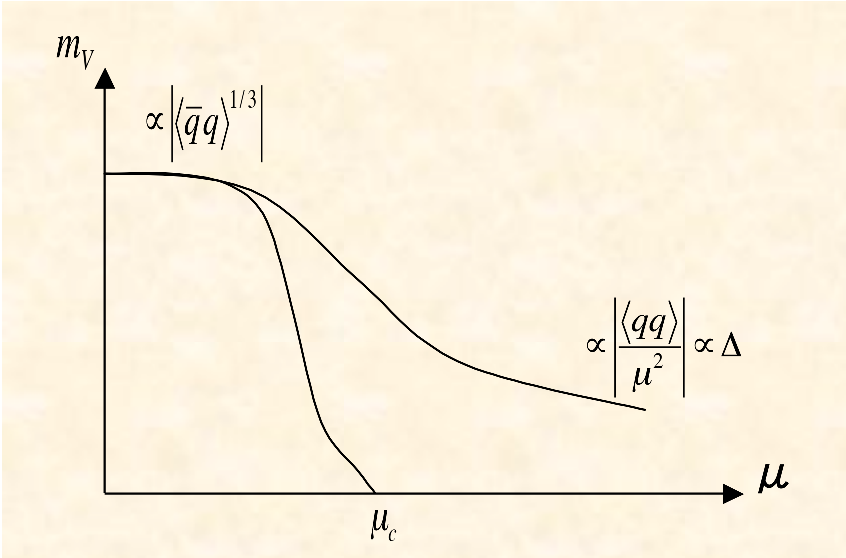
Spectral Continuity in Dense QCD

(有限密度QCDにおけるスペクトル連続性)



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arXiv:0802.4143[*hep-ph*]
Phys.Rev.D.78:011501,2008

Key words

"**Spectral Continuity**" of hadrons

(in-medium) QCD **sum rules**

Different roles of **vacuum condensates**

Quark-Hadron continuity

Plan of this talk

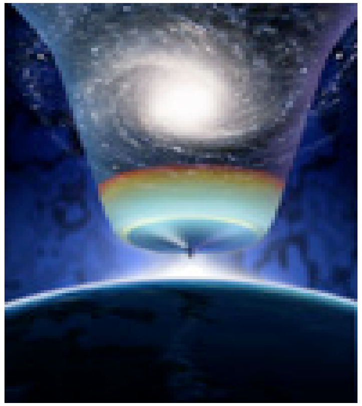
1. Introduction and motivations
2. In-medium QCD Sum Rules (QSR)
3. Flavor-octet vector mesons in QSR
4. Flavor-singlet vector meson in QSR
5. “Spectral Continuity” of vector mesons
6. Sum-mary and Per-spect-ives

1. Introduction and motivations

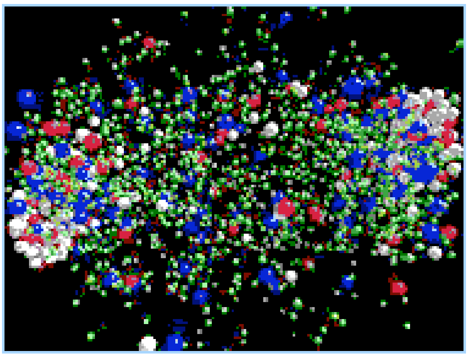
Exploring the QCD phase diagram is ***challenging***

- Big-Bang Cosmology, Heavy Ion Collisions (Little-Bang)
- Compact Stars (Neutron Stars, Quark Stars)
- Strongly-coupled Quantum Field Theory (QCD)

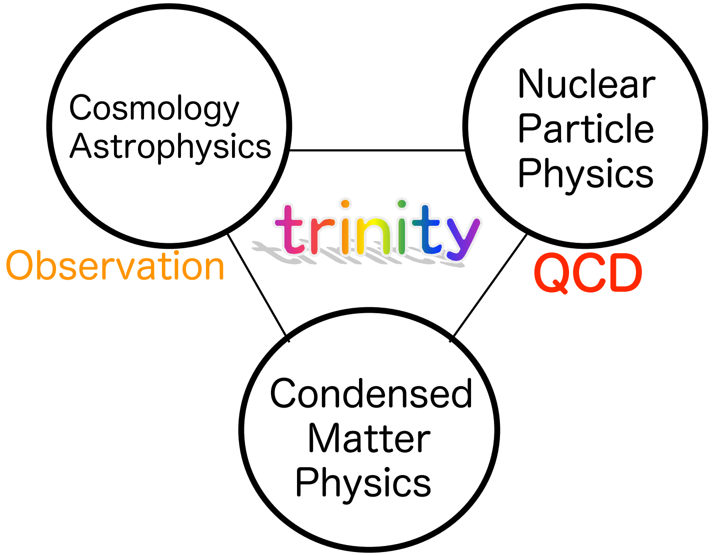
new state of matter



Big-Bang Cosmology



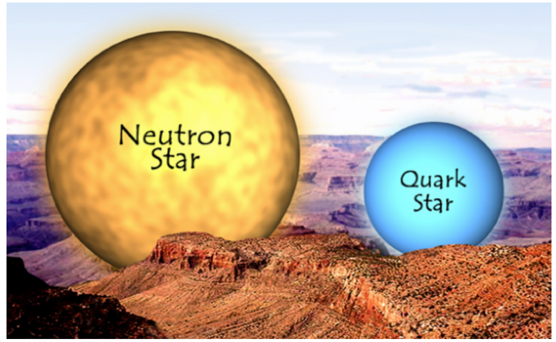
Heavy Ion Collisions

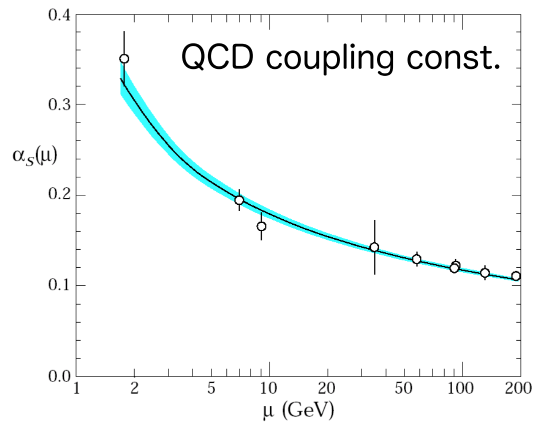


Many-body problem
Critical phenomena

QCD Phase Diagram

Compact Stars





QCD phase transition and the phase diagram

QCD @ **high** temperature(T) / density(ρ) [Collins-Perry (1975)]

- **asymptotic freedom**
- **screening** of color force

$$\Rightarrow \quad V(r) \sim \frac{\alpha_s(T,\mu)}{r} e^{-m_{sc} r}$$

"**weakly** interacting gas of quarks/gluons"

$\therefore$ QCD vacuum undergoes **a phase change** at some values of T and ρ!

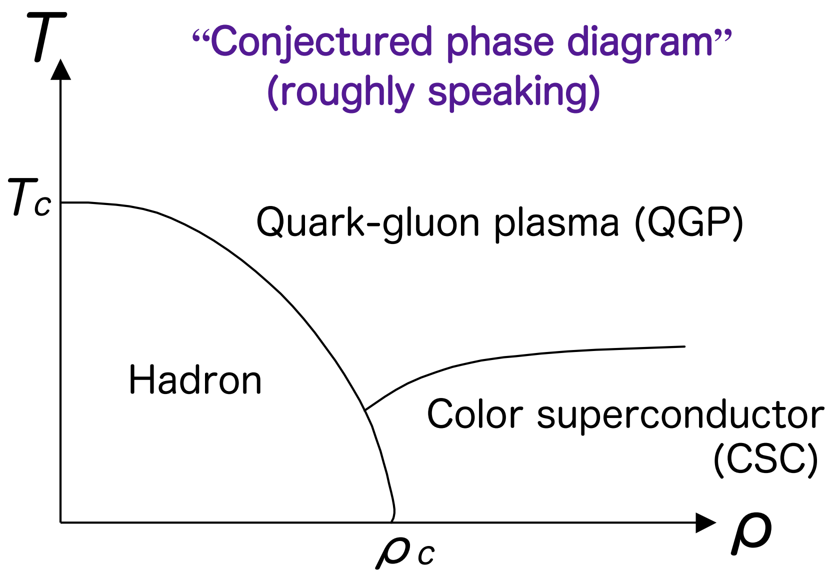
Model calculations and numerical simulations

