The interplay of flavour- and Polyakov-loop-degrees of freedom

A PNJL model analysis

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The QCD Critical Point
Institute for Nuclear Theory, Seattle

Outline

Connections between colour and flavour ($N_f = 2$ thermodynamics)

- Flavour blind dofs couple to up- and down- quark densities
- Up- and down- quark densities couple to flavour blind dofs
- Up- and down- quarks *communicate* via an intermediary:

Polyakov loop dofs

Quantitative investigation of induced flavour mixing:

- NJL-model $+$ Polyakov-loop model ➠ PNJL model
- A perturbative approach[1] to investigate:
  - The Polyakov loop $\langle \Phi \rangle$ and its conjugate $\langle \Phi^* \rangle$
  - Non-vanishing up- and down-quark susceptibilies χ_{uu} and χ_{ud}
- Conclusion & Outlook

[1] R., Hell, Ratti, Weise arXiv:0712.3152 [hep-ph], [RHRW07]

QCD thermodynamics

Symmetry breaking patterns of QCD at finite T

Chiral symmetry

1. Explicit breaking $m_q > 0$
2. *Dynamic* breaking at low T
3. Order parameter:
   Chiral condensate $\langle \bar{q}q \rangle$
4. Quarks are
   coloured objects

Confinement-deconfinement

1. Explicit breaking $m_q < \infty$
2. *Dynamic* breaking at high T
3. Order parameter:
   Polyakov loop $\langle \Phi^* \rangle$ and $\langle \Phi \rangle$
4. Colour *screening* by vacuum fluctuations of quarks

Dynamic quark masses ⇔ Colour confinement

⇒ Chiral symmetry breaking $+$ Z(3) symmetry breaking are closely linked

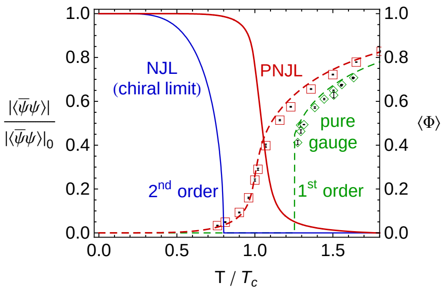
⇒ Joint crossover transition

Modelling colour and flavour dynamics

The Polyakov loop extended Nambu and Jona-Lasinio model (PNJL model)

NJL-model + Polyakov loop model = PNJL model

Joint crossover of $\langle\Phi\rangle$ and $\langle\bar{q}q\rangle$



R., Hell, Ratti, Weise arXiv:0712.3152 [hep-ph]

[RHRW07]

- *PNJL*: Joint effects of quarks and Polyakov loop
- ➠ Confinement (colour) *affecting* quark densities

Modelling colour and flavour dynamics

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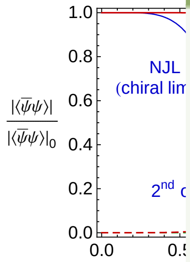

R., Hell, Ratti, Weis

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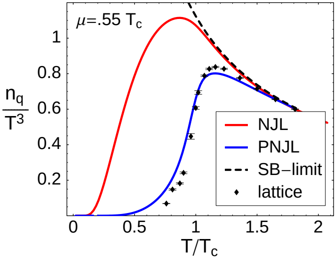
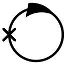
Quark densities



Ratti, Thaler, Weise
PRD 73 (2006) 014019 [RTW06]

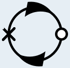
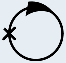
- *PNJL*: Joint effects of quarks and Polyakov loop
- ➠ Confinement (colour) *affecting* quark densities

$$n_{q_x} = \frac{\partial \Omega}{\partial \mu_x} = \sum_{\omega_n} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \mathrm{tr} \left[\frac{\partial S^{-1}}{\partial \mu_x} S \right] =$$ 

Ω : thermodynamic potential $\qquad \Omega = \sum_{\omega_n} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \mathrm{tr} \ln \left[\beta S^{-1} \right]$

$\times$: $\gamma_0\, \tau_x$, where τ_x is a matrix in flavour space

Quark number susceptibilities

$$\chi_{\mathrm{u}x} \propto \langle n_\mathrm{u} n_x \rangle - \langle n_\mathrm{u} \rangle \langle n_x \rangle =$$  $-$  $\times$ 

