Numerical study of the critical point in lattice QCD at high temperature and density

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Existence of the critical point in finite density lattice QCD
Physical Review D77 (2008) 014508 [arXiv:0706.3549]

Canonical partition function and finite density phase transition in lattice QCD
arXiv:0804.3227

The QCD Critical Point (1^{st} week), July 28-August 1, 2008

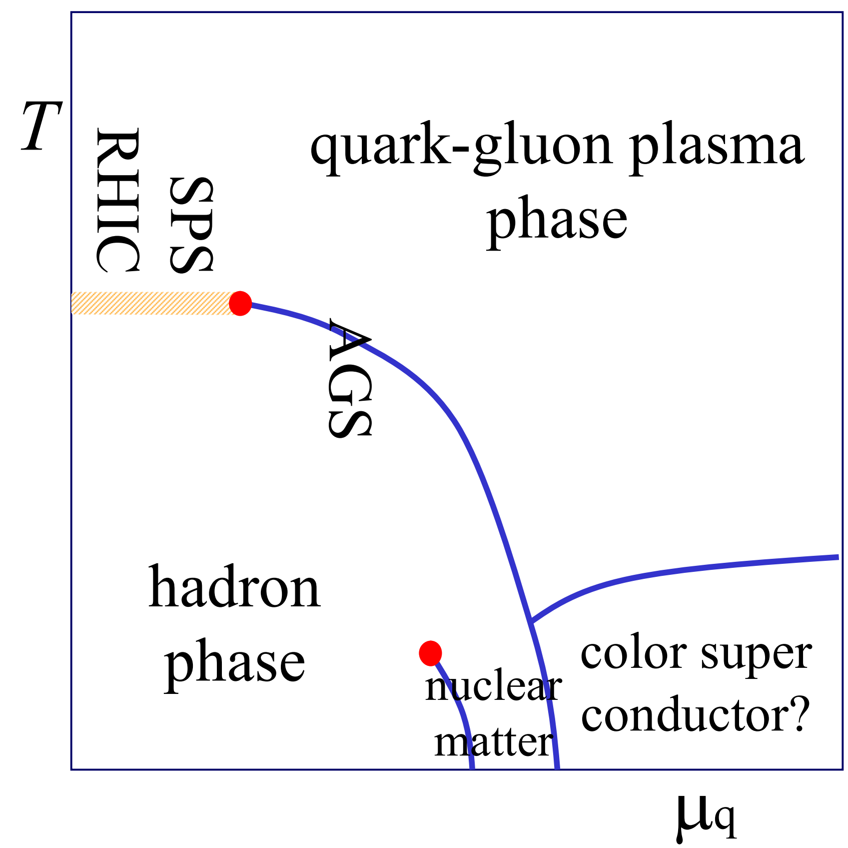
QCD thermodynamics at $\mu \neq 0$

- Interesting properties of QCD

  Measurable in heavy-ion collisions

  Critical point at finite density

- Important roles of lattice QCD study
  - Location of the critical point ?
  - Properties of the critical point ?
    - Large fluctuation in quark number ?
    - Large bulk viscosity ?



Location of the critical point

- Distribution function of plaquette value
- Distribution function of quark number density

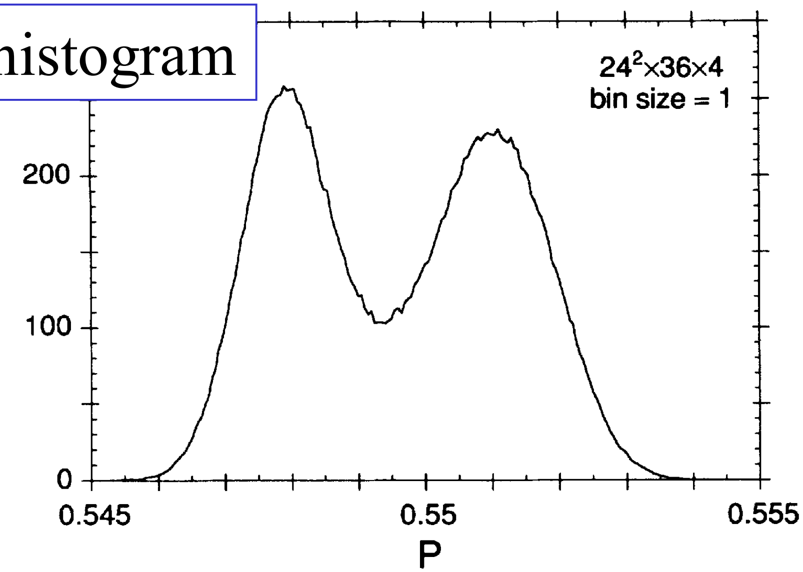
- Simulations:
  - Bielefeld-Swansea Collab., PRD71,054508(2005).
  - 2-flavor p4-improved staggered quarks with $m_\pi \approx 770$MeV
  - 16^3x4 lattice
  - ln det M: Taylor expansion up to $O(\mu^6)$

Effective potential of plaquette $V_{\mathrm{eff}}(P)$

Plaquette distribution function (histogram)

SU(3) Pure gauge theory
QCDPAX, PRD46, 4657 (1992)

histogram



- First order phase transition

  Two phases coexists at T_c

  e.g. SU(3) Pure gauge theory

- Gauge action $\quad S_g = -6N_{site}\beta P$

  $(\beta = 6/g^2)$

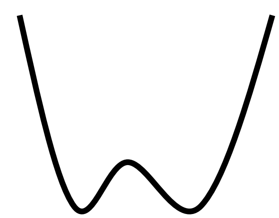
- Partition function

$$Z(\beta,\mu) = \int dP\, \underline{W(P,\beta,\mu)}$$

histogram $\quad W(P',\mu) = \int DU\, (\det M(\mu))^{N_f}\, e^{-S_g}\, \delta(P-P')$

Effective potential

$$V_{\mathrm{eff}}(P) \equiv -\ln(W(P))$$



Problem of complex quark determinant at $\mu\neq 0$

- Problem of Complex Determinant at $\mu\neq 0$

$$\left(M(\mu)\right)^{+} = \gamma_5 M(-\mu)\gamma_5 \qquad (\gamma 5\text{-conjugate})$$

$$\Rightarrow \underline{\left(\det M(\mu)\right)^{*} = \det M(-\mu) \neq \det M(\mu)}$$

- Boltzmann weight: complex at $\mu\neq 0$
  - Monte-Carlo method is not applicable.
  - Configuration cannot be generated.