↓

Strongly indicate the existence of such a ***transition*** from hadron to quark-gluon phase

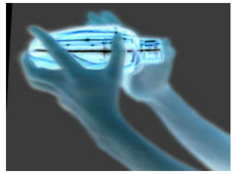
$$T_c \sim (150-200)\,MeV \sim 10^{12}\,K$$

$$\rho_c \sim several \times \rho_{nm} \sim 10^{12}\,kg \cdot cm^{-3}$$



Possible *New* Critical Point in Dense QCD

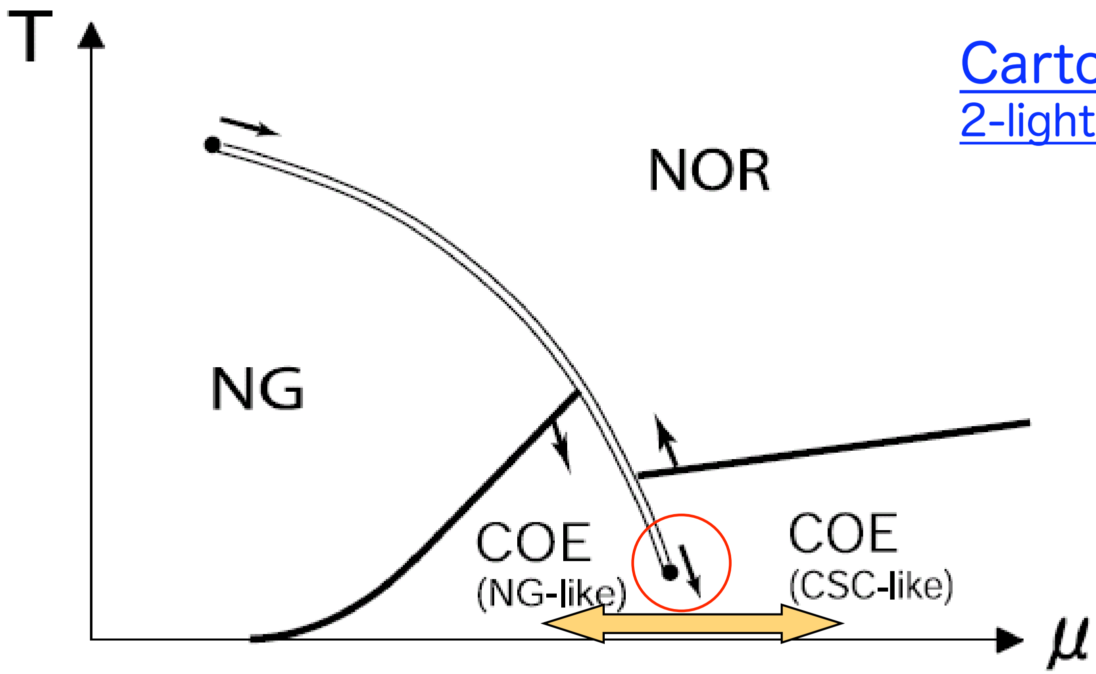
Hatsuda-Yamamoto-Baym-M.T., Phys. Rev. Lett. 97 (2006) 122001



@ $\mu \neq 0$
- Interplay b/w ***chiral*** & ***diquark*** *condensates*
- Presence of the ***U*(1)** ***axial anomaly***

Ginzburg-Landau (GL) model

➡ ***New critical point*** & ***Crossover*** from hadron-to-CSC !!



Cartoon phase diagram in 2-light +1-medium flavors

: critical point

: crossover

Appearance of a new critical point

σ : chiral condensate $\qquad$ d : diquark condensate

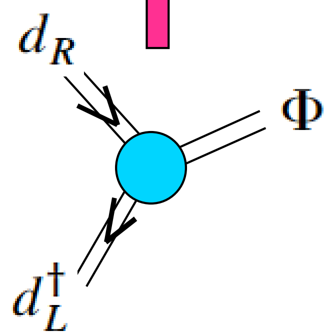
Ginzburg-Landau free energy in **massless 3 flavor** quark matter

$$\boxed{\Omega_{3F} = \left(\frac{a}{2}\sigma^2 - \frac{c}{3}\sigma^3 + \frac{b}{4}\sigma^4\right) + \left(\frac{\alpha}{2}d^2 + \frac{\beta}{4}d^4\right) - \gamma d^2 \sigma}$$

$a, b, c, \alpha, \beta, \gamma$: GL parameters

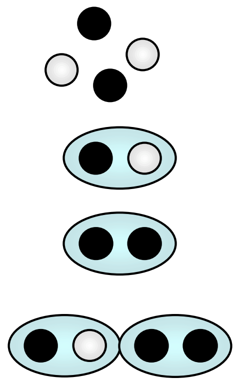
$\gamma \sim c > 0, \quad \beta,\ b > 0$

***anomaly**-driven*
'tHooft interaction



Possible phases

$$\left\{\begin{array}{c} \sigma = d = 0 \\ \sigma \neq 0 \\ d \neq 0 \\ \sigma \neq 0, d \neq 0 \end{array}\right.$$



- Mass term for d
- external field for σ

(equivalent to Ising Ferro-magnet)

Comments

1. All the lines and the points characterizing the whole phase boundaries can be determined analytically.

2. A similar critical point at low temperature has been derived by Kitazawa et al. [PTP108(2002)929], using the 2-flavor NJL model with scalar and vector type 4-fermion interactions. However since the axial anomaly does *not* produce a triple boson coupling in 2-flavors, the origin of their critical point will be different with that discussed here.

3. We performed the similar analysis in 2 flavor case and this case is found in an anisotropic anti-ferromagnet in reality such as *GdAlO3* (e.g., see Chaikin-Lubensky's textbook)

Interplay between chiral and diquark condensates

Intriguing!!

Excitation spectra

Hatsuda-Yamamoto-Baym-M.T., Phys. Rev. D76 (2007) 074001

Low energy excitations ∼ Nambu-Goldstone **(NG)** bosons associated with ***chiral symmetry breaking*** (in both hadronic and CSC phases !)

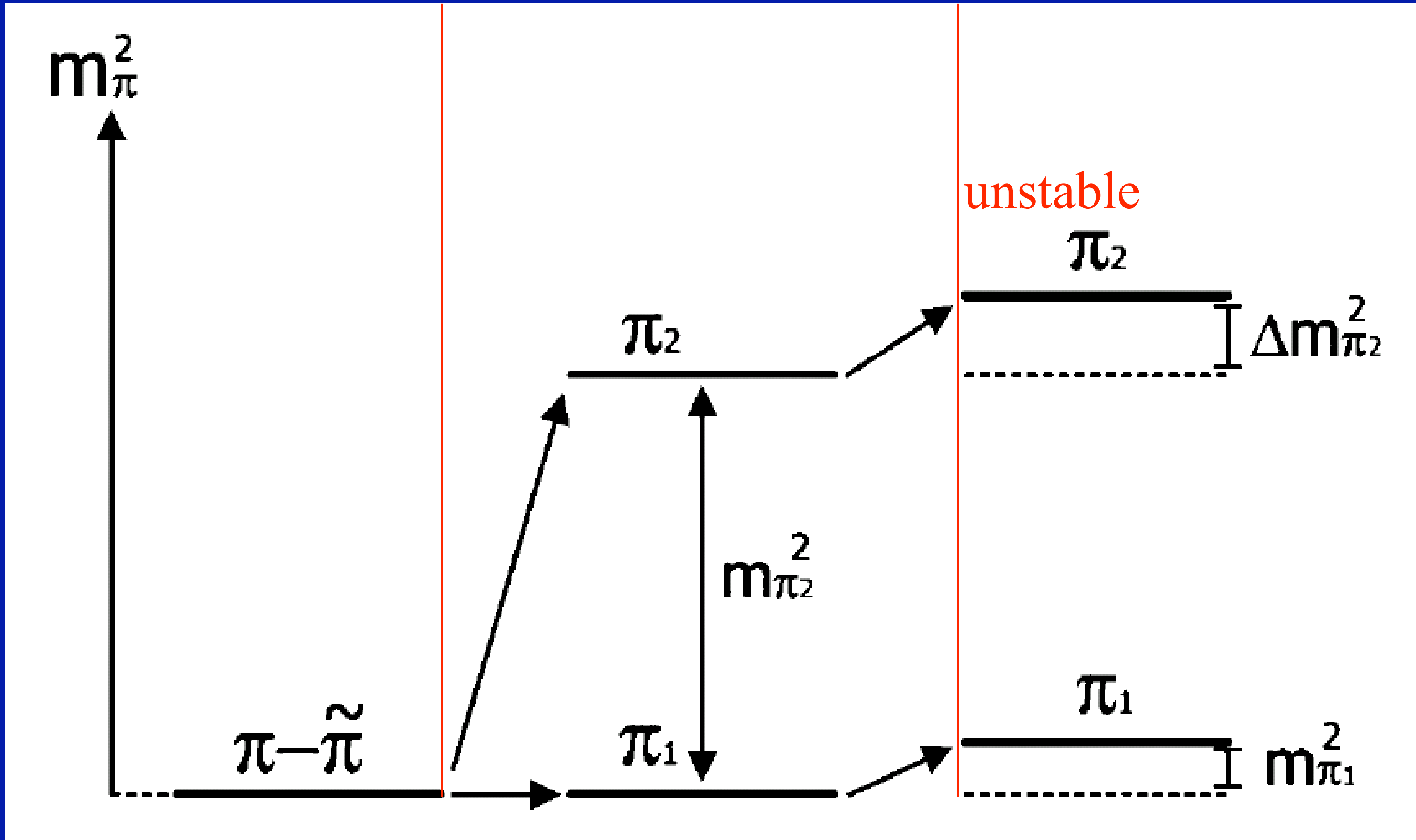
Generalized *Gell-Mann-Oaks-Renner (GOR) relation*

