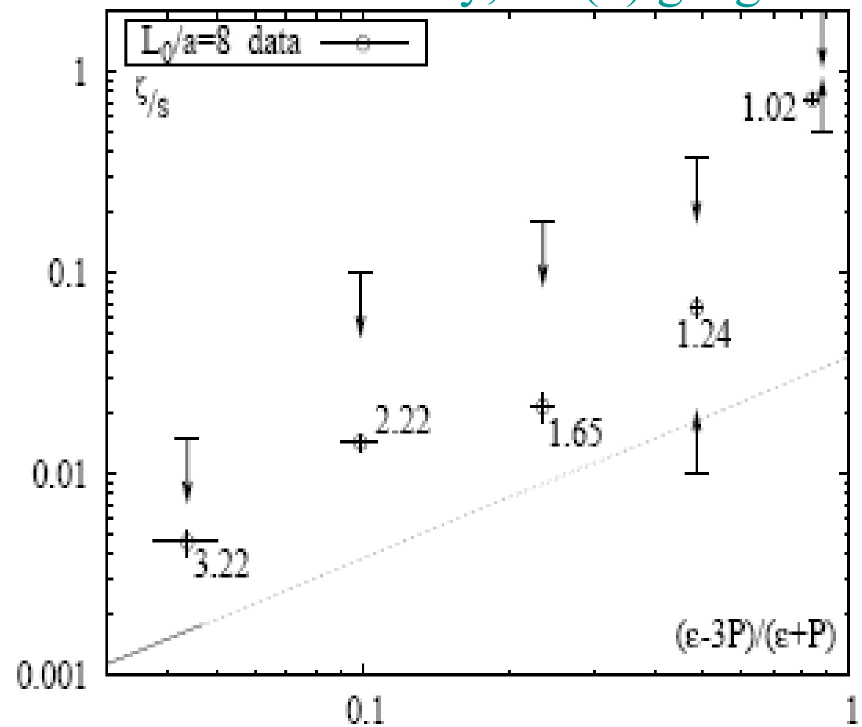
Transport Coefficient near phase transition

L. Csernay, J. Kapusta & L. McLerran 06

R. Lacey et al. 07: data for shear viscosity



Harvey B. Meyer 08 LGT results for bulk viscosity, SU(3) gauge th.



Shear viscosity to entropy ratio in LGT

$$\frac{\eta}{s} = \begin{array}{l} 0.134(33) \quad for \quad T = 1.65 T_c \\ 0.102(56) \quad for \quad T = 1.24 T_c \end{array}$$

Transport coefficients from kinetic theory:

- Energy momentum tensor

$$T^{\mu\nu} = \int \frac{d^3p}{(2\pi)^3} p^\mu p^\nu \frac{1}{E}[f + \overline{f}] \qquad E^2 = \vec{p}^{\,2} + M^2(T,\mu)$$

- Assume small deviations from equilibrium $\delta f = f - f_0$ with $f^{-1} = \exp(E - \vec{p}\vec{u} \mp \mu) \pm 1$, consequently

$$\delta T^{\mu\nu} = \int \frac{d^3p}{(2\pi)^3} p^\mu p^\nu \frac{1}{E}[\delta f + \delta \overline{f}]$$

- To get δf use Boltzmann equation under relaxation time approximation

$$\frac{\partial f}{\partial t} + \vec{v}_p \vec{\nabla} f = -C[f] \simeq -\frac{f - f_0}{\tau}$$

$$\Rightarrow \quad p^\mu \partial_\mu f_0 = -\frac{E}{\tau} \delta f$$

with the collision time obtained from the particle density and cross section

$$\tau_f{}^{-1} = n_f < \sigma v_{rel} >$$

derivation of transport coefficients:

use:

- δf from Boltzmann equation: $\delta f = -\tau E^{-1} p^{\mu} \partial_{\mu} f_0$
- energy conservation: $\partial_0 T^{00} = 0$
- charge conservation: $\partial_0 j^0 = 0$
- stationary condition : $\partial P / \partial M = 0$
- thermodynamics relation: $\varepsilon = T \partial P / \partial T - P + \mu \partial P / \partial \mu$

get:

$$\delta T^{ij} = -\varsigma \delta_{ij} \partial_k u^k - \eta W_{ij}$$

bulk viscosity

shear viscosity

- shear viscosity $\eta(T,\mu)$ is not modified by thermal change of quasi-particle energy

$$\eta = \frac{g}{15T}\int \frac{d^3 p}{(2\pi)^3}\frac{\vec{p}^{\,4}}{E^2}[\tau f_0(1\pm f_0)+\bar{\tau}\bar{f}_0(1\pm \bar{f}_0)]$$