Distribution function and Effective potential at μ≠0

(S.E., Phys.Rev.D77, 014508(2008))

- Distributions of plaquette P (1x1 Wilson loop for the standard action)

$$Z(\mu) = \int dP \, \underline{R(P,\mu) W(P,\beta)} \qquad\qquad S_g = -6 N_{site} \beta P$$

$$W(\bar{P},\beta) \equiv \int DU \delta(P\text{-}\bar{P}) (\det M(0))^{N_f} \, e^{-S_g}$$ (Weight factor at μ=0)

$$R(\bar{P},\mu) \equiv \frac{\int DU \, \delta(P\text{-}\bar{P}) (\det M(\mu))^{N_f}}{\int DU \, \delta(P\text{-}\bar{P}) (\det M(0))^{N_f}} = \frac{\left\langle \delta(P\text{-}\bar{P}) \left(\frac{\det M(\mu)}{\det M(0)} \right)^{N_f} \right\rangle_{(\beta,\mu=0)}}{\left\langle \delta(P\text{-}\bar{P}) \right\rangle_{(\beta,\mu=0)}}$$ (Reweight factor)

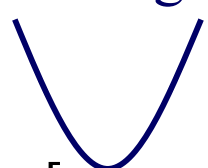
$R(P,\mu)$: independent of β, → $R(P,\mu)$ can be measured at any β.

Effective potential:

μ=0 crossover
non-singular

1st order phase transition?

$$V_{\rm eff}(P) = -\ln\left[R(P,\mu) W(P,\beta)\right] =$$



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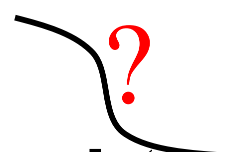

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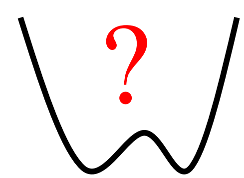
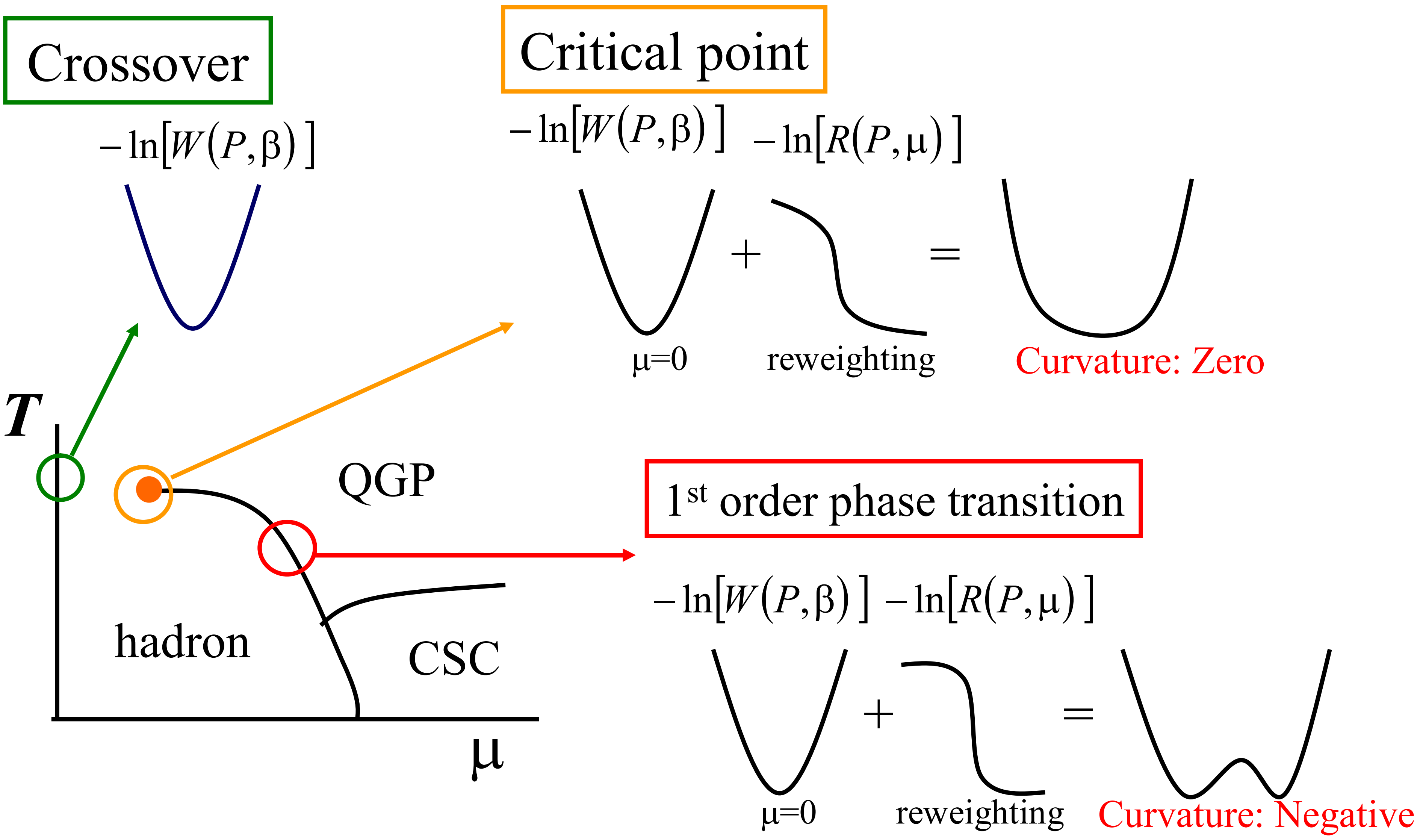


$-\ln[W(P,\beta)]$ $\quad -\ln[R(P,\mu)]$

μ-dependence of the effective potential



Sign problem and phase fluctuations

- Complex phase of det$M$ $\quad \theta = N_f \,\mathrm{Im}\left[\ln \det M(\mu)\right]$
  – Taylor expansion: odd terms of ln det M (Bielefeld-Swansea, PRD66, 014507 (2002))

$$\theta = N_f \mathrm{Im}\left[\frac{\mu}{T}\frac{\mathrm{d}\ln\det M}{\mathrm{d}(\mu/T)} + \frac{1}{3!}\left(\frac{\mu}{T}\right)^3 \frac{\mathrm{d}^3\ln\det M}{\mathrm{d}^3(\mu/T)} + \frac{1}{5!}\left(\frac{\mu}{T}\right)^5 \frac{\mathrm{d}^5\ln\det M}{\mathrm{d}^5(\mu/T)} + \cdots\right]$$

➡ θ: NOT in the range of $[-\pi, \pi]$

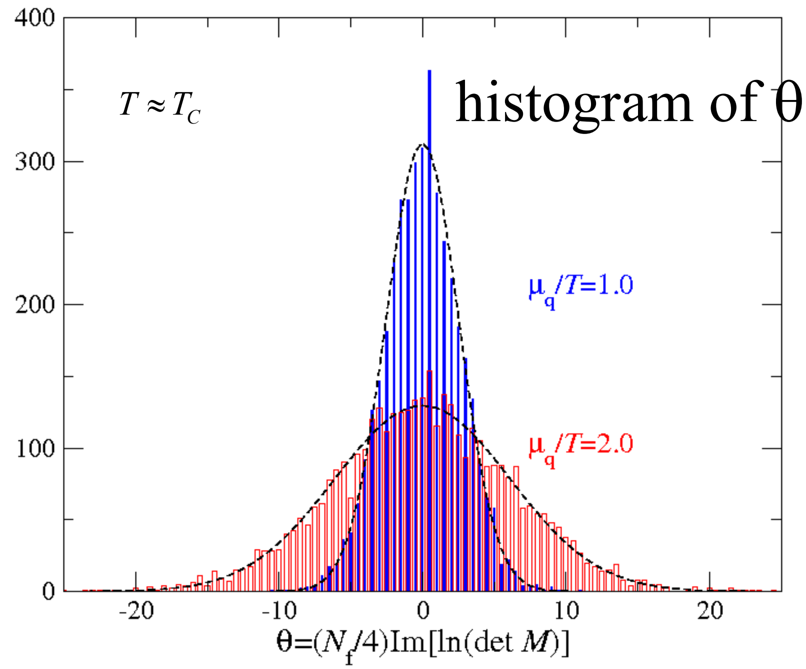
- $|\theta| > \pi/2$: Sign problem happens.