$$\boxed{\left(f_\pi^2 + f_{\tilde{\pi}}^2\right) m_\Pi^2 = -m_q \left(\langle \bar{q} q \rangle + \Gamma \langle q q \rangle^2\right)}$$

- An example of ”***spectral continuity*** ” of hadrons
- A concrete realization of “***quark-hadron continuity*** ”

(Schäfer-Wilczek)

Pion mass splitting



$\mathcal{L}^{\mathrm{kin}}$

$\mathcal{L}^{\mathrm{kin}} + \mathcal{L}^{\mathrm{mix}}$

$\mathcal{L}^{\mathrm{kin}} + \mathcal{L}^{\mathrm{mix}} + \mathcal{L}^{\mathrm{mass}}$

Taken from Gordon's talk in QM08

Quark-Hadron (QH) continuity

T. Schäfer and F. Wilczek, Phys. Rev. Lett. 82 (1999) 3956

excitations	low μ (hadron)	high μ (CFL)
NG bosons	π (& H)	π' & H
Fermions	Baryons	(gapped) quarks
Vector mesons	ρ, ω, ϕ, K*	(massive) gluons

However…

$N_f = 3$

vector mesons (8+1) $\longleftrightarrow$ gluons (8)
baryons (8) $\longleftrightarrow$ quarks (9)

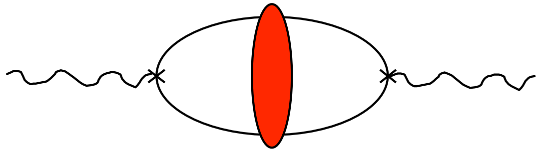
mismatch?

$\Rightarrow$ Investigating some general aspects of hadron spectrum in medium

QCD sum rules (QSR)

Shifman-Vainshtein-Zakharov, Nucl.Phys.B147 (1979) 385.

2. QCD sum rule (QSR)



Current correlators

$$\Pi_L^{AB}(\omega) = \lim_{\vec{k}\to 0} \frac{i}{\vec{k}^2} \int d^4x e^{ikx} \left\langle R J_0^A(x) J_0^B(0) \right\rangle$$

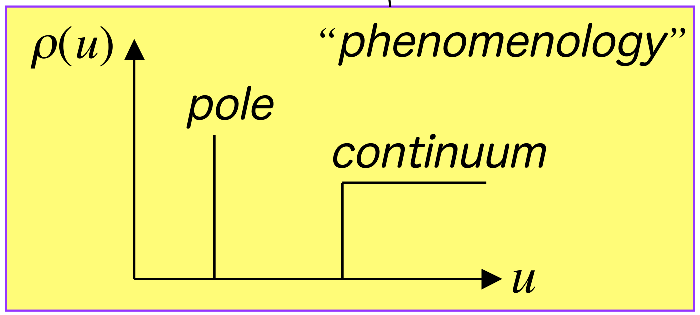
Dispersion relations

"spectral function"

$$\Pi_L(\omega) = \int_0^\infty \frac{\rho(u)}{u^2 - (\omega + i\varepsilon)^2} du^2$$

Operator product expansion (OPE):

$$\Pi_L(Q^2 \to \infty) \sim \sum_n C_n(Q^2) \frac{\langle O_n \rangle}{Q^{2n}}$$



QSR (cont'd)

Comments

1. In medium, not only the Lorentz scalar but also the ***tensor*** operators in OPE contribute to the correlation functions.
(Hatsuda-Lee '92, Hatsuda-Koike-Lee '93)

2. The spectral function $\rho(u)$ is just *phenomenologically* given in terms of so called "**resonance parameters**".

In the following, we focus on flavor octet and singlet vector mesons in *3 flavor quark matter at zero temperature and finite density with $m_q = 0$.*

$$J_\mu^A = \bar{q}\tau^A\gamma_\mu q \quad \text{with} \quad q = (u,d,s)$$

τ^A : *U(3)* generators with $tr\left[\tau^A\tau^B\right] = 2\delta^{AB}$ $(A = 0,\cdots,8)$

Warm-up --non-interacting quark matter@$\mu \neq 0$ --

$$\Pi_L^{(free)}(Q) = -\frac{1}{2\pi^2}\log Q^2 + \frac{16}{9}\frac{\langle q^+ i\partial_0 q\rangle}{Q^4} + \frac{64}{9}\frac{\langle q^+ i\partial_0^3 q\rangle}{Q^6}$$

Lorentz **non-scalar** operators

Spectral function in free quark matter

$$\rho^{(free)}(u) = F\delta(u^2) + A\theta(u^2 - S_0)$$

$S_0 = (2\mu)^2$

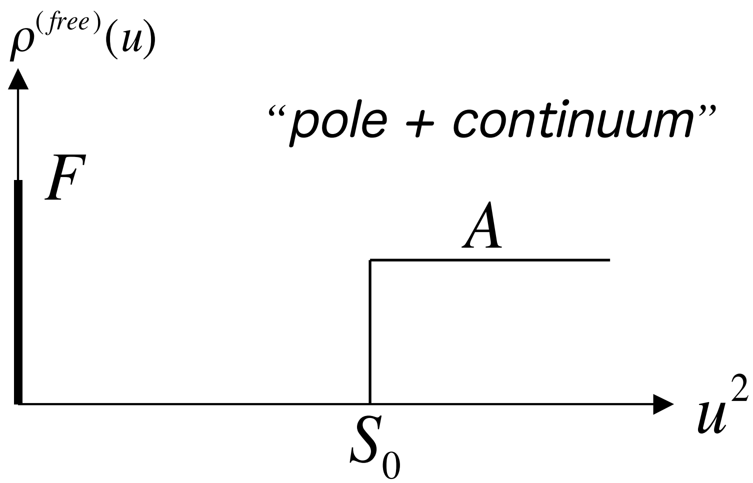
$A = 1/(2\pi^2)$

$F = AS_0$

"resonance parameters"

scattering of quarks on the Fermi surface with external current, i.e., the *Laundau damping* term (pole part)