- the above coincides with Hosoya & Kajantie (1985),

  however, the bulk viscosity

$\varsigma(T,\mu)$ is modified by thermal change of quasi-particle energy through: $\partial E/\partial T$ and $\partial E/\partial \mu$

$$\varsigma = -\frac{g}{3T}\int \frac{d^3p}{(2\pi)^3}\Big[\frac{M^2}{E}\left(\tau f_0(1\pm f_0)+\bar{\tau}\bar{f}_0(1\pm \bar{f}_0)\right)$$

$$\times\left(\frac{\vec{p}^{\,2}}{3E}-\left(\frac{\partial P}{\partial \varepsilon}\right)_n\left(E-T\frac{\partial E}{\partial T}-\mu\frac{\partial E}{\partial \mu}\right)+\left(\frac{\partial P}{\partial n}\right)_\varepsilon\frac{\partial E}{\partial \mu}\right)$$

$$-\frac{M^2}{E}\left(\tau f_0(1\pm f_0)-\bar{\tau}\bar{f}_0(1\pm \bar{f}_0)\right)\left(\frac{\partial P}{\partial n}\right)_\varepsilon\Big]$$

- for $\partial E/\partial T(\partial\mu)=0$ the above coincides with Hosoya & Kajantie (1985)
- For $\partial E/\partial \mu = 0$ the above coincides with Arnold, Dogan & Moore (06)

- Near phase transition the bulk viscosity can be singular through derivatives terms:

$$\frac{\partial E}{\partial x} = \frac{1}{2E}\frac{\partial M^2}{\partial x} \qquad \chi_{xy} = \frac{\partial^2 P}{\partial x \partial y} \quad \text{with} \quad x, y = (T, \mu)$$

$$\frac{\partial P}{\partial \varepsilon}\Big|_n = \frac{1}{C_V \chi_{\mu\mu}}(s\chi_{\mu\mu} - n\chi_{\mu T})$$

$$\frac{\partial P}{\partial n}\Big|_\varepsilon = \frac{1}{C_V \chi_{\mu\mu}}(nT\chi_{TT} + (n\mu - sT)\chi_{\mu T} - s\mu\chi_{\mu\mu})$$

$$C_V = T\left(\frac{\partial s}{\partial T}\right)\Big|_V = T\left(\chi_{TT} - \frac{\chi_{\mu T}{}^2}{\chi_{\mu\mu}}\right)$$

which all can diverge at the critical points !

critical behavior of bulk viscosity:

- consider a system where dynamical mass acts as an order parameter

- Mean Field Scaling from Ginzburg-Landau potential

$$\Omega(T,\mu,M)=\Omega_{M=0}+a(T,\mu)M^2+b(T,\mu)M^4-hM$$

=> 2nd order: a=0 and b>0 TCP at a=b=0

with $a(T,\mu)=\alpha\,|T-T_c|+\beta\,|\mu-\mu_c|$ and from gap equation

$$\varsigma^{singular}\,|_{TCP}\sim\frac{M^3}{C_V}\left(\frac{\partial M}{\partial T}-\frac{\alpha}{\beta}\frac{\partial M}{\partial \mu}\right)\sim M^4\chi_\sigma\times 0\sim t^{2\cdot\frac{1}{2}-1}\times 0\approx 0$$

$$\varsigma^{\sin gular}\,|_{2^{nd}}\sim M^2=0$$

Conclusions: under MF approximation there is no singularity of bulk viscosity at the TCP and 2nd order phase transition.

actually : in the chiral limit the bulk viscosity vanishes at the transition point

O(4) scaling of bulk viscosity: use the free energy

Ejiri, Karsch & K.R. 06; Hatta & Ikeda 04

$$F_S(T,\mu) = t^{2-\alpha} f_S(t^{-\beta\delta} h) \quad t \equiv \bar{t} + A\bar{\mu}^2 \ , \ \bar{t} = |T - T_c| / T_c \ , \ \bar{\mu} = \mu / T_c$$

that gives $\chi_{\mu\mu} \sim t^{1-\alpha} \ , \ \chi_{TT} \sim t^{-\alpha} \ , \ C_V \sim t^{-\alpha} \ , \ M \sim t^{\beta}$

consequently $\varsigma^{singular} \sim \dfrac{M^3}{C_V} \dfrac{\partial M}{\partial T} \sim t^{\alpha+4\beta-1}$

for $\alpha = -0.24 \ , \ \beta = 0.38$ => $\varsigma^{singular} \sim t^{+0.28} \to 0$