➡ $e^{i\theta}$ changes its sign.

$$\left\langle \left(\frac{\det M(\mu)}{\det M(0)}\right)^{N_f} \right\rangle \equiv \left\langle e^{i\theta} F \right\rangle \ll \text{(statistical error)}$$

- Gaussian distribution
  – Results for p4-improved staggered
  – Taylor expansion up to $O(\mu^5)$
  – Dashed line: fit by a Gaussian function

Well approximated $\boxed{W(\theta) \approx \sqrt{\frac{\alpha}{\pi}} e^{-\alpha\theta^2}}$

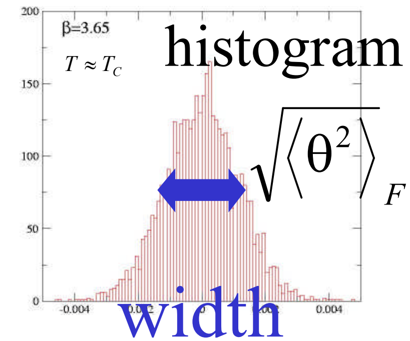


Complex phase distribution

Assume: Gaussian distribution ➡ Sign problem is avoided.

(S.E., Phys.Rev.D77, 014508(2008))

- Sign problem: $\left\langle e^{i\theta} F \right\rangle$ << (statistical error)
- Gaussian integral: $W(F,\theta) \approx \sqrt{\frac{\alpha}{\pi}} e^{-\alpha\theta^2} W'(F)$



$$\left\langle e^{i\theta} F \right\rangle = \int dF \int d\theta \, e^{i\theta} F W\left(F,\theta\right) \approx \int dF \, e^{-1/(4\alpha)} F W'(F)$$

$$\frac{1}{2\alpha(F')} = \frac{\int \theta^2 W(F',\theta) d\theta}{\int W(F',\theta) d\theta} \equiv \left\langle \theta^2 \right\rangle_F$$

➡ $$\boxed{\left\langle e^{i\theta} F \right\rangle \approx \left\langle e^{-\left\langle \theta^2 \right\rangle_F / 2} F \right\rangle}$$

real and positive (No sign problem)

Why Gaussian distribution?

Taylor expansion: $$\theta = N_f \mathrm{Im}\left[\frac{\mu}{T}\frac{\mathrm{d}\ln\det M}{\mathrm{d}(\mu/T)} + \frac{1}{3!}\left(\frac{\mu}{T}\right)^3 \frac{\mathrm{d}^3\ln\det M}{\mathrm{d}^3(\mu/T)} + \frac{1}{5!}\left(\frac{\mu}{T}\right)^5 \frac{\mathrm{d}^5\ln\det M}{\mathrm{d}^5(\mu/T)} + \cdots\right]$$

– e.g. 1st term: $$\mathrm{Im}\left[\frac{d\ln\det M}{d(\mu/T)}\right] = \mathrm{Im}\left[Tr\left(M^{-1}\frac{\partial M}{\partial(\mu/T)}\right)\right]$$

$\boxed{\text{Diagonal element: local density operator}}$

– If density correlation: not long & volume: large,

Central limit theorem ➡ θ: Gaussian distribution

- Valid for large volume (except on the critical point)
- Splittorff and Verbaarschot, arXiv:0709.2218

Gaussian distribution: suggested from chiral perturbation theory

For the case: $$W(\theta) \approx \sqrt{\frac{\alpha}{\pi}}\left(1 - \frac{3\alpha_4}{4\alpha_2^2} + \cdots\right)^{-1} \exp\left(-\alpha_2\theta^2 \underline{\underline{-\alpha_4\theta^4}} + \cdots\right),$$

$$\boxed{\frac{\alpha_4}{\alpha_2} \ll 1}$$

$$\int d\theta\, e^{i\theta} W(\theta) \to \exp\left(-\frac{1}{2}\left\langle\theta^2\right\rangle_F + \underline{\underline{\frac{1}{16\alpha_2^3}}}\frac{\alpha_4}{\alpha_2} + O\left[\left(\frac{\alpha_4}{\alpha_2}\right)^2\right]\right)$$

$$\boxed{\sim O(\mu^6)}$$

because $$1/\alpha_2 \sim 2\left\langle\theta^2\right\rangle_F \sim O(\mu^2)$$

- Valid for low density (expansion of μ^2)

Effective potential at $\mu \neq 0$

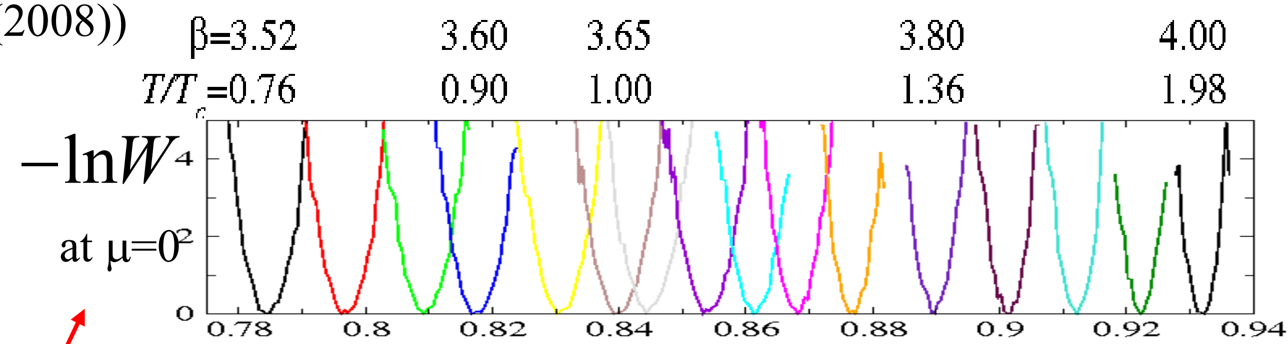
$$V_{\text{eff}}(P,\beta,\mu) = -\ln W(P,\beta) - \ln R(P,\mu)$$