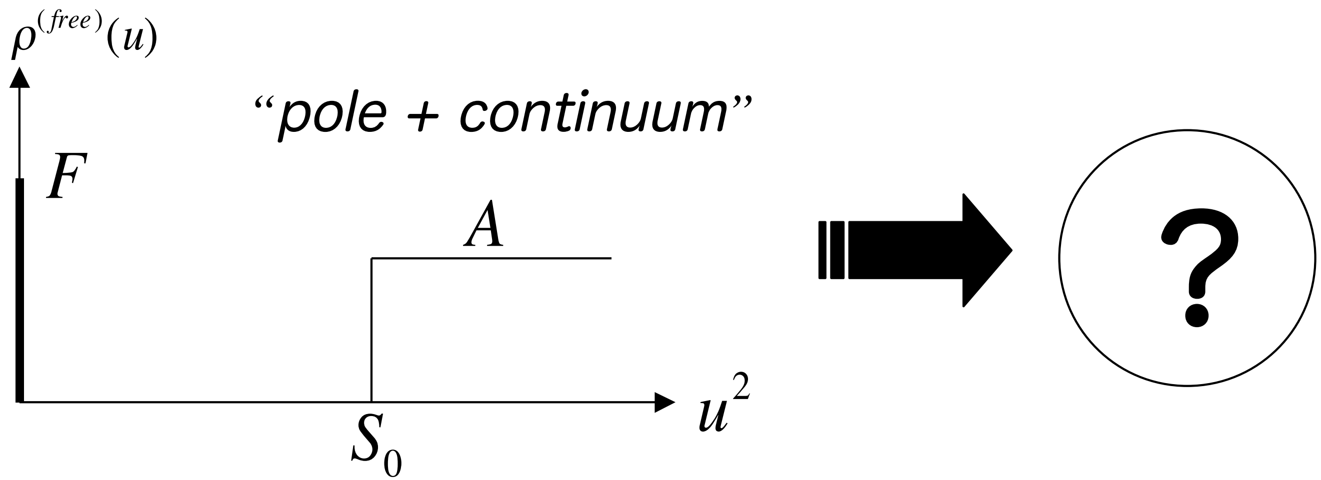
decay of the external current into $q\bar{q}$-pair with *Pauli blocking* effect (continuum part)



Comment

Corrections of the form $\alpha_s \ln Q^2$ and $\alpha_s (\mu/Q)^n$ to this case can be taken into account in *perturbation theory* and are compensated by the *perturbative* corrections to the **resonance parameters** and the shape of the spectral function.

How the ***genuine nonpertubative*** effects such as the *condensates* affect properties of hadrons from QSR?



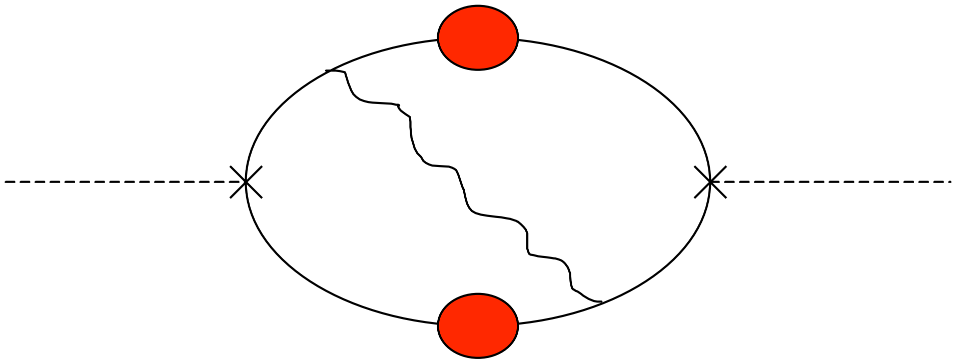
In-medium OPE (vector mesons)

$$J_\mu^{(8)}(x) \equiv \bar{q}(x)\tau^a\gamma_\mu q(x), \qquad J_\mu^{(1)}(x) \equiv \bar{q}(x)\tau^0\gamma_\mu q(x)$$

$$\Pi_L^{(8,1)} = \Pi_L^{(free)} + \delta\Pi_L^{(8,1)},$$

$$\delta\Pi_L^{(8)} = -\frac{\pi\alpha_s}{Q^6}\left[\left\langle \frac{1}{4}(\bar{q}\gamma_\mu\gamma_5\tau^a\lambda^{a'}q)^2 + \frac{8}{27}(\bar{q}\gamma_\mu\lambda^{a'}q)^2 \right\rangle\right]$$

$$\delta\Pi_L^{(1)} = -\frac{\pi\alpha_s}{Q^6}\left[\left\langle 2(\bar{q}\gamma_\mu\gamma_5\tau^0\lambda^{a'}q)^2 + \frac{8}{27}(\bar{q}\gamma_\mu\lambda^{a'}q)^2 \right\rangle\right]$$



$$\Pi_L^{(8)} \equiv \frac{1}{8}\sum_{A=1}^{8}\Pi_L^{AA}, \quad \Pi_L^{(1)} \equiv \Pi_L^{00}$$

λ^a : color *SU(3)* generators

τ^a : flavor *SU(3)* generators

$(a = 1 \sim 8)$

Comments

1. There are several operators neglected up to $O(1/Q^6)$, whose explicit forms are given in
Hatsuda-Lee '92, Hatsuda-Koike-Lee '93.

2. Among others, the *gluon condensate* $\left\langle \frac{\alpha_s}{\pi} TrF_{\mu\nu}^2 \right\rangle$, if it exists, affects the octet and singlet mesons in the *same* way.

3. The non-scalar operators like the *quark-gluon mixed* operators and the *twist-4* quark operators do *not* produce the chiral and diquark condensates and lead only to *perturbative* corrections to $\Pi_L^{(free)}$.

『twist(τ) ≡ cano. dim.(d)-spin』 $\boxed{\mathrm{O}^{d,\tau}}$

e.g.) $\mathrm{O}^{4,4} = \frac{\alpha_s}{\pi} TrF_{\mu\nu}^2$ $\mathrm{O}^{6,4} = \bar{q}\{D_\mu, {}^* G_{\nu\lambda}\}\gamma^\lambda\gamma_5 q$

Pairing patterns and factorization

Let us consider here chiral condensate $\langle \bar{q}q \rangle$ and diquark condensate $\langle qq \rangle$ which are in ***the most attractive channels*** (*MAC*):

$$\left\langle \bar{q}_i^{\alpha} q_j^{\alpha} \right\rangle = diag(\sigma,\sigma,\sigma)$$

$$\frac{1}{4}\varepsilon_{ijk}\varepsilon_{\alpha\beta\gamma}\left\langle q_j^{\beta} C\gamma_5 \Lambda_+ q_k^{\gamma} \right\rangle = diag(\varphi,\varphi,\varphi)$$

i,j,k : flavor

α,β,γ : color

Λ_+ : positive energy projection op.

After rewriting 4 quark ops. in chiral basis and making the *Fierz rearrangement*

together with the **factorization ansatz**: $\langle O \rangle = \sum_l \langle P_l \cdot P_l \rangle \cong \sum_l \langle P_l \rangle^2$, we obtain

$$\delta\Pi_L^{(8,1)} \cong \Pi_\sigma + \Pi_\varphi^{(8,1)},$$ where

$$\Pi_\sigma = -\frac{448\pi\alpha_s}{81Q^6}\sigma^2$$

$$\Pi_\varphi^{(8)} = -\frac{5}{22}\Pi_\varphi^{(1)} = -\frac{320\pi\alpha_s}{27Q^6}\varphi^2$$

Comments

1. The qualitative conclusion in the present work do *not* depend on the factorization ansatz.

2. Since the chiral condensate is *flavor-diagonal* , it does not distinguish between octet and singlet.

3. While, the diquark condensate has *color-flavor* structure so that it can *smell* flavors differently.

This is why the flavor-octet and -singlet vector mesons, which are almost degene-rate at low density, tend to split at high density due to the appearance of *diquark condensates.*

3. Flavor-octet vector mesons in QSR

Spectral function

$$\rho^{(8)}(u) = F\delta(u^2 - m_V^2) + A\theta(u^2 - S_0)$$

Plugging this into the dispersion relation, carrying out the *asymptotic expansion* in terms of $1/Q^2$, and comparing the result with the OPE expression:

$$F - AS_0 = 0$$

$$2Fm_V^2 - AS_0^2 = -\frac{(2\mu)^4}{2\pi^2}$$

$$3Fm_V^4 - AS_0^3 = -\frac{(2\mu)^6}{2\pi^2} + \left\langle O^{(8)} \right\rangle$$

Finite energy sum rules (*FESR*)

$$\left\langle O^{(8)} \right\rangle = -\frac{448\pi\alpha_s}{27}\left(\sigma^2 + \frac{15}{7}\varphi^2\right) < 0$$

Solutions of *FESR*s

$\mu=0$ $(\varphi=0)$

$$\boxed{\left(m_V^{(8)}\right)^2 \to \left(\frac{448\pi^3\alpha_s}{27}\sigma^2\right)^{1/3}}$$

N. V. Krasnikov et al., Z. Phys.C19 (1983) 301

$\mu\neq 0$ From *FESR*s, we have the following ***quartic*** equation:

$$t^4+6t^2-4(1+r^{(8)})t-3=0$$

$$t=\frac{S_0}{(2\mu)^2}$$

$$r^{(8)}=-\frac{2\pi^2}{(2\mu)^6}\langle O^{(8)}\rangle$$

In a situation where $0<r^{(8)}<<1$, one finds **a *unique* solution**:

$S_0\cong(2\mu)^2+\left(m_V^{(8)}\right)^2$ and

$$\boxed{\left(m_V^{(8)}\right)^2 \cong \frac{56\pi^3\alpha_s}{81\mu^4}\left(\sigma^2+\frac{15}{7}\varphi^2\right)}$$

a *new* formula relating octet vector meson mass to chiral condensate (σ) and diquark one (φ) !!