Conclusions: bulk viscosity is non-singular at O(4) critical point

- Z(2) scaling of bulk viscosity: use the free energy

Stephanov, Rajagopal & Shuryak 1998
Karsch et. al. (08)

$$F_S(T,\mu) = h^{\frac{1+\delta}{\delta}} f_S(h^{-1/\beta\delta} t)$$

$$t \equiv A_{t,h}\bar{t} + B_{t,h}\bar{\mu} \ , \quad \bar{x} = |x - x_c| / x_c$$

that gives

$$\chi_{\mu\mu,\mu T,TT} \sim h^{-\gamma/\beta\delta} \ , \ C_V \sim h^{-\gamma/\beta\delta} \ , \ M \sim h^{1/\delta}$$

consequently

$$\varsigma^{\sin gular} \sim \frac{M^3}{C_V} \frac{\partial M}{\partial T} \sim h^{\gamma/\beta\delta + 4/\delta - 1}$$

for $\gamma = 1.25 \ , \beta = 0.31 \ , \delta = 5.2$ => $\varsigma^{\sin gular} \sim h^{+0.54} \rightarrow 0$

Conclusions: bulk viscosity is non-singular at Z(2) critical point within the relaxation time approximation

- Beyond the relaxation time approximation
  - modes with long wave-length could stay in non-equil.
  - ς modified by dynamic critical exponents "z"

Onuki, "Phase transition dynamics", Cambridge Univ. Press (02)

For O(4) : $\varsigma^{singular} \sim t^{-z\nu+\alpha}$

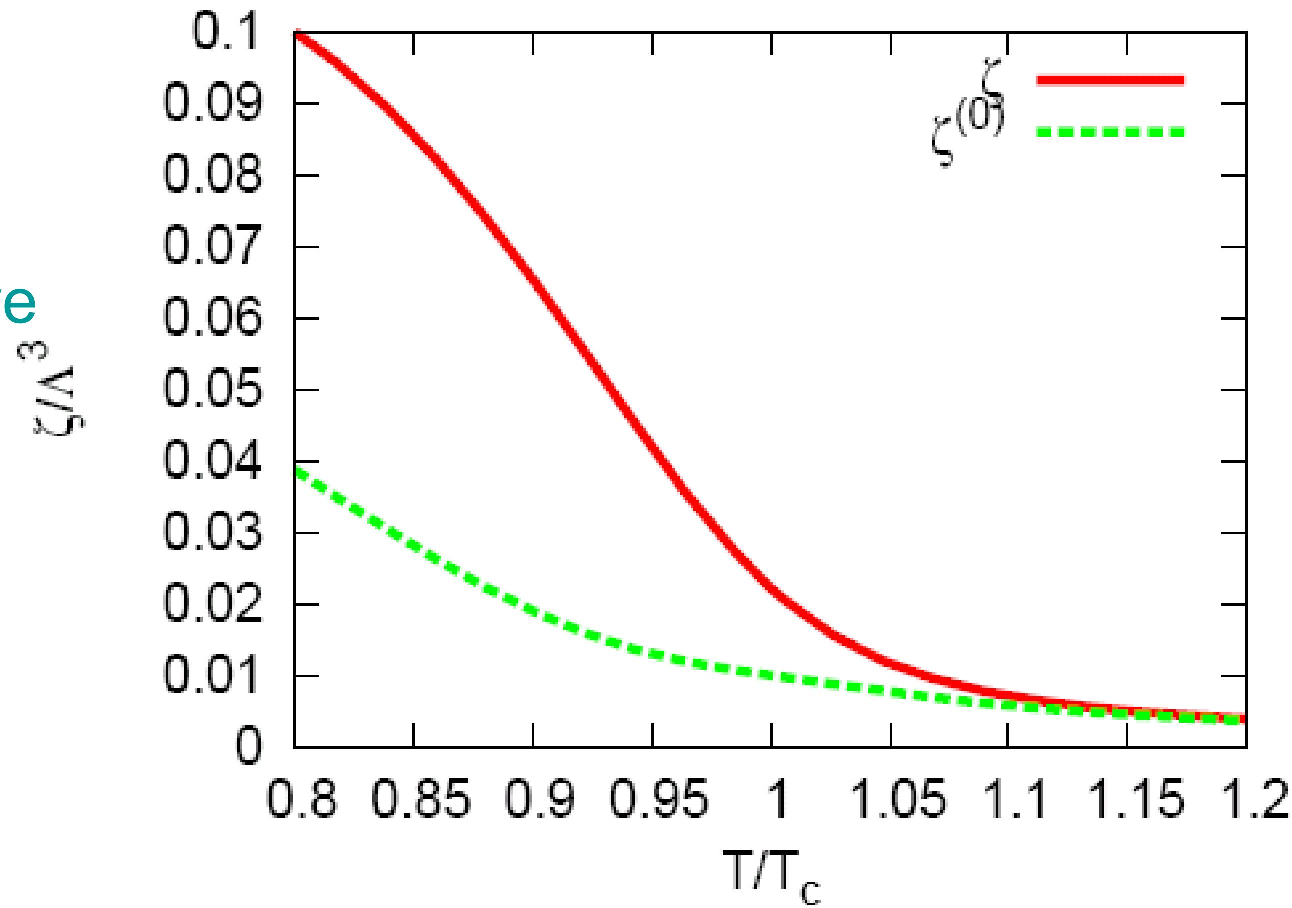
For Z(2) : $\varsigma^{singular} \sim t^{-z^{*}+\alpha}$