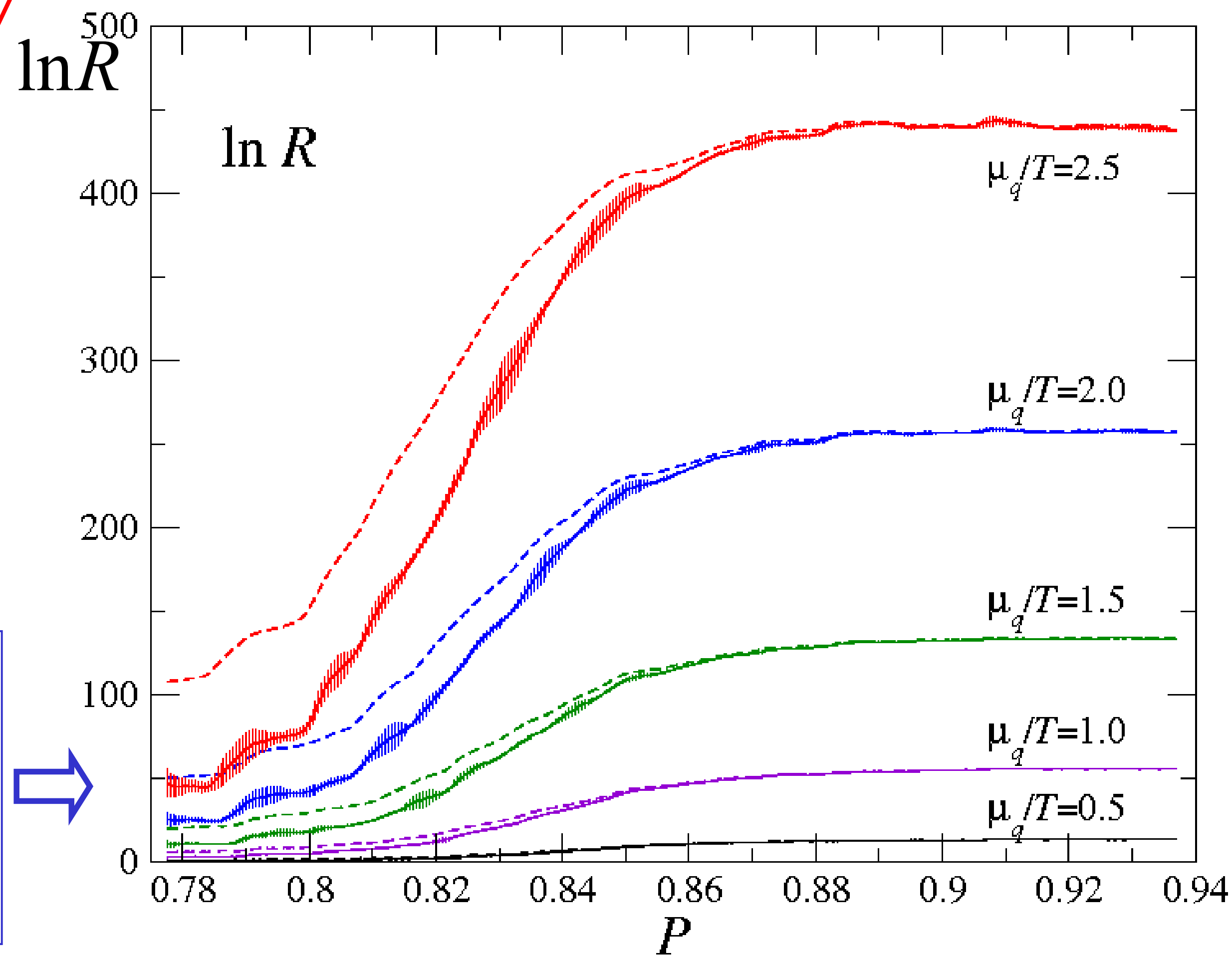
(S.E., Phys.Rev.D77, 014508(2008))

Results of Nf=2 p4-staggared, $m_\pi / m_\rho \approx 0.7$

[data in PRD71,054508(2005)]

- det$M$: Taylor expansion up to $O(\mu^6)$
- The peak position of $W(P)$ moves left as β increases at $\mu=0$.



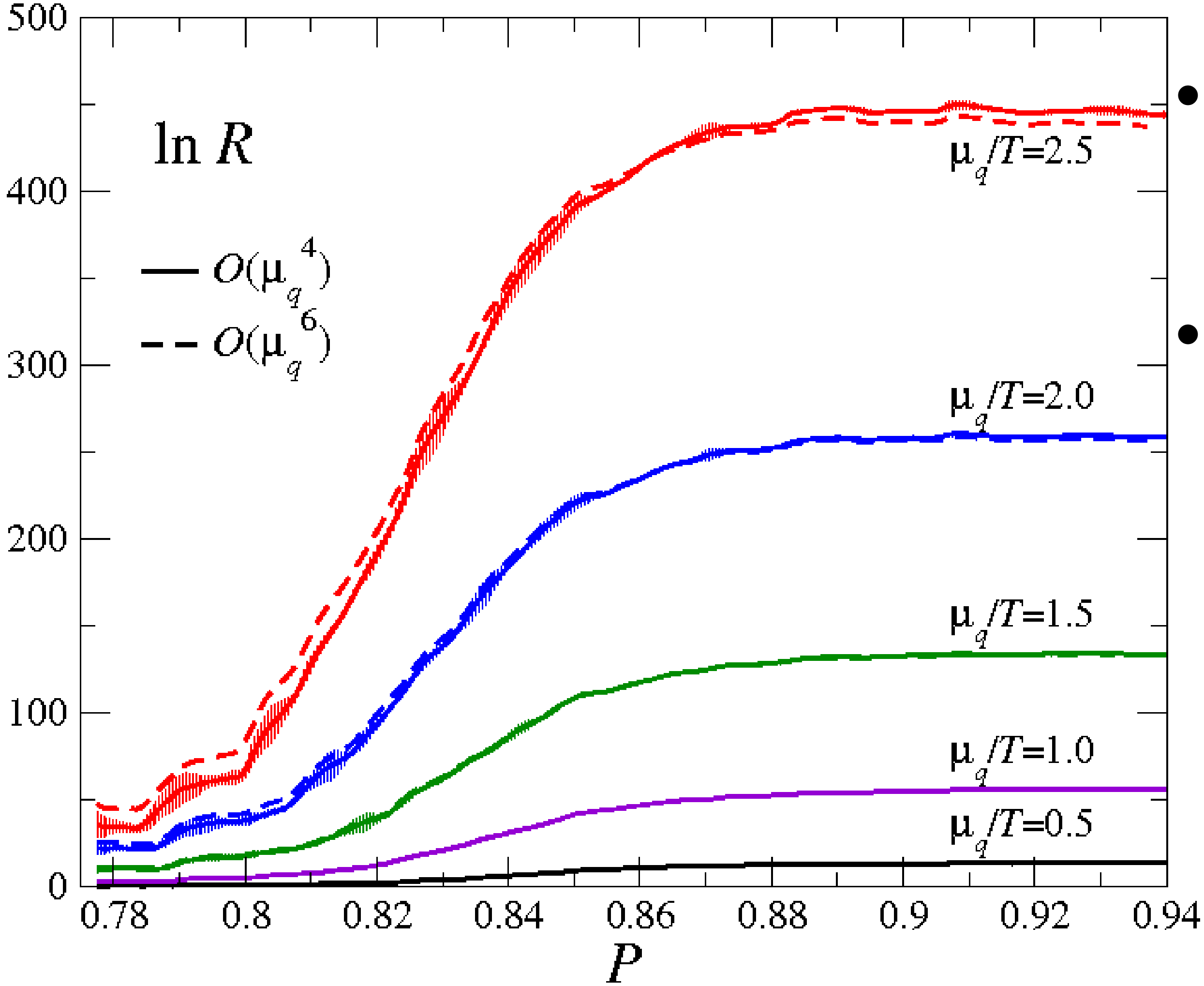


Solid lines: reweighting factor at finite μ/T, $R(P,\mu)$

Dashed lines: reweighting factor without complex phase factor.