Octet (cont'd)

$$\boxed{\left(m_V^{(8)}\right)^2 \cong \frac{56\pi^3\alpha_s}{81\mu^4}\left(\sigma^2 + \frac{15}{7}\varphi^2\right)}$$

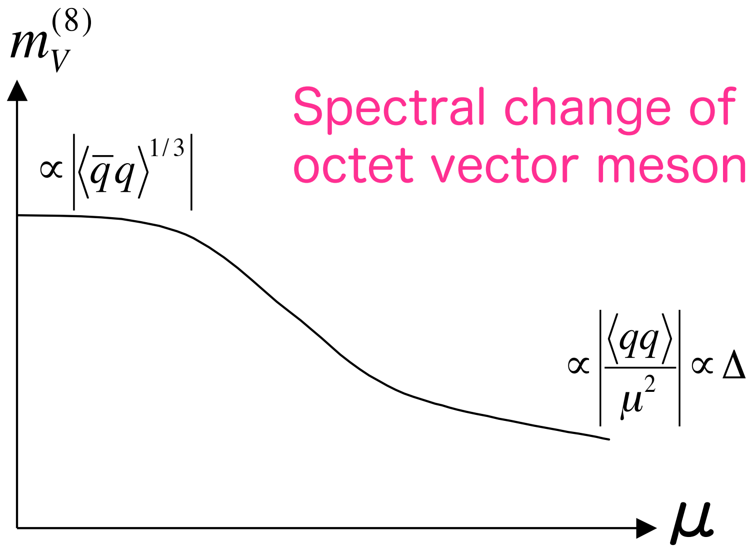
① For $\mu = 500\,MeV$, $\alpha_s \sim 1$ and $\sigma \sim \varphi \sim (150\,MeV)^3$, $m_V^{(8)} \cong 100\,MeV$.

② At asymptotic high density, we have $\sigma \sim 0$ and the weak coupling relation :

(T. Schäfer, Nucl.Phys.B575('00)269)

$$\varphi = \frac{3\mu^2\Delta}{\sqrt{2\pi^3\alpha_s}} \quad (\Delta\text{: fermion gap})$$

$$\boxed{m_V^{(8)} \to \sqrt{\frac{20}{3}}\Delta \cong 2.6\Delta}$$



Spectral change of octet vector meson

4. Flavor-singlet vector mesons in QSR

In this case, *unlike the flavor octet*, the NG scalar boson associated with $U(1)_B$ breaking contributes to the spectral function

(cf. this is analogous to the situation where not only a_1 meson but also the pion contribute to *the axial-vector current correlation* in the vacuum.)

Spectral function

$$\rho^{(1)}(u) = F_H \delta(u^2) + F\delta(u^2 - m_V^2) + A\theta(u^2 - S_0)$$

FESRs

$$F + F_H - AS_0 = 0$$

$$2Fm_V^2 - AS_0^2 = -\frac{(2\mu)^4}{2\pi^2}$$

$$3Fm_V^4 - AS_0^3 = -\frac{(2\mu)^6}{2\pi^2} + \left\langle O^{(1)} \right\rangle$$

$$\left\langle O^{(1)} \right\rangle = -\frac{448\pi\alpha_s}{27}\left(\sigma^2 - \frac{66}{7}\varphi^2\right)$$

This could be either positive or negative!!

Singlet (cont'd)

Although sum rules are *not* closed due to the extra parameter F_H, one can still select a possible solution under physical constraints such as ***positivity*** of the spectral function $\rho(u)$ and the small magnitude of $r^{(1)} \equiv -\frac{2\pi^2}{(2\mu)^6}\langle O^{(1)} \rangle$.

From *FESR*s, we have the following set of equations:

$$\boxed{\begin{aligned} &2t^3 - 3m^2t^2 + 3m^2 - 2 - 2r^{(1)} = 0 \\ &2fm^2 - t^2 + 1 = 0 \end{aligned}}$$

could be positive or negative!!

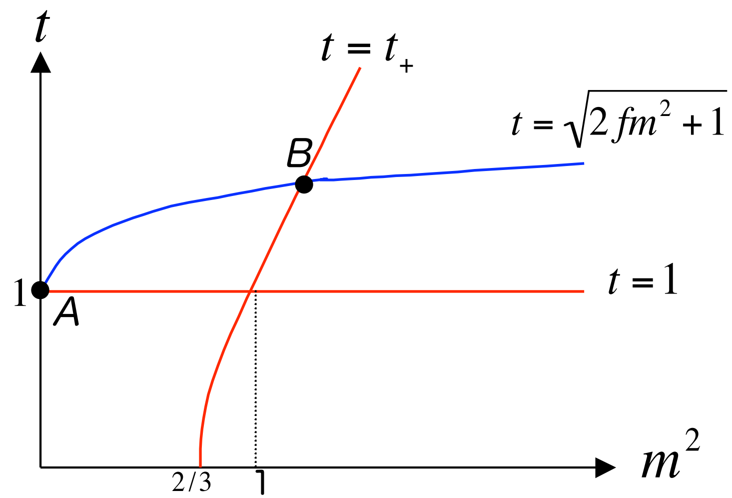
where

$$t \equiv \frac{S_0}{(2\mu)^2}, \quad m \equiv \frac{m_V^{(1)}}{2\mu}, \quad f \equiv \frac{F}{F_H}, \quad r^{(1)} \equiv -\frac{2\pi^2}{(2\mu)^6}\langle O^{(1)} \rangle$$

Level crossing/repulsion

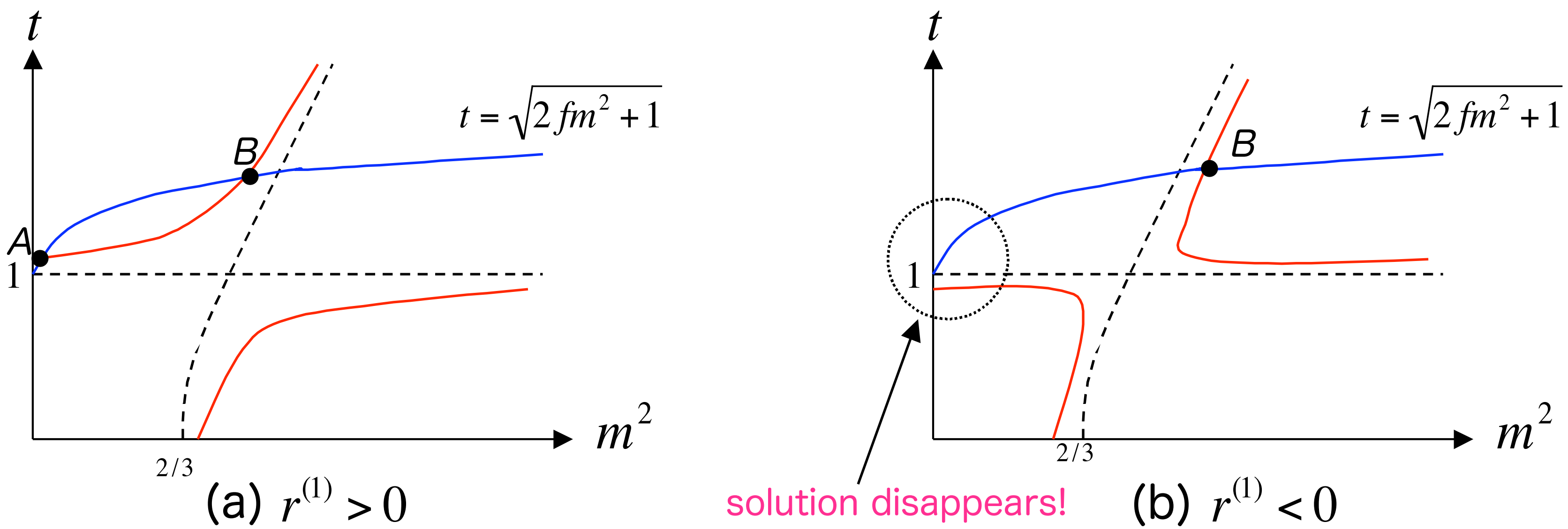
[the case with $r^{(1)} = 0$] **"level crossing"**

t=1 is a trivial solution from the cubic eq. and there is another positive solution (t_+) . As the result we find two solutions for $m_V^{(1)}$: one is massless (*A*) and the other is heavier than 2μ (*B*). ***(see the right cartoon)***