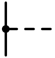
$\times = \gamma_0\, \tau_\mathrm{u}$ $\qquad \circ = \gamma_0\, \tau_x$

No explicit isospin breaking (thoughout this presentation)

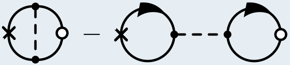
Introduction of a perturbative interaction $\delta\mathcal{U}$

$$\Omega = \Omega_{\mathrm{MF}} + \delta\mathcal{U}(\zeta) \qquad (\Omega_{\mathrm{MF}} \text{ indep. of } \zeta)$$

- Propagators remain unchanged
  - ➠ Use the same quasiparticles as in MF
- Quark density operators induce new Feynman rules:

 $= \dfrac{\partial S^{-1}}{\partial \zeta}$ $= \left[\dfrac{\partial^2 \delta\mathcal{U}}{\partial\zeta\partial\zeta}\right]^{-1}$

Corrections to the susceptibilities:

$\delta\chi_{\mathrm{ux}} =$ 

- ζ couples to n_{q_x}
  - ➠ $\zeta \xrightarrow[\text{conjugation}]{\text{charge}} -\zeta$
- $\chi_\zeta \propto \left[\frac{\partial^2 \delta\mathcal{U}}{\partial\zeta\partial\zeta}\right]^{-1} \neq 0$

ζ-susceptibility ⇨ $\chi_\zeta \propto \left[\frac{\partial^2 \delta\mathcal{U}}{\partial\zeta\partial\zeta}\right]^{-1} \propto^{a} \delta\chi_{\mathrm{ud}}$ ⇦ n_{q_x}-susceptibility

[a] The mean field contribution of χ_{ud} vanishes due to flavour symmetry.

Polyakov loop degrees of freedom

- Polyakov loop $\Phi(\vec{x})$ is the trace of a time-like Wilson-line

$$\Phi(\vec{x}) = \frac{1}{N_c}\mathrm{tr}_c L(\vec{x}) \qquad L(\vec{x}) = \mathcal{P}\exp\left\{ i\int_0^\beta \mathrm{d}\tau\, A_4^a(\vec{x})\, t_a \right\}$$

- Order parameter for de-confinement

  ➠ $\langle\Phi\rangle = 0 \iff$ confined ➠ $\langle\Phi\rangle \neq 0 \iff$ deconfined

- Define Polyakov loop fields with good charge conjugation parity:

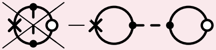
$$\Phi^+ = \frac{1}{2}\langle\Phi^* + \Phi\rangle \qquad \Phi^- = \frac{1}{2}\langle\Phi^* - \Phi\rangle$$

QCD toy model (Ginzburg-Landau-type)

$$\mathcal{S}_{\mathrm{eff}}^{\mathrm{QCD}} = \underbrace{\mathcal{S}_{\mathrm{eff},\,0}^{\mathrm{QCD}}}_{\text{indep. of }\Phi^-} + \delta\mathcal{U}(\Phi^-)$$

- Treat $\mathcal{S}_{\mathrm{eff},\,0}^{\mathrm{QCD}}$ in mean field
- Treat $\delta\mathcal{U}$ pertubatively

$$\chi_{\mathrm{ud}} = \cancel{\chi_{\mathrm{ud}}^{\mathrm{MF}}} + $$



$$\propto -\frac{\partial^2\mathcal{S}_{\mathrm{eff}}}{\partial\mu_u\partial\Phi^-}\left[\frac{\partial^2\delta\mathcal{U}}{\partial\Phi^-\partial\Phi^-}\right]^{-1}\frac{\partial^2\mathcal{S}_{\mathrm{eff}}}{\partial\mu_d\partial\Phi^-} < 0$$

Part 1: NJL model

$$\mathcal{L}_{\mathrm{NJL}} = \bar{\psi}\left(\not{p} - m_0\right)\psi - g\left(\bar{\psi}\gamma^{\mu}\lambda_a\psi\right)\left(\bar{\psi}\gamma_{\mu}\lambda_a\psi\right)$$