Truncation error of Taylor expansion

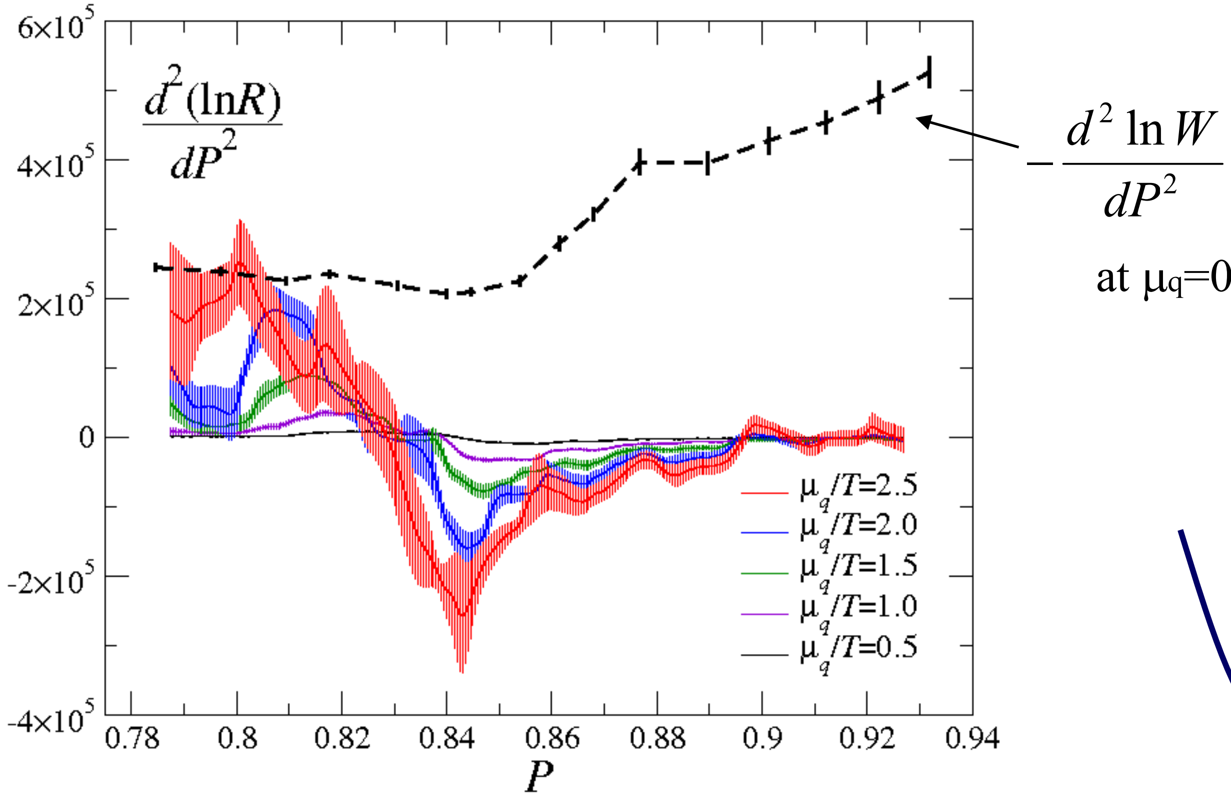
$$N_f \ln\det M(\mu) = N_f \sum_{n=0}^{N}\left[\frac{1}{n!}\left(\frac{\mu}{T}\right)^n \frac{d^n \ln\det M}{d(\mu/T)^n}\right]$$



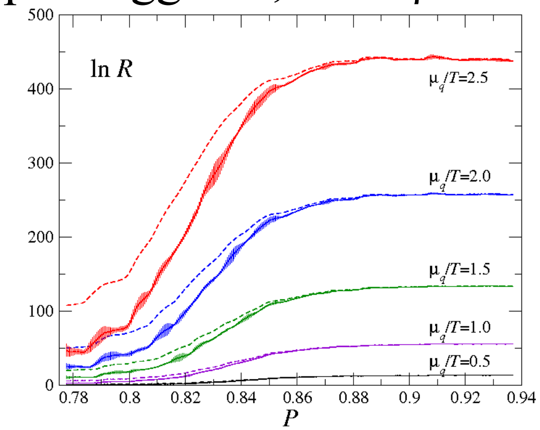
- Solid line: $O(\mu^4)$
- Dashed line: $O(\mu^6)$

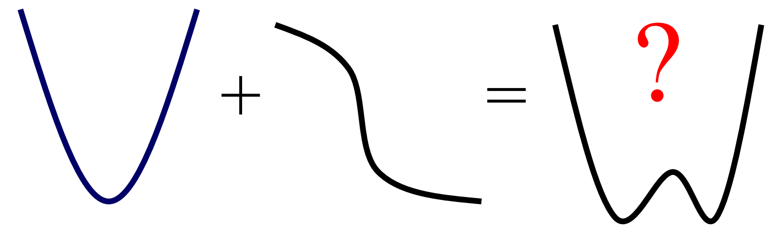
- The effect from 5th and 6th order term is small for $\mu_q/T \leq 2.5$.