[the case with $r^{(1)} \neq 0$] **"level repulsion"**

In this case, *t=1* is not a solution any more. According to the sign of $r^{(1)}$, the resultant picture gets dramatically different.



(a) $r^{(1)} > 0$ (b) $r^{(1)} < 0$

Physical interpretations

Let us assume that chiral condensate (σ) is an ***increasing*** function and diquark condensate (φ) is a ***decreasing*** function of μ. Then $r^{(1)}$ starts from some positive value at low density and will change its sign to negative at a certain intermediate chemical potential μ_c, which is given by the condition, $\sigma^2 = (66/7)\varphi^2$.

At lower density, as seen from the left panel of the previous cartoon, a light solution (*A*) exists. However as soon as $r^{(1)}$ changes the sign, the light solution is **gone** (right panel) and only the heavy solution (*B*) exists. Since this heavy solution is above the threshold for $\bar{q}q$ decay, it is not expected to appear as a sharp resonance in reality.

Therefore, **unlike the octet case,**

the singlet vector meson disappears

from the low-energy spectrum at high density !

5. "Spectral Continuity" of vector mesons

Our results

Flavor-octet vector mesons $\longrightarrow$ survive $\left(m_V^{(8)} \cong O(\Delta)\right)$

Flavor-singlet vector meson $\longrightarrow$ disappears at high density

On the other hand…

1. Flavors and colors are **mixed** in the CFL phase

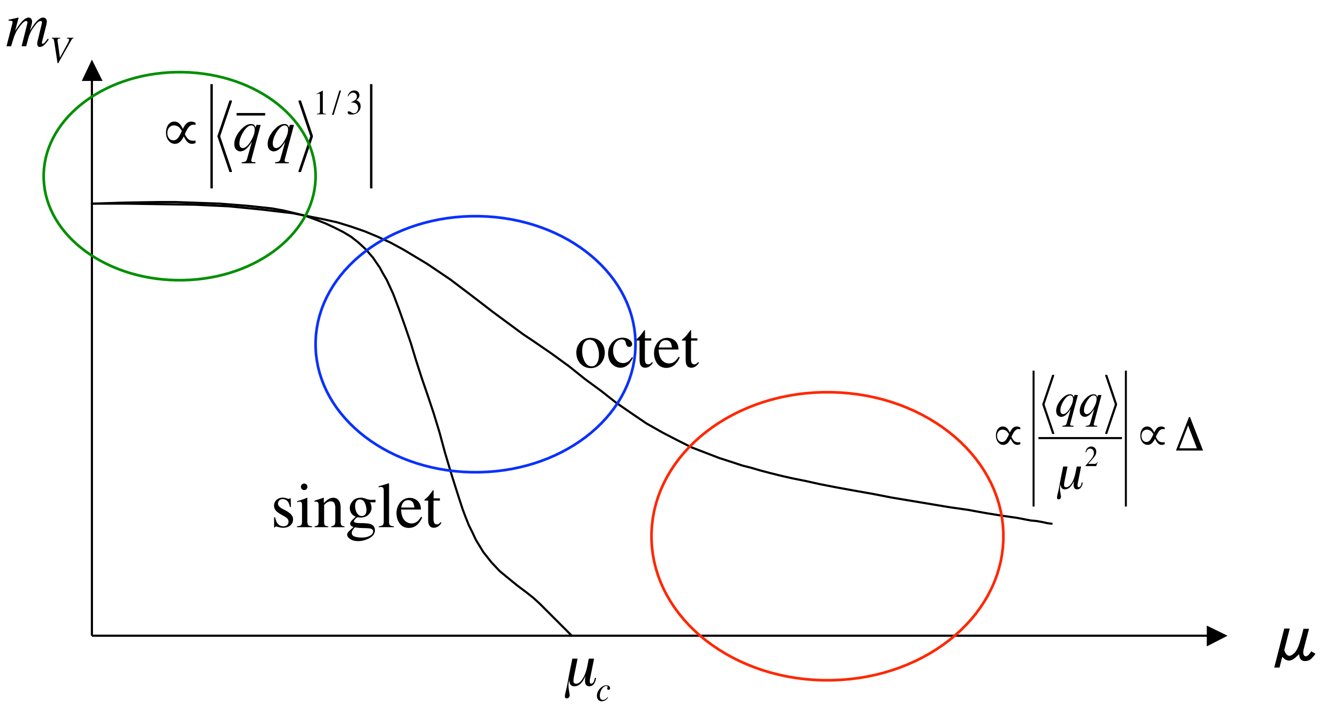
2. The light gluonic mode **(the CFL plasmon)**

H. Malekzadeh and D. Rischke,
Phys.Rev.D73 (2006) 114006

$$\boxed{m_{pl.}^{CFL} \cong O(\Delta)}$$

Strongly suggest **the QH continuity** of vector mesons?

Schematic plot of the vector meson masses



lower density

almost degenerate (*nonet*) with the mass governed by **chiral condensate** $\langle \bar{q}q \rangle$

medium density

mass splitting b/w octet and singlet developed due to **diquark condensates** $\langle qq \rangle$

higher density

octet **survive** as light modes, while singlet **disappears** from the low-energy spectrum

6. Sum-mary and Per-spect-ives

Summary

- ”**Spectral Continuity**” of hadrons
- **Vector mesons** in quark matter using in-medium QSR
- **New mass formula** (diquark condensate)
- Fate of **octet/singlet** vector mesons at high density

octet ~ **survive** as the light excitations of $O(\Delta)$

singlet ~**disappears** from the low-energy spectrum

Diquark says “Yeah, I can smell you”

- Possible connection to the **quark-hadron continuity**

Perspectives

- Spectral continuity of **baryons** (connected to gapped quarks?)
- **Width** of the excitations
- Incorporating **nonzero quark masses**
- Effect of **confinement** (OPE is a local-operator expansion)
- Utilizing **gauge/gravity duality** (field-operator correspondence)

Back-up slides

T. Schafer and F. Wilczek, Phys.Rev.Lett.82 (1999) 3956

...This superfluidity, whatever its source, supplies us with the key to the riddle of the missing vector meson. For once there is a massless singlet scalar, the putative singlet vector becomes radically unstable, and should not appear in the effective theory.

... Finally, there is the question of the “extra” singlet baryon. This is the most straightforward. In the original calculations, it was found that the singlet gap is much larger than the octet gap. Thus the singlet baryon is predicted to be considerably heavier than the octet.

How about photon? (Jean-Paul Blaizot)

Zero density

ρ-ω—photon mixing (Vector dominance)

High density (CFL)

gluon—photon mixing $SU(3)_c \times U(1)_{em} \rightarrow \tilde{U}(1)_{em}$

"Weinberg-Salam"

How the photon branch is connected from low to high density?

What governs the QCD phase diagram?