- Free quarks
- Integrated out gluons
- Local color current interaction
- Chiral symmetry

➠ Local $SU(3)_c \xrightarrow{QCD \to NJL}$ Global $SU(3)_c$ ➠ No confinement

Spontaneous chiral symmetry breaking

$$\Omega_{\mathrm{NJL}} = \frac{\sigma^2 + N^2}{2G} - \frac{T}{2}\sum_{\omega_n}\int \frac{d^3p}{(2\pi)^3}\,\mathrm{Tr}\log\frac{S^{-1}(\omega_n, \vec{p})}{T}$$

- Hartree-Fock approximation using Fierz-transformations
- Bosonization in channels with large 4-quark coupling

➠ $\sigma = G\,\langle\bar{\psi}\psi\rangle$ ➠ $M = m_0 - \sigma$ ➠ $N = -G\,\langle\bar{\psi}i\gamma_5\tau_1\psi\rangle$

➠ $$S^{-1} = \begin{pmatrix} \not{p} - M + \gamma_0(\mu + \mu_I) & -i\gamma_5 N \\ -i\gamma_5 N & \not{p} - M + \gamma_0(\mu - \mu_I) \end{pmatrix}$$

- No explicit isospin breaking terms (Zhang, Liu [ZL07]) ➠ $N = 0$

Part 2: Polyakov loop model

- Model for SU(3)$_c$-gauge theory ➠ Confinement
  ➠ 1$^{\text{st}}$-order ➠ Spontaneous breakdown of $Z(3)$-center sym.

Order parameter for de-confinement – Polyakov loop

- Polyakov loop $\Phi(\vec{x})$ is the trace of a time-like Wilson-line

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➠ $\langle \Phi \rangle = 0 \iff$ confined ➠ $\langle \Phi \rangle \neq 0 \iff$ deconfined

Ginzburg-Landau effective potential $U = U(\Phi, \Phi^*, T)$

- Simplified loop: $\Phi = \frac{1}{N_c} \mathrm{Tr} \exp\left\{ i \frac{A_4^a t_a}{T} \right\} \quad a \in \{3, 8\}$
- Integrate out all dofs that do not change order parameters

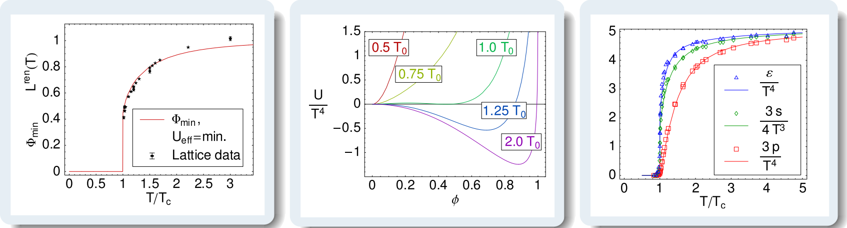
$$\int \mathcal{D}\Phi \int \mathcal{D}\Phi^*\, e^{-U(\Phi,\, \Phi^*, T)} = \int \mathcal{D}A\, e^{-S_{\mathrm{eff}}(\Phi(A), \Phi^*(A), T)}$$

$$\frac{U(\Phi,\Phi^*,T)}{T^4} = -\frac{1}{2}b_2(T)\ \Phi^*\Phi + \frac{1}{4}b_4(T)\ \log\left[J(\Phi,\Phi^*)\right]$$

$$J(\Phi,\Phi^*) = 1 - 6\Phi^*\Phi + 4\left({\Phi^*}^3 + \Phi^3\right) - 3\left(\Phi^*\Phi\right)^2$$

$$b_4(T) = b_4\left(\frac{T_0}{T}\right)^3 \qquad b_2(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2$$

- Temperature dependent coupling strength $b_2 = b_2(T)$



G. Boyd et. al. [B+96], O. Kaczmarek et. al. [KKPZ02, KZ05]

- $\Phi^* = \Phi$ at $N_f = 0$: $\quad U(\Phi,\Phi^*,T)$ only fixed in $\frac{1}{2}(\Phi^* + \Phi)$
- Stiffness of $U(\Phi,\Phi^*,T)$ in $\frac{1}{2}(\Phi^* - \Phi)$ is *free* to be adjusted