Curvature of the effective potential



Nf=2 p4-staggared, $m\pi/m\rho \approx 0.7$



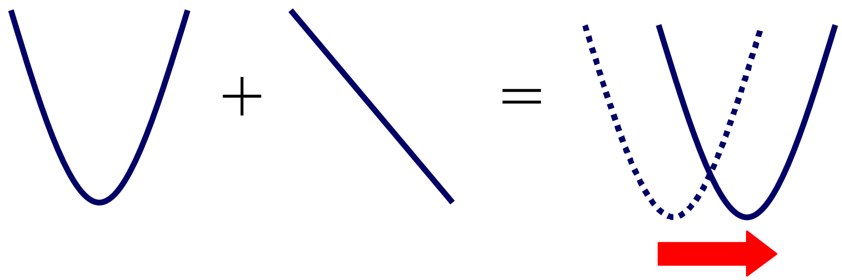


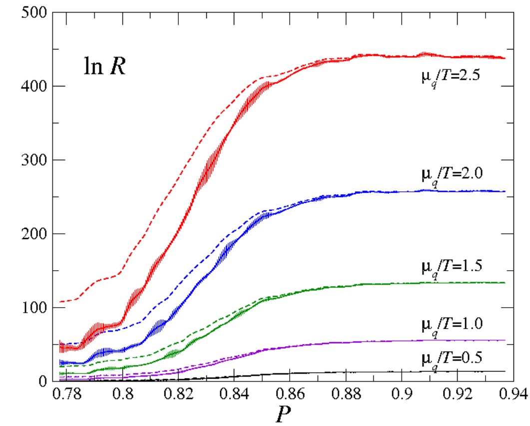
Critical point: $$\frac{d^2 V_{\text{eff}}(P,\beta,\mu)}{dP^2} = -\frac{d^2 \ln W(P,\beta)}{dP^2} - \frac{d^2 \ln R(P,\mu)}{dP^2} = 0$$

- First order transition for $\mu_q/T \geq 2.5$
- Existence of the critical point: suggested
  - Quark mass dependence: large
  - Study near the physical point is important.

Slope of ln$R(P,\mu)$ at low density

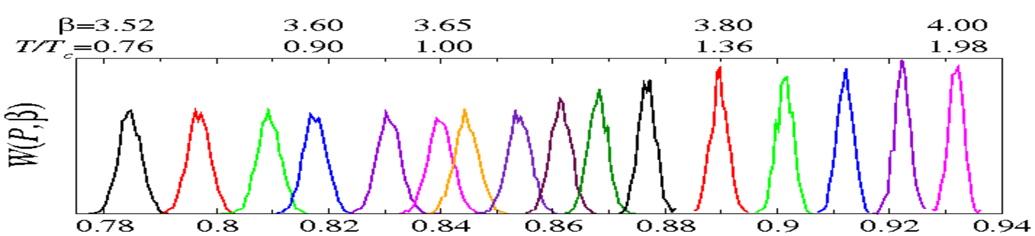
$$-\ln W(P,\beta) - \ln R(P,\mu)$$





- Minimum point moves, $P \to$ large
- Same effect as

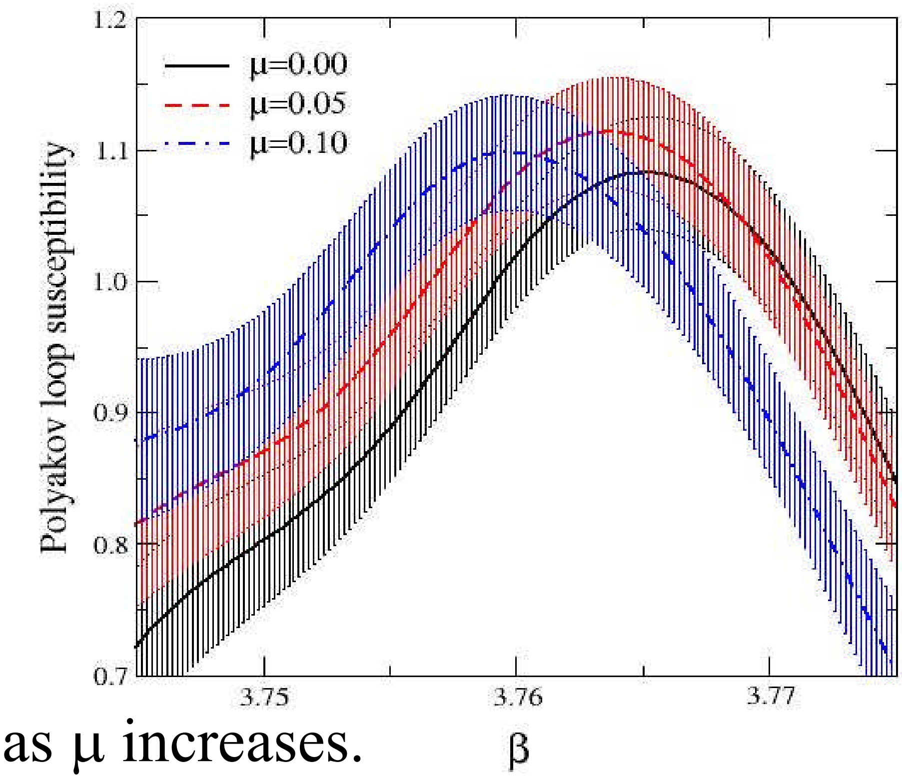
$$\beta \Rightarrow \beta_{\text{eff}} \equiv \beta + \frac{1}{6N_{\text{site}}} \frac{\partial(\ln R)}{\partial P}$$



Low T phase High T phase

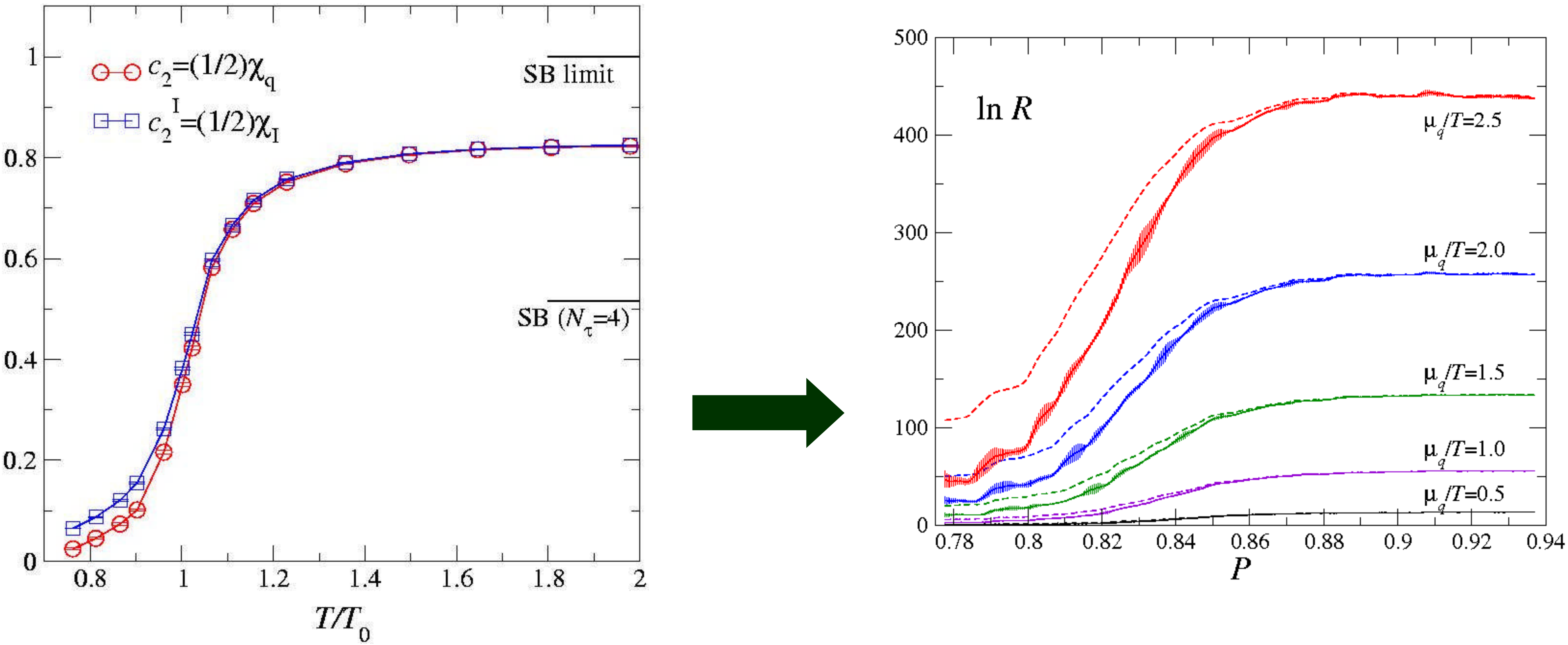
μ-dependence of βc

Bielefeld-Swansea Collab., PRD66,074507 ('02)