1. Structures of interactions and their changes in T and ρ
2. Order parameters, i.e., vacuum condensates
3. Existence of external parameters (e.g., quark mass)
4. Physical constraints (neutrality conditions,etc.)
5. Universality arguments in critical phenomena

⋮

6. Models to be applied
7. Assumptions and approximations to be made

⋮

Each person may propose its own phase diagram…

蓋然性？

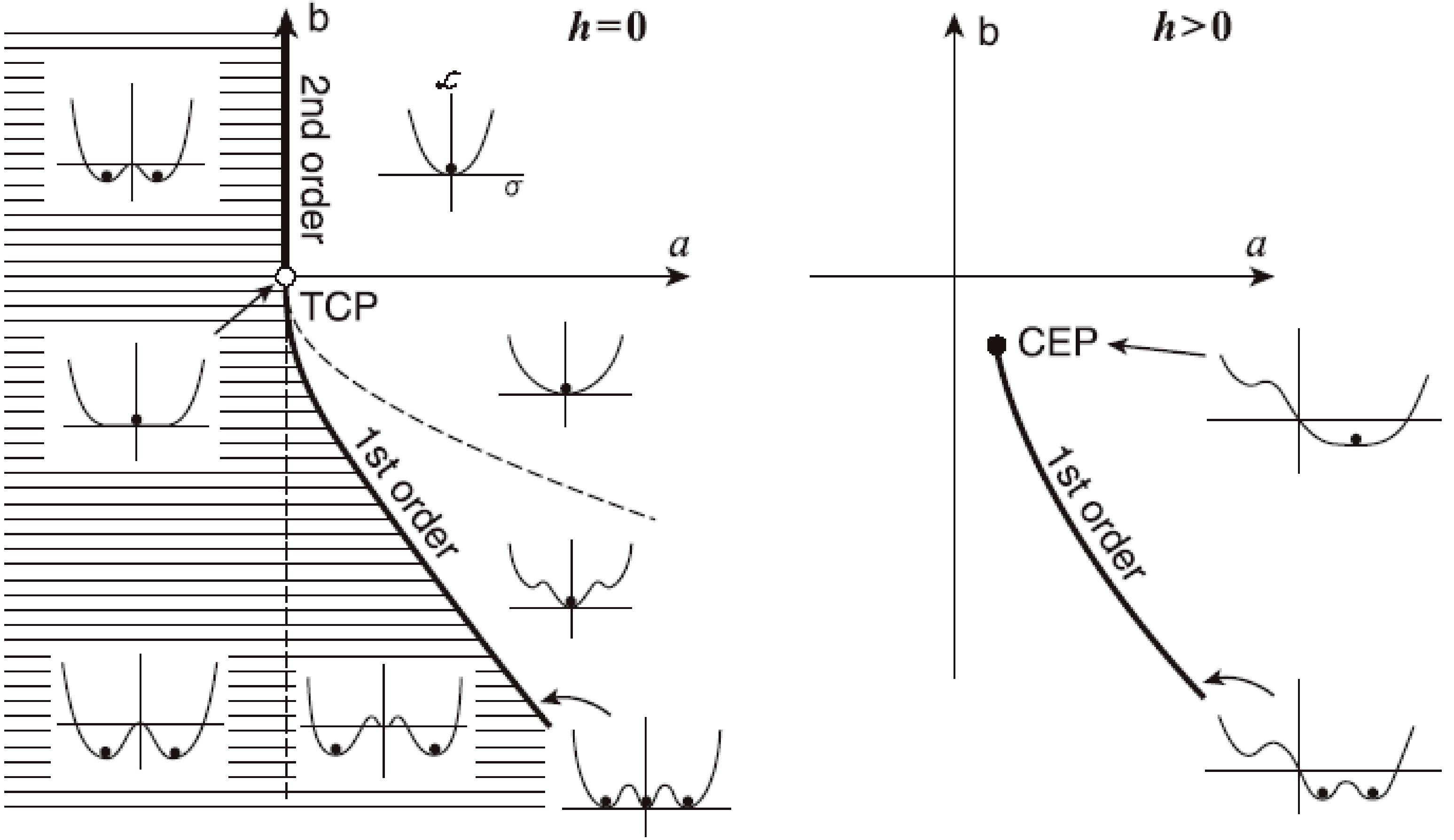
Ginzburg-Landau theory of tricritical/critical point

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}a\sigma^2 + \frac{1}{4}b\sigma^4 + \frac{1}{6}c\sigma^6 - h\sigma$$

Taken from “Quark-Gluon Plasma”

Yagi, Hatsuda and Miake

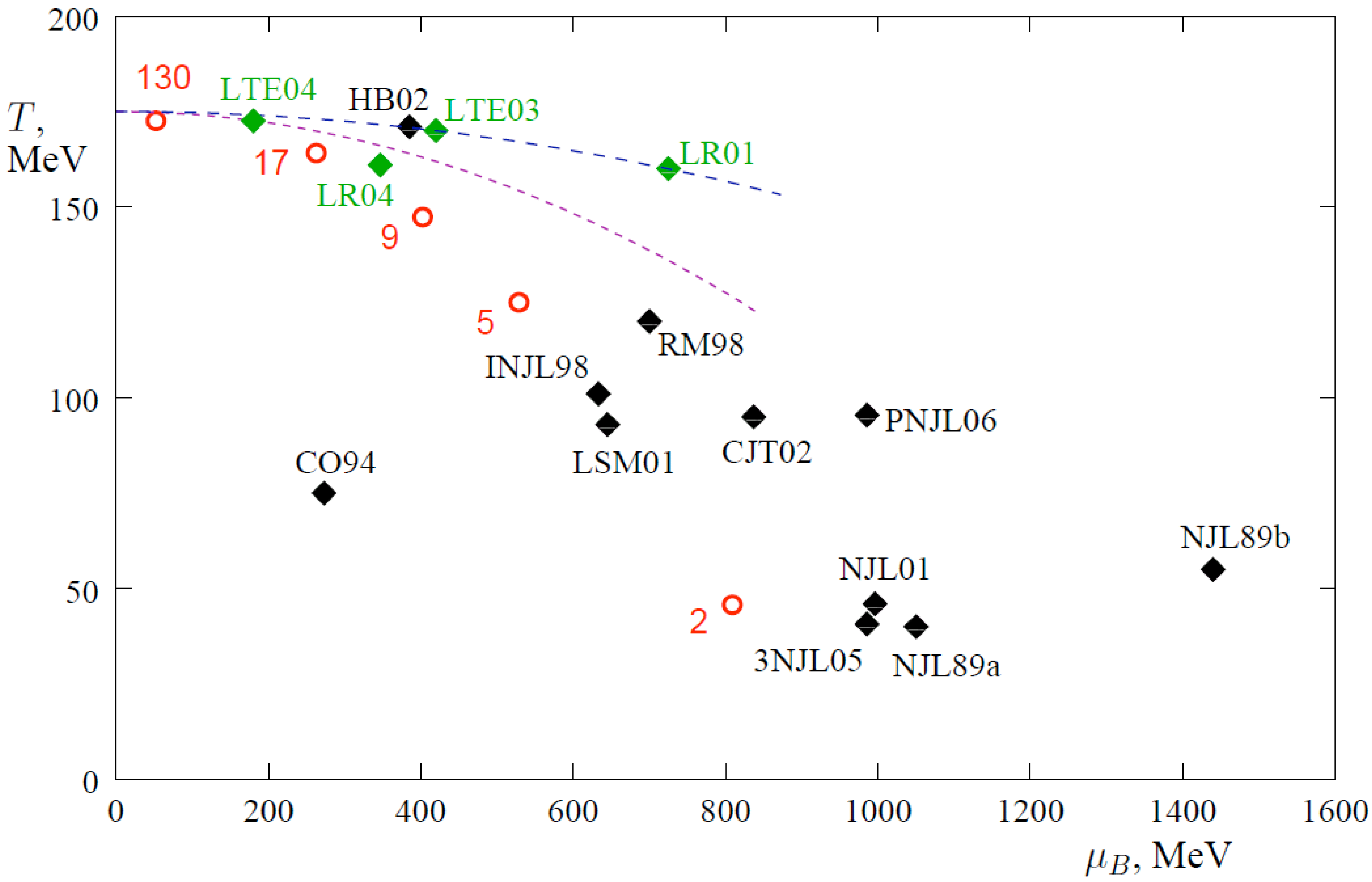
(Cambridge Univ. Press, 2005)



(a)

(b)

Prediction for the location of the critical point



Taken from hep-lat/0701002, M. Stephanov

Recipe to construct Ginzburg-Landau (GL) free-energy

Chiral condensate

$$\Phi_{ij} \sim -\langle \bar{q}_R^j q_L^i \rangle$$

$$\Phi \to \mathrm{e}^{-2i\alpha_A} V_L \Phi V_R^\dagger$$

$$\begin{cases} \Phi^\dagger \Phi \to V_R (\Phi^\dagger \Phi) V_R^\dagger \\ \det\Phi \to \mathrm{e}^{-6i\alpha_A} \det\Phi \end{cases}$$