↓

Singularity in bulk viscosity along O(4) critical line and at the TCP due to dynamic critical exponents

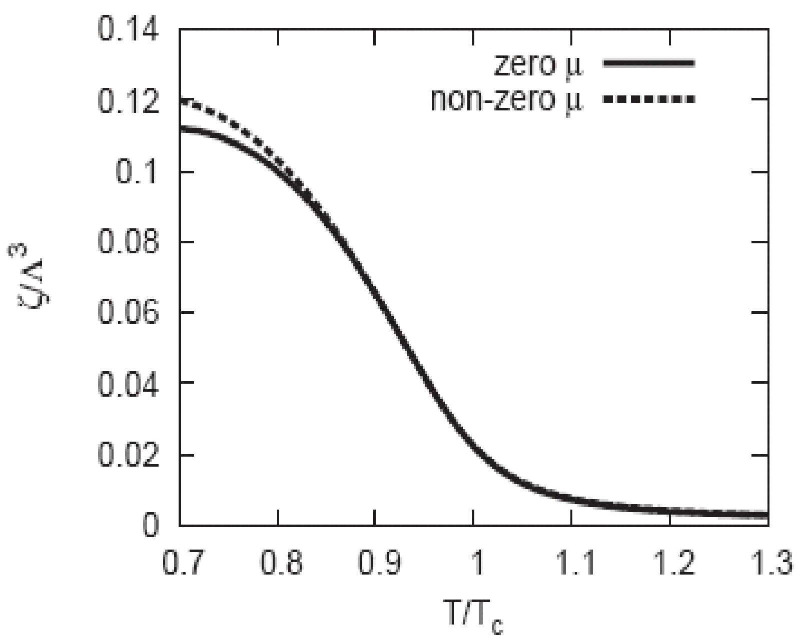
- how relevant $\partial M/\partial T$ is: ζ vs. $\zeta^{(0)}$ w/o $\partial M/\partial T$
  a demonstration in NJL model under MF approximation at $\mu = 0$

- Large change of bulk viscosity due to contribution from temperature derivative of dynamical quark mass, $\partial M / \partial T$