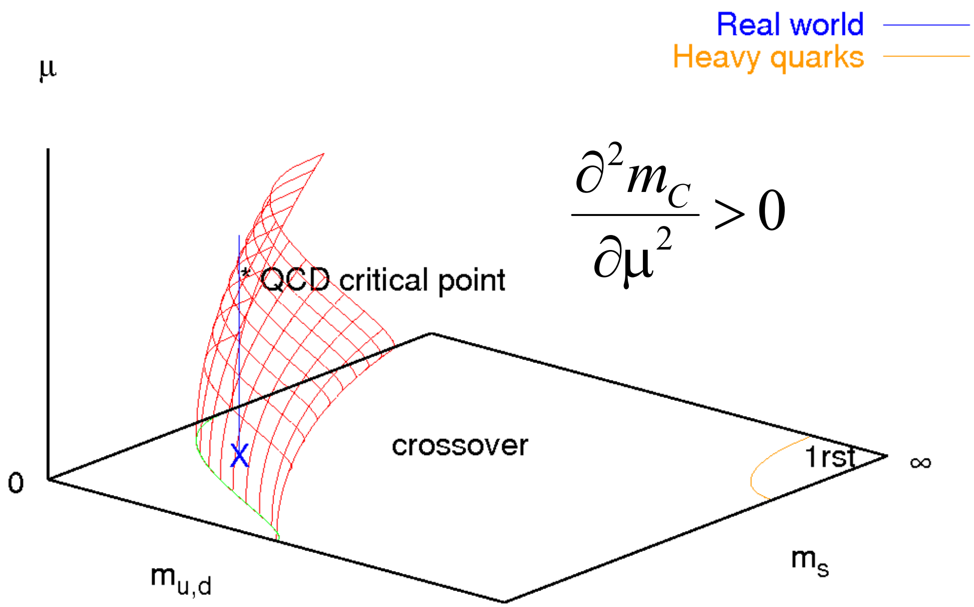
- The phase transition point becomes lower as μ increases.

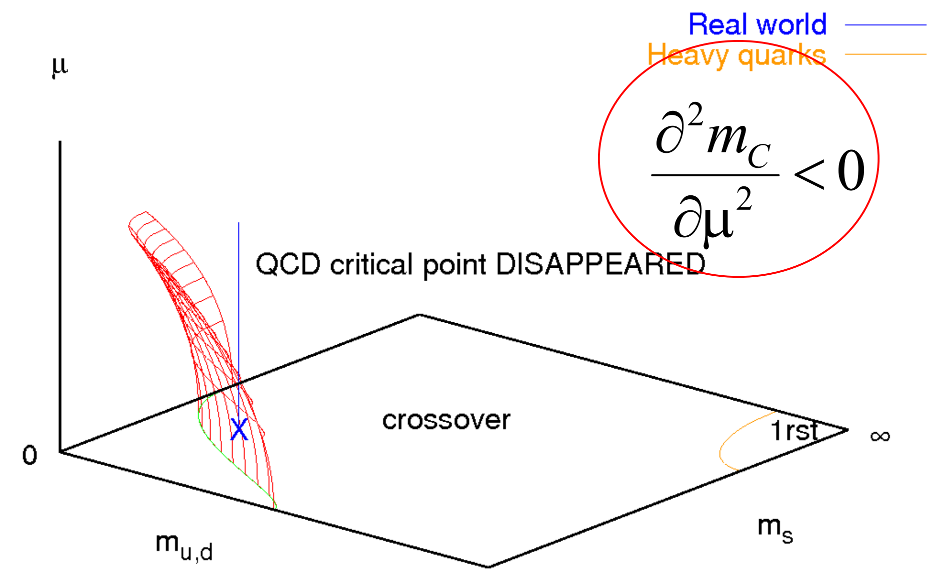
Relation to quark number susceptibility



- Up to $O(\mu^2)$: $T \rightarrow P$
- Curvature in χ_q makes the curvature in ln $R(P)$.

Curvature of the critical surface





- Usual expectation
- Critical point: exists

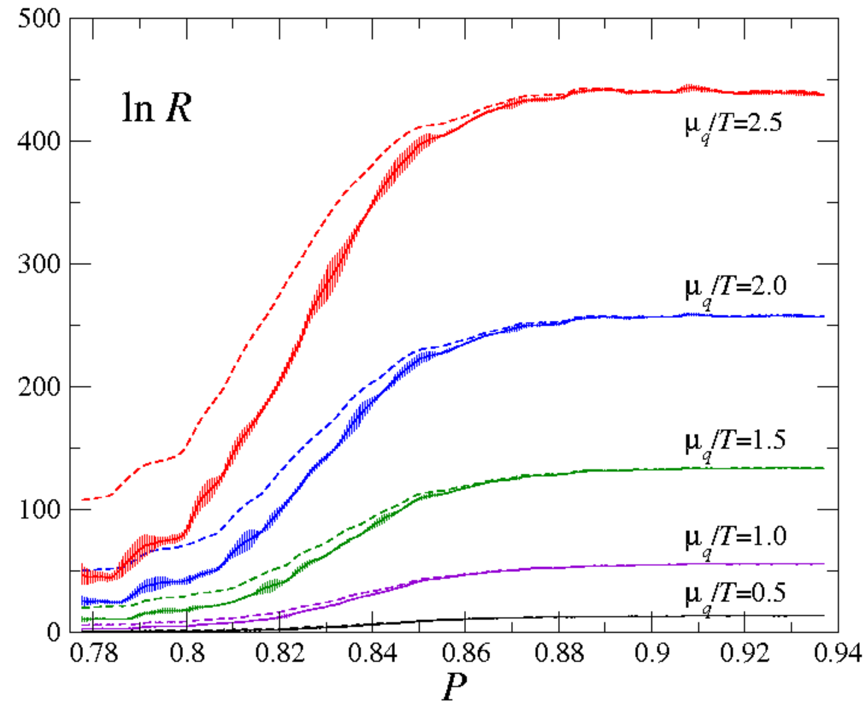
- de Forcrand - Philipsen, JHEP01(2007)077; PoS(LAT2007)178
- Curvature: slightly negative.
  (3-flavor, 8^3x4 and 12^3x4 lattices)

Why the curvature is so small.