Polyakov loop extended NJL (PNJL)

- Substitute the Matsubara frequencies ω_n by $\omega_n + A_4$
  - Formal substitution $\mu \to \mu - iA_4$ *after* Matsubara summation

$$\Omega_0 = \Omega_{\text{NJL}}\big|_{\mu \to \mu - iA_4} + U(\Phi, \Phi^*, T)$$

Defining mean field as 0th perturbative order

- Fermion sign problem: $\mu \to \mu - iA_4$ ➠ $\Omega_0 = \frac{T}{V} S_{\text{E}} \in \mathbb{C}$
- Identification in 0th order: $p_{(T)} = -\Omega_{\text{MF}(T)} + \Omega_{\text{MF}(T=0)}$
- Mean field: $\Omega_{\text{MF}} = \text{Re}\,\Omega_0$

➠ Maximization of $\left|e^{-S_{\text{E}}/T}\right|$ ("quenched" mean field)

$$\frac{\partial\,\text{Re}\,\Omega_0}{\partial\sigma} = \frac{\partial\,\text{Re}\,\Omega_0}{\partial\Delta} = \frac{\partial\,\text{Re}\,\Omega_0}{\partial\mathcal{A}_4^{(3)}} = \frac{\partial\,\text{Re}\,\Omega_0}{\partial\mathcal{A}_4^{(8)}} = 0$$

➠ Constraints: $\Omega_{\text{MF}} \overset{!}{\in} \mathbb{R}$ ➠ $\Phi_{\text{MF}} = \Phi^*_{\text{MF}} \ldots$

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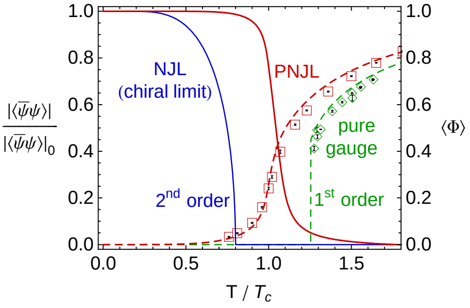

R., Hell, Ratti, Weise arXiv:0712.3152 [hep-ph], [RHRW07]

	T_c in MeV
Kaczmarek et al. [KZ05] ($N_f = 2$)	202
Cheng et al. [C$^+$06] ($N_f = 2+1$)	192
PNJL ($N_f = 2$)	215

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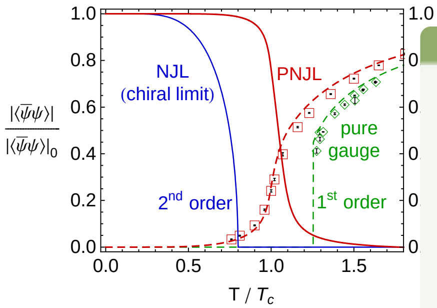


R., Hell, Ratti, Weise arXiv:0712.3152 [hep-
[RHRW07]

"PNJL-Confinement"

- Confinement at $T < T_c$:
  - Polyakov loop $\langle\Phi\rangle \ll 1$
- Free quarks suppressed
- Statisical confinement:
  - Active (color-neutral) quasi-particles:

$$m = 3\,M \approx M_N$$

Goal: Release constraints on the mean fields

➠ Integrate out the *approximate* order parameters (mean fields)

- Subtile cancellation of imaginary parts is guaranteed

How? Perturbative expansion about the (constraint) MF solution

➠ Taylor expansion of the action with respect to the fields

$$S = \frac{V}{T}\Omega_0 = \frac{V}{T}\sum_k \frac{1}{k!}\omega_k \,\vec{\xi}^k \quad \text{with } \vec{\xi} = \vec{\theta} - \vec{\theta}_0$$