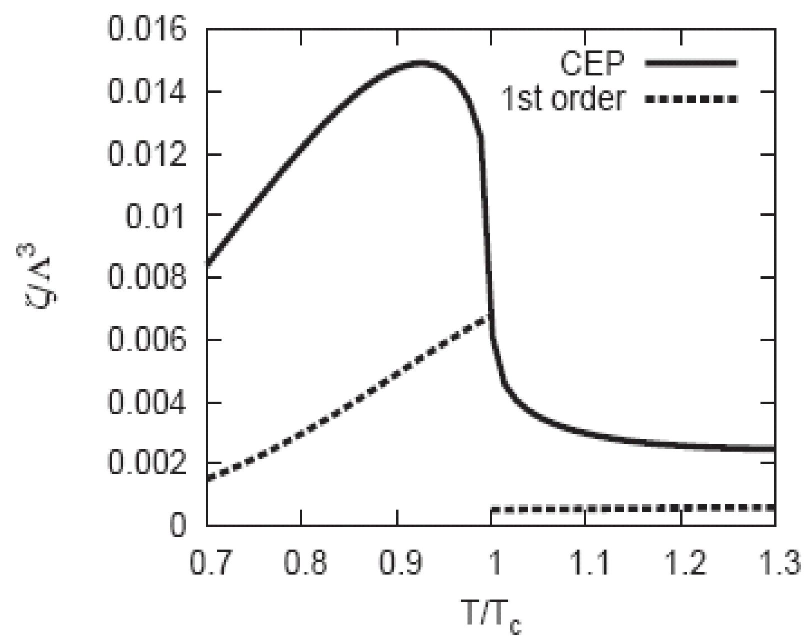


Bulk Viscosity across the phase transition in the NJL model

- Cross over region

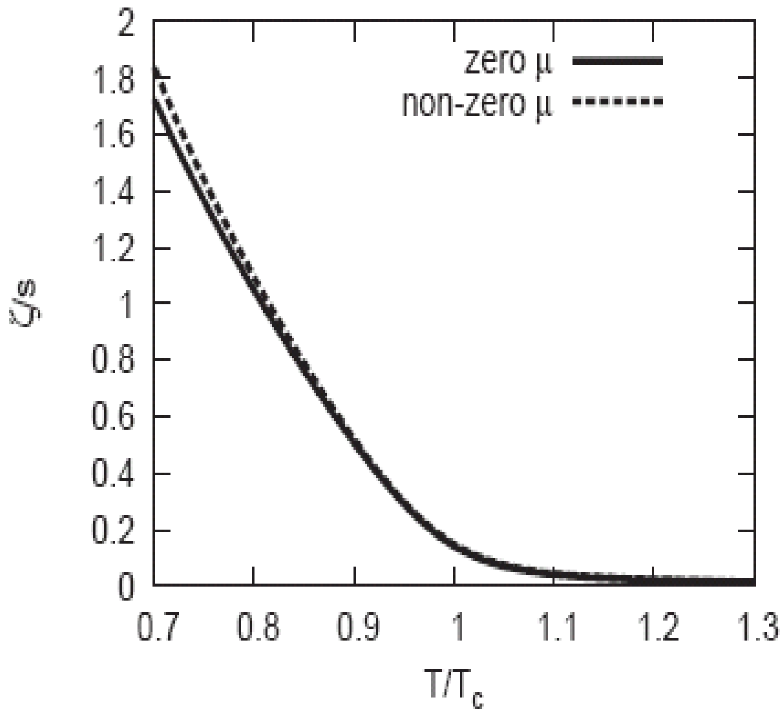


- CEP and 1st order line

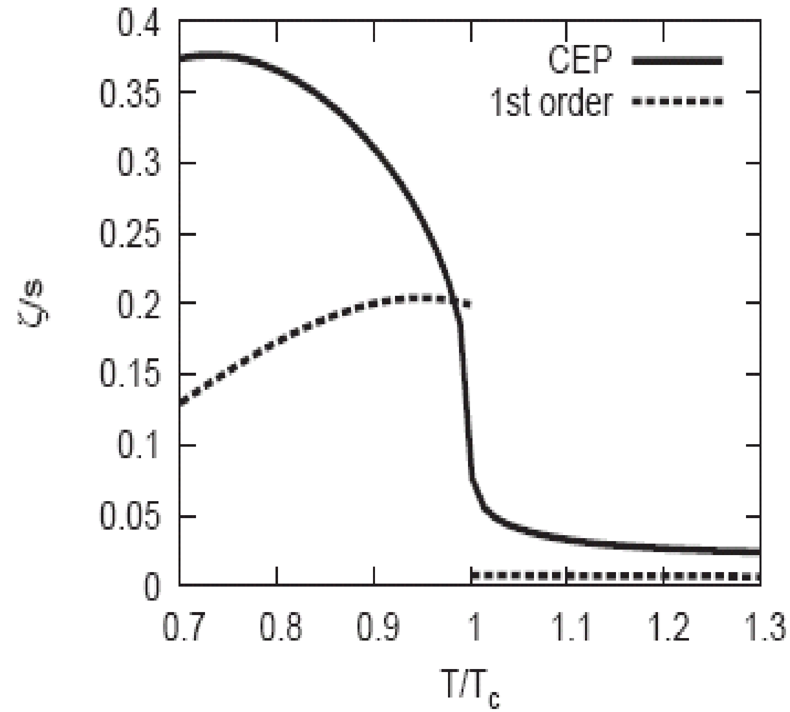


Bulk viscosity to entropy ratio

- Cross over line

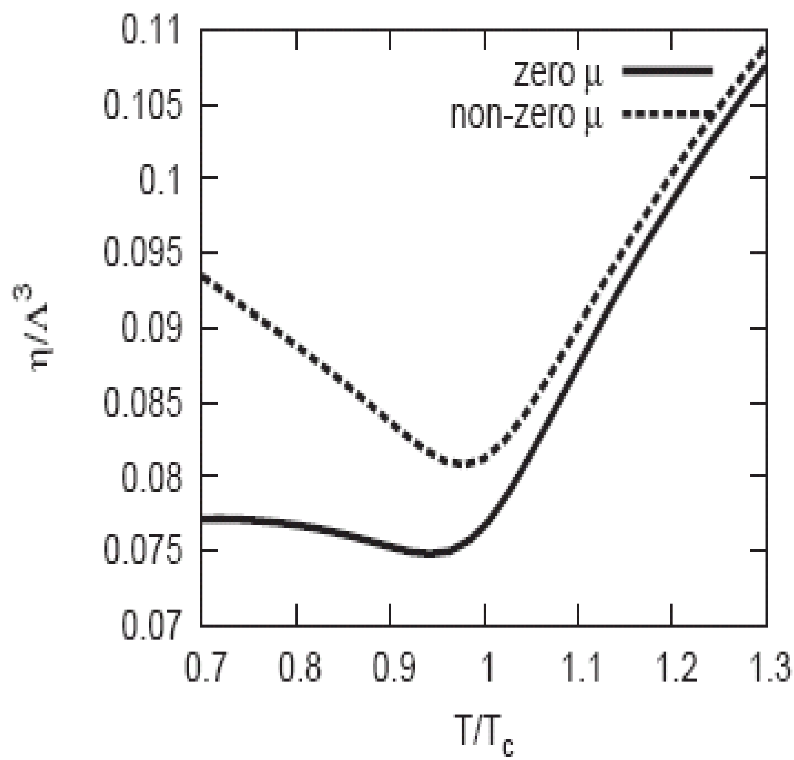