Diquark condensate

$$\langle (q_L)_b^j C (q_L)_c^k \rangle \sim \epsilon_{abc}\epsilon_{ijk} [d_L^\dagger]_{ai}$$

$$d_L \to e^{2i\alpha_A} e^{2i\alpha_B} V_L d_L V_C^{\mathrm{T}}$$

$$\begin{cases} d_L d_L^\dagger \to V_L (d_L d_L^\dagger) V_L^\dagger \\ d_L d_R^\dagger \to e^{4i\alpha_A} V_L (d_L d_R^\dagger) V_R^\dagger \\ \det d_L \to e^{6i\alpha_A} e^{6i\alpha_B} \det d_L \end{cases}$$

Terms to appear in GL expansions

Breaking $U(1)_A$ (Axial anomaly)

Chiral: $\mathrm{Tr}(\Phi^\dagger\Phi)^n$, $(\mathrm{Tr}\Phi^\dagger\Phi)^n$, $\boxed{\det\Phi}$, $\cdots$

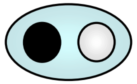
Diquark:
$$\begin{cases} \mathrm{Tr}[(d_L d_L^\dagger)^n],\ [\mathrm{Tr}(d_L d_L^\dagger)]^n, \\ \mathrm{Tr}[(d_L^\dagger d_L)^n (d_R^\dagger d_R)^m],\ \ \mathrm{Tr}(d_L d_L^\dagger)^n \mathrm{Tr}(d_R d_R^\dagger)^m, \cdots \end{cases}$$

Ginzburg-Landau effective Lagrangian

Pion at low density

$$\Phi = \sigma \Sigma e^{-2i\theta}$$

$$\Sigma = \exp\left(i \frac{\lambda^I \pi^I}{f_\pi} \right)$$

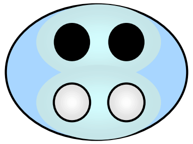


$$\Sigma \to V_L \Sigma V_R^\dagger$$

Generalized pion at high density

$$d_L = dU_L e^{2i\tilde{\theta}+2i\phi}, \quad d_R = -dU_R e^{-2i\tilde{\theta}+2i\phi}$$

$$\tilde{\Delta} = U_L U_R^\dagger = \exp\left(i \frac{\lambda^I \tilde{\pi}^I}{f_{\tilde{\pi}}} \right)$$



$$\tilde{\Delta} \to V_L \tilde{\Delta} V_R^\dagger$$

Effective Lagrangian

- Kinetic term $\mathcal{L}^{\rm kin} = f_\pi^2 {\rm Tr}\left(g_\pi^{\mu\nu}\partial_\mu \Sigma \partial_\nu \Sigma^\dagger\right) + f_{\tilde{\pi}}^2 {\rm Tr}\left(g_{\tilde{\pi}}^{\mu\nu}\partial_\mu \tilde{\Delta}\partial_\nu \tilde{\Delta}^\dagger\right)$

  $g_\pi^{\mu\nu} = {\rm diag}\left(1, \vec{v}_\pi^{\,2}\right), \quad g_{\tilde{\pi}}^{\mu\nu} = {\rm diag}\left(1, \vec{v}_{\tilde{\pi}}^{\,2}\right)$

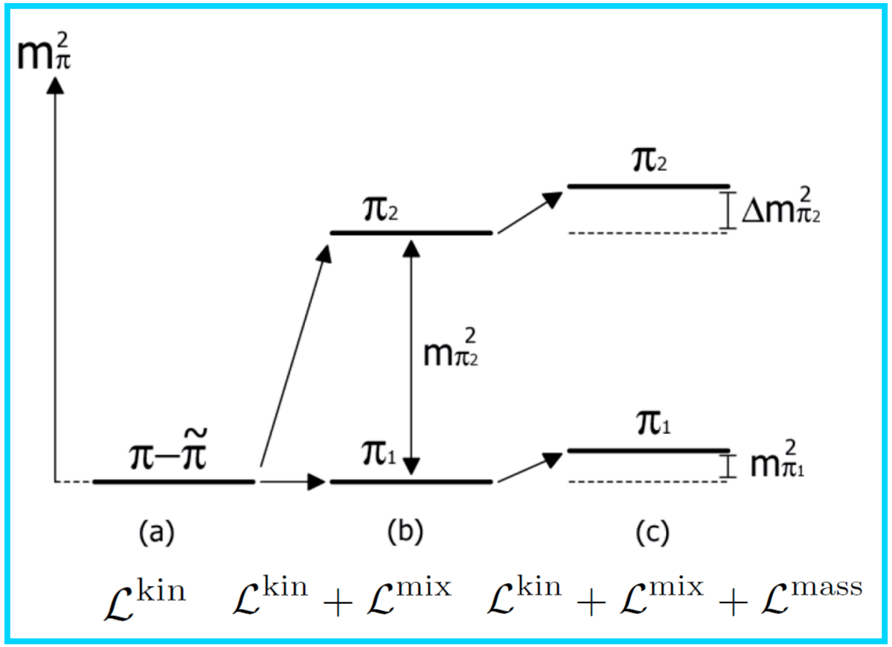
- Mixing term $\mathcal{L}^{\rm mix} = \boxed{-\gamma d^2 \sigma \left[{\rm Tr}\left(\tilde{\Delta}^\dagger \Sigma\right) + {\rm h.c.}\right]}$

- Mass term: $\mathcal{L}^{\rm mass} = A_0 \left[{\rm Tr}\left(M\Sigma^\dagger\right) + {\rm h.c.}\right] + \boxed{\Gamma_1 \left[{\rm Tr}\left(M\tilde{\Delta}^\dagger\right) + {\rm h.c.}\right]}$

A toy model

Lagrangian: $\mathcal{L} \;=\; \mathcal{L}_{\rm kin} + \mathcal{L}_{\rm mix} + \mathcal{L}_{\rm mass}$

$$\begin{aligned}
\mathcal{L}_{\rm kin} &= \frac{f^2}{2}\left|\partial_\mu \alpha\right|^2 + \frac{\tilde{f}^2}{2}\left|\partial_\mu \tilde{\alpha}\right|^2 , \\
\mathcal{L}_{\rm mix} &= \frac{A}{2}\left(\alpha\tilde{\alpha}^\dagger + {\rm h.c.}\right) , \\
\mathcal{L}_{\rm mass} &= \frac{m}{2}\left(B\alpha + C\tilde{\alpha} + {\rm h.c.}\right)
\end{aligned}$$



$$\alpha = \exp\left(i\frac{\varphi}{f}\right), \quad \tilde{\alpha} = \exp\left(i\frac{\tilde{\varphi}}{\tilde{f}}\right)$$

diagonalize: $\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} = \begin{pmatrix} \cos\Theta & \sin\Theta \\ -\sin\Theta & \cos\Theta \end{pmatrix} \begin{pmatrix} \varphi \\ \tilde{\varphi} \end{pmatrix}$

Mass formula: $\boxed{\left(f^2 + \tilde{f}^2\right) m_{\varphi_1}^2 \;=\; m\left(B + C\right)}$

Axial-vector mes[redacted]high density [preliminary]

Finite Energy Sum [redacted]ules:

$$\begin{cases} F_\pi + F_A = \dfrac{1}{2\pi^2} S_A, & \longleftrightarrow \ 1/Q^2 \\ 2F_A m_A^2 = \dfrac{1}{2\pi^2}\left(S_A^2 - (2\mu)^4\right), & \longleftrightarrow \ 1/Q^4 \\ 3F_A m_A^4 = \dfrac{1}{2\pi^2}\left(S_A^3 - (2\mu)^6\right) + \langle O_A \rangle & \longleftrightarrow \ 1/Q^6 \end{cases}$$

- Weak coupling QCD Son-Stephanov ('01)

$$F_\pi = \frac{4(21 - 8\ln 2)}{27\pi^2}\mu^2 \approx 1.14\frac{2\mu^2}{\pi^2}$$

- Solution: $m_A^2 = 8.51\mu^2, \quad S_A = 11.8\mu^2$

expected not to appear as a sharp resonance in reality.