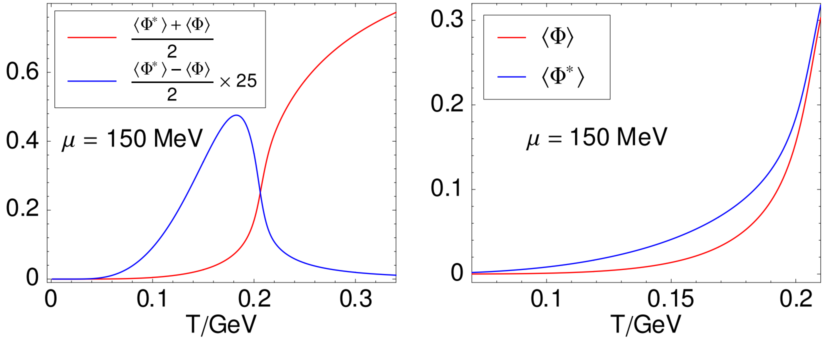
- The vector arrow " $\vec{}$ " ➠ Set of all fields $\vec{\theta} = (\sigma, N, A_4^{(3)}, A_4^{(8)})^T$
- $\vec{\theta}_0$ is the new minimum after SSB

➠ Separate free and perturbative parts:

- Free part: $k = 0, 1, 2$
- Interactions: $k \geq 3$

Note: $\mathrm{Im}[\omega_1] \neq 0$ for the (former) gauge field $A_4^{(8)}$

Expectation values of the Polyakov loop $\langle\Phi\rangle$ and $\langle\Phi^*\rangle$



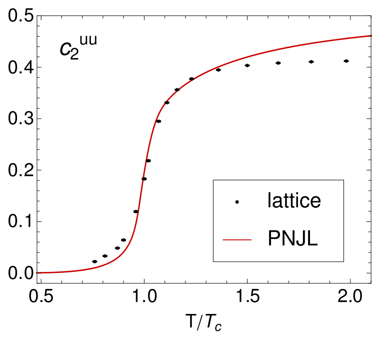
In mean field	MF + corrections
$\langle\Phi\rangle_{\text{MF}} = \langle\Phi^*\rangle_{\text{MF}}$	$\langle\Phi\rangle \in \mathbb{R}$ and $\langle\Phi^*\rangle \in \mathbb{R}$
No split of $\langle\Phi\rangle$ and $\langle\Phi^*\rangle$	$\langle\Phi\rangle \neq \langle\Phi^*\rangle$ at $\mu \neq 0$

➠ Fluctuation effects beyond mean field produce $\langle\Phi\rangle \neq \langle\Phi^*\rangle$

R., Hell, Ratti, Weise arXiv:0712.3152 [hep-ph], [RHRW07]

Susceptibilities: c_2^{uu}, c_2^{ud}, c_4^{uu}, c_4^{ud} beyond mean field

c_2^{uu}



c_2^{ud} in MF

$$c_n(T) = -\frac{1}{n!}\frac{\partial^n(\Omega/T^4)}{\partial(\mu/T)^n}\bigg|_{\mu=\mu_I=0} \qquad c_n^I(T) = -\frac{1}{n!}\frac{\partial^n(\Omega/T^4)}{\partial(\mu_I/T)^2\partial(\mu/T)^{(n-2)}}\bigg|_{\mu=\mu_I=0}$$

$$c_n^{uu} = \frac{1}{4}\left(c_n + c_n^I\right) \qquad c_n^{ud} = \frac{1}{4}\left(c_n - c_n^I\right)$$

- Isovector moments in agreement with lattice data as well

Lattice data: Allton et al. [A$^+$05], Bielefeld-Swansea coll.

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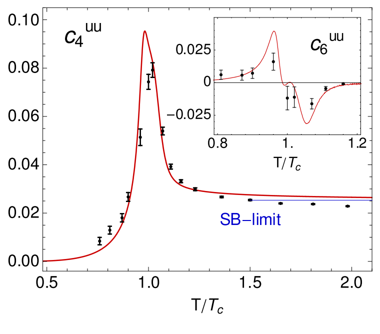

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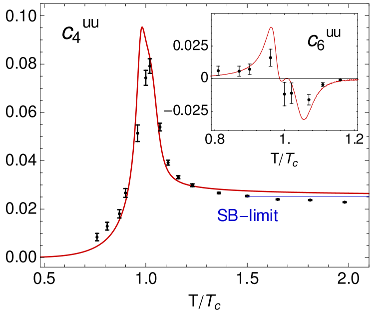