- CEP and 1st order line

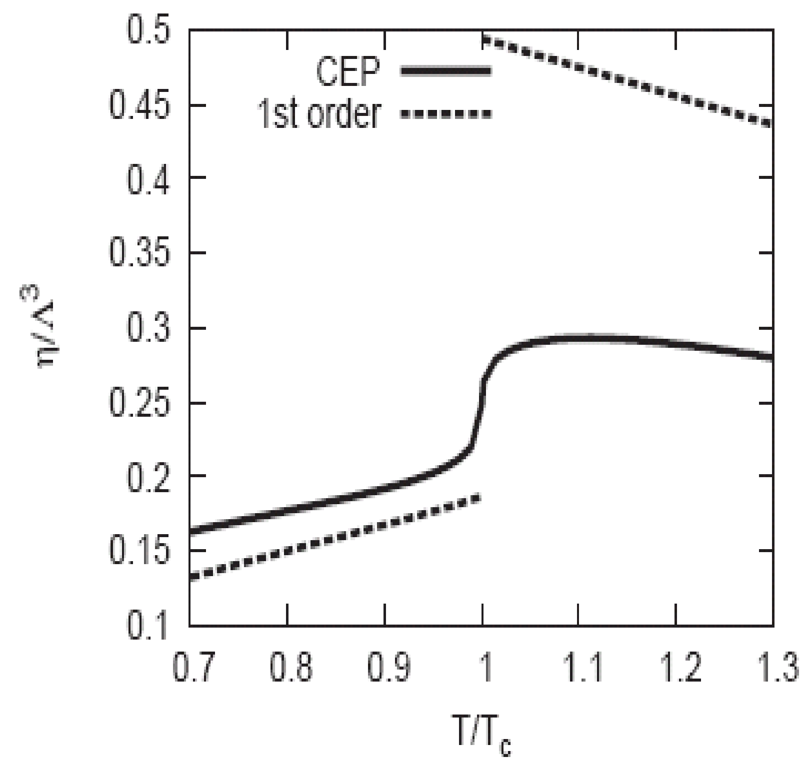


Shear viscosity normalized by momentum cutoff

- Cross over transition

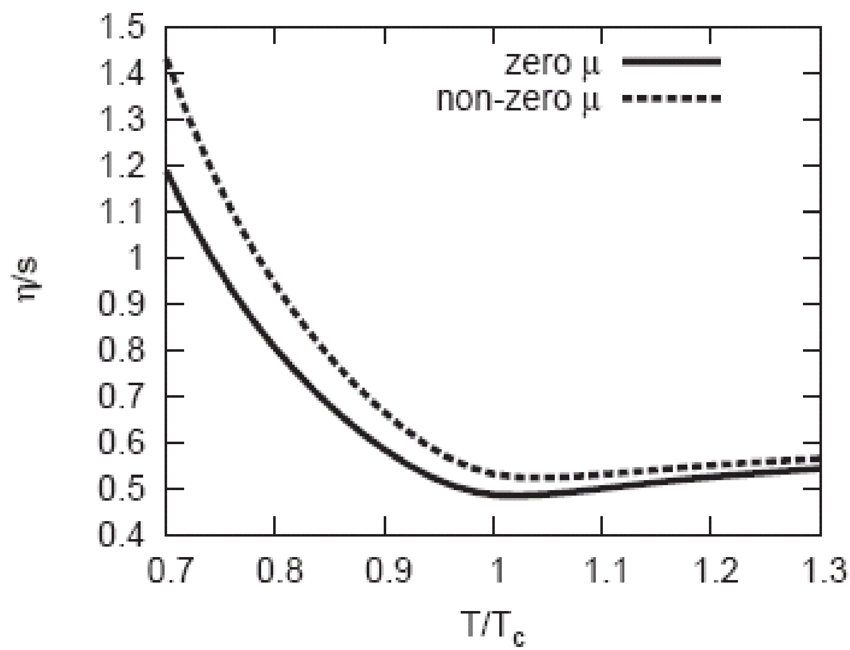


- CEP and 1st order line

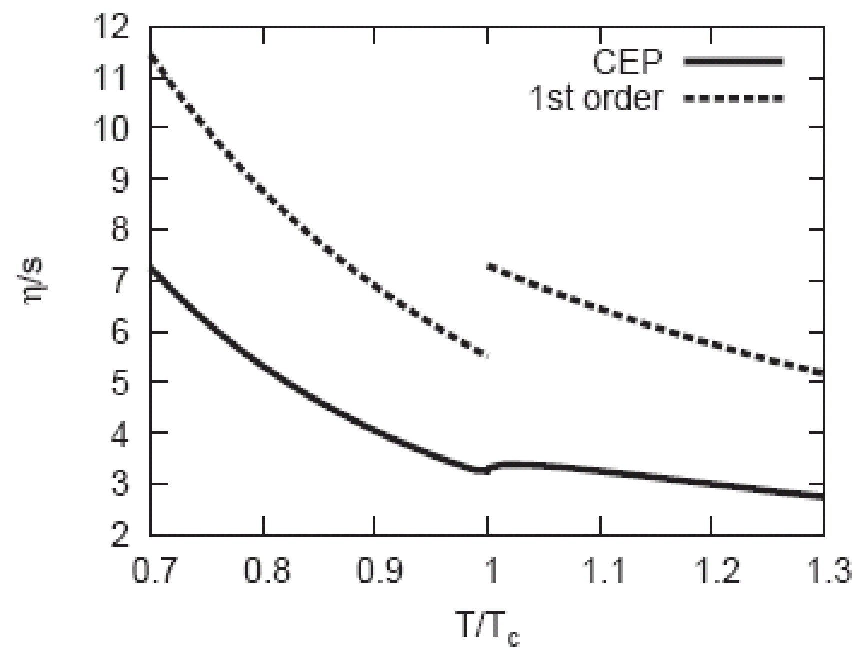


Shear viscosity per entropy

- Crossover