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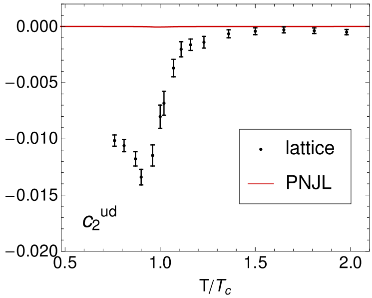

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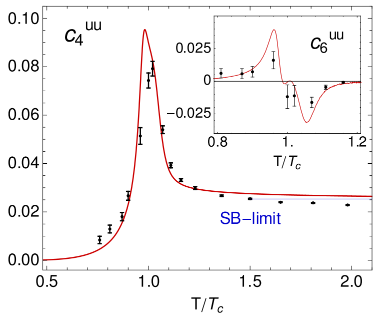
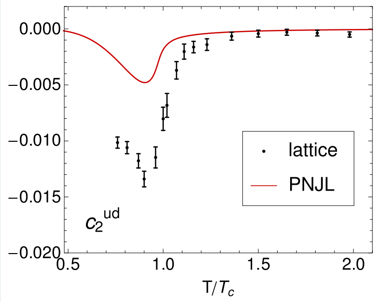


c_2^{ud} beyond MF



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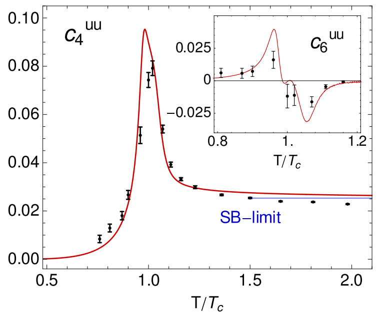
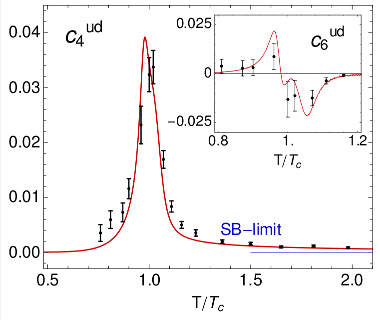


c_4^{ud}



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Adjusted Polyakov loop effective potentials

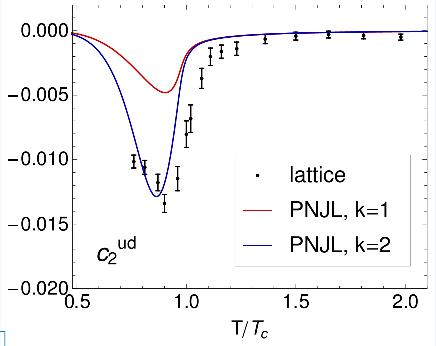
Connection to the Polyakov loop degrees of freedom

- Polyakov loop effective potential adjusted at $N_{\mathrm{f}} = 0$

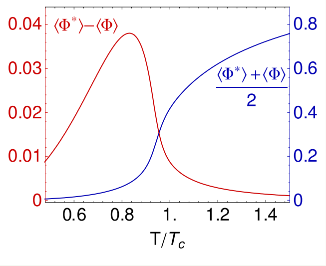
$$\frac{\delta U(\Phi, \Phi^*, T)}{T^4} \propto \Phi^*\Phi = \frac{{\Phi^+}^2 - {\Phi^-}^2}{4} \longrightarrow \frac{{\Phi^+}^2 - k\,{\Phi^-}^2}{4}$$

➡ Thermodynamics unchanged by varying k

c_2^{ud} with modified Φ^--potential



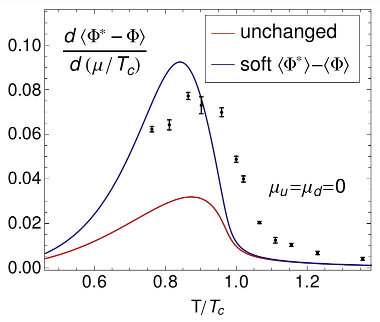
Φ^- at $\mu > 0$



- Φ^--dof ➡ $c_2^{ud} < 0$

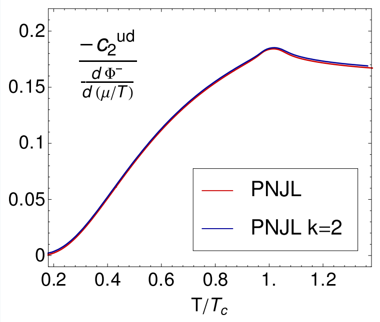
c_2^{ud} controlled by $\Phi^- = \frac{1}{2}\langle \Phi^* - \Phi \rangle$

Φ^- around $\mu = 0$



- Lattice: Döring [PhD Thesis]

c_2^{ud} normalized to Φ^--variation

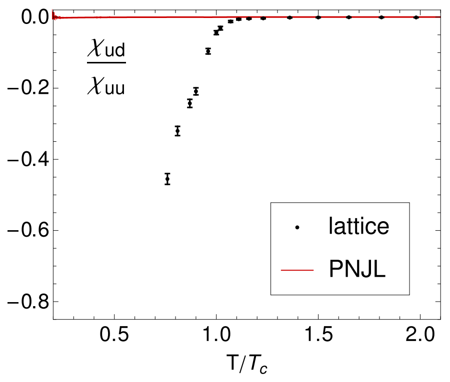


- Universal for varying k

➠ Variations of Φ^- are responsable for $c_2^{ud} < 0$.

Susceptibilities χ_{ud} beyond mean field

$\chi_{\mathrm{ud}}/\chi_{\mathrm{uu}}$ in MF



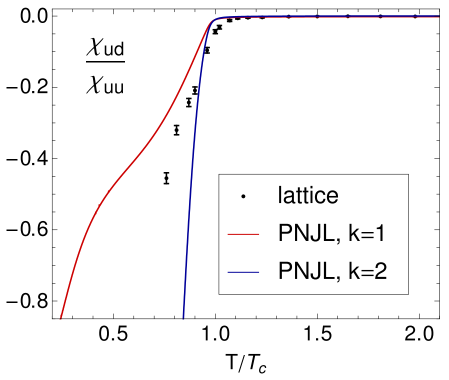
$$\frac{\chi_{\mathrm{ux}}(T,\mu)}{T^2} = 2c_2^{\mathrm{ux}} + 12c_4^{\mathrm{ux}}\left(\frac{\mu}{T}\right)^2 + 30c_6^{\mathrm{ux}}\left(\frac{\mu}{T}\right)^4 + \cdots \quad \text{with } x \in \{u, d\}$$

- Fluctuation effects: $\chi_{\mathrm{ud}} \neq 0$

Lattice data: Allton et al. [A+05], Bielefeld-Swansea coll.

Susceptibilities χ_{ud} beyond mean field

$\chi_{\mathrm{ud}}/\chi_{\mathrm{uu}}$ beyond MF



$$\frac{\chi_{\mathrm{ux}}(T,\mu)}{T^2} = 2c_2^{\mathrm{ux}} + 12c_4^{\mathrm{ux}}\left(\frac{\mu}{T}\right)^2 + 30c_6^{\mathrm{ux}}\left(\frac{\mu}{T}\right)^4 + \cdots \quad \text{with } x \in \{u, d\}$$

- Fluctuation effects: $\chi_{\mathrm{ud}} \neq 0$

Lattice data: Allton et al. [A$^+$05], Bielefeld-Swansea coll.

Conclusion

- PNJL:
  - Chiral symmetry breaking
  - Confinement
  - Entanglement of chiral and deconfinement crossover
- Perturbative approach used to investigate
  - ✔ Polyakov loop: $\langle\Phi\rangle \neq \langle\Phi^*\rangle$ at $\mu \neq 0$
  - ✔ Isovector susceptibilities
- Variations of Φ^- are responsable for $c_2^{\mathrm{ud}} < 0$
  - ➠ Stiffness of Polyakov loop effective potential directly governs c_2^{ud}
  - ➠ c_2^{ud} contains information about Polyakov loop effective potential
- Outlook
  - ✘ $2+1$ flavors
  - ✘ Extract Polyakov loop effective potential using c_2^{ud} from the lattice

Thank you for your attention

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