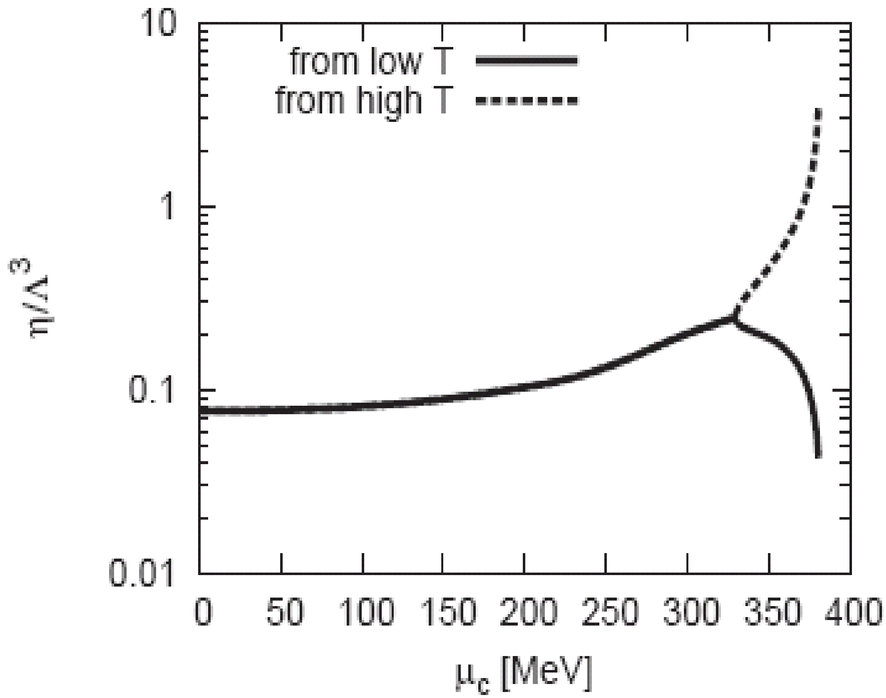


- CEP and 1st order line

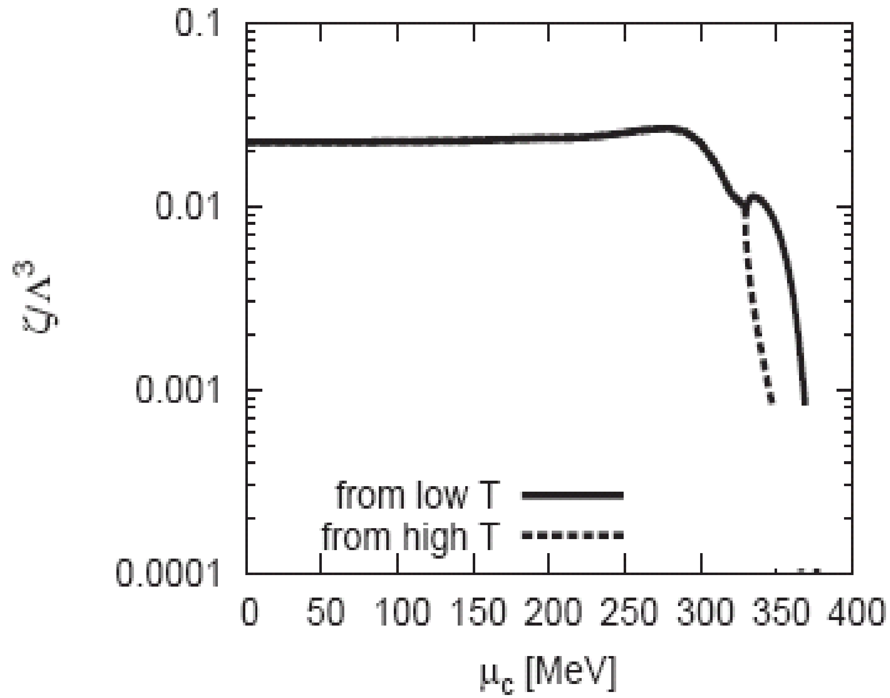


Bulk and shear viscosity along the phase boundary

- shear viscoisity



- bulk viscoisity



Conclusions

- A non-monotonic change of the net-quark susceptibility probes the existence of CEP However in non-equilibrium: due to spinodal instabilitie the charge fluctuations diverge at 1st order critical line

=> Large fluctuations signals 1st order transition

- Under relaxation time approximation the bulk viscosity is finite at CEP and O(4) line

⇒ Divergence of bulk viscosity controlled by the dynamical, rather then static critical exponents