Curvature of V_{eff} near μ=0

- Slope of ln$R$(P) maximum at the critical P.
- Curvature of ln$R$(P) zero at the critical P.

- Curvature of $V_{\mathrm{eff}}(P,\beta,\mu)$ does not change at the critical point.
- Critical point $\left(\partial^2 V_{\mathrm{eff}} / \partial P^2 = 0\right)$ does not move in the leading order.



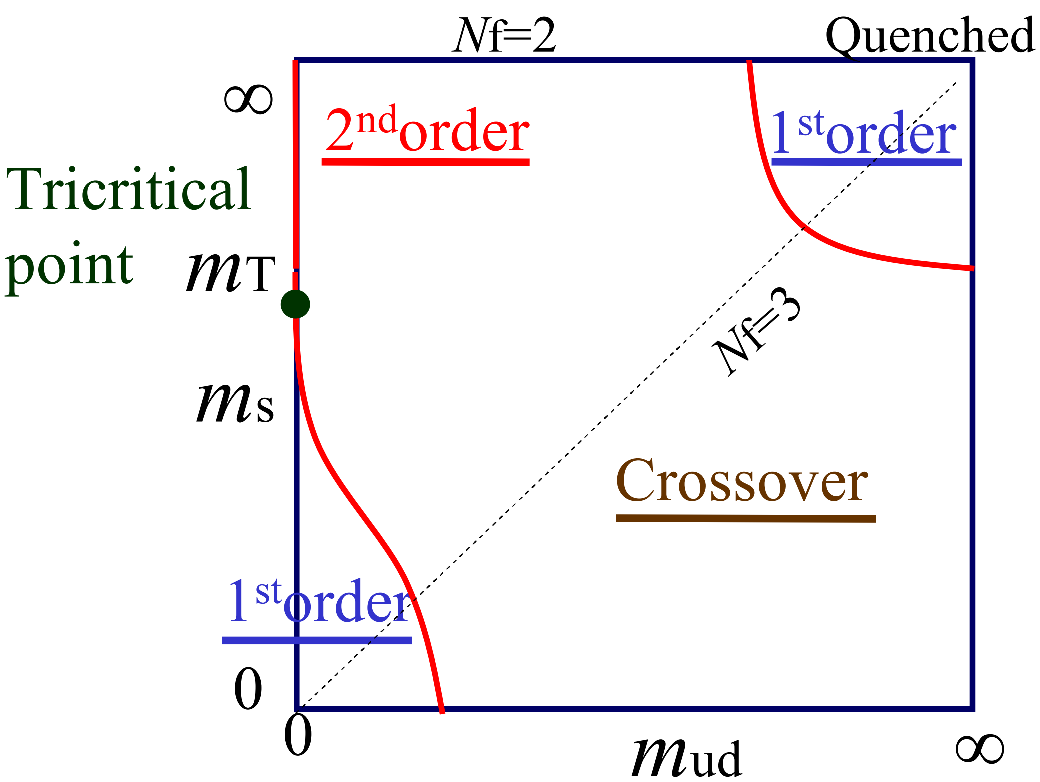
Low T phase High T phase

Mean field argument

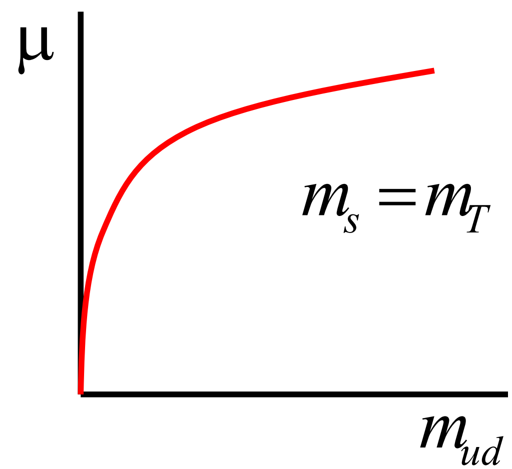
- Sigma model prediction near tri-critical point on the ms axis.

$$V_{\text{eff}}(\sigma)=\frac{1}{2}a\sigma^2+\frac{1}{4}b\sigma^4+\frac{1}{6}c\sigma^6-h\sigma$$

Critical point: $\frac{\partial^n V_{\text{eff}}}{\partial\sigma^n}=0 \quad (n=1,2,3)$



$b\sim(m_T-m_s)$ ⇒ $b\sim\mu^2$



$$m_{ud}^{\text{crit}}\sim(m_T-m_s)^{5/2}$$ ⇒ $$m_{ud}^{\text{crit}}\sim\mu^5$$

Canonical approach

- Canonical partition function

$$Z_{GC}(T,\mu)=\sum_N Z_C(T,N)\exp(N\mu/T)\equiv\sum_N W(N)$$

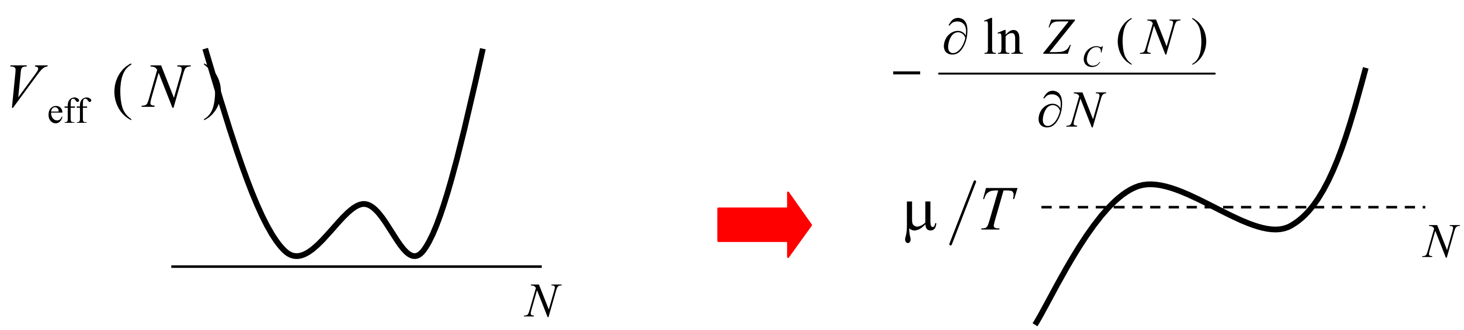
- Effective potential as a function of the quark number *N*.

$$V_{\text{eff}}(N)=-\ln W(N)=-\ln Z_C(T,N)-N\mu/T$$

- At the minimum,

$$\frac{\partial V_{\text{eff}}(N)}{\partial N}=-\frac{\partial\ln W(N)}{\partial N}=-\underline{\underline{\frac{\partial\ln Z_C(T,N)}{\partial N}}}-\frac{\mu}{T}=0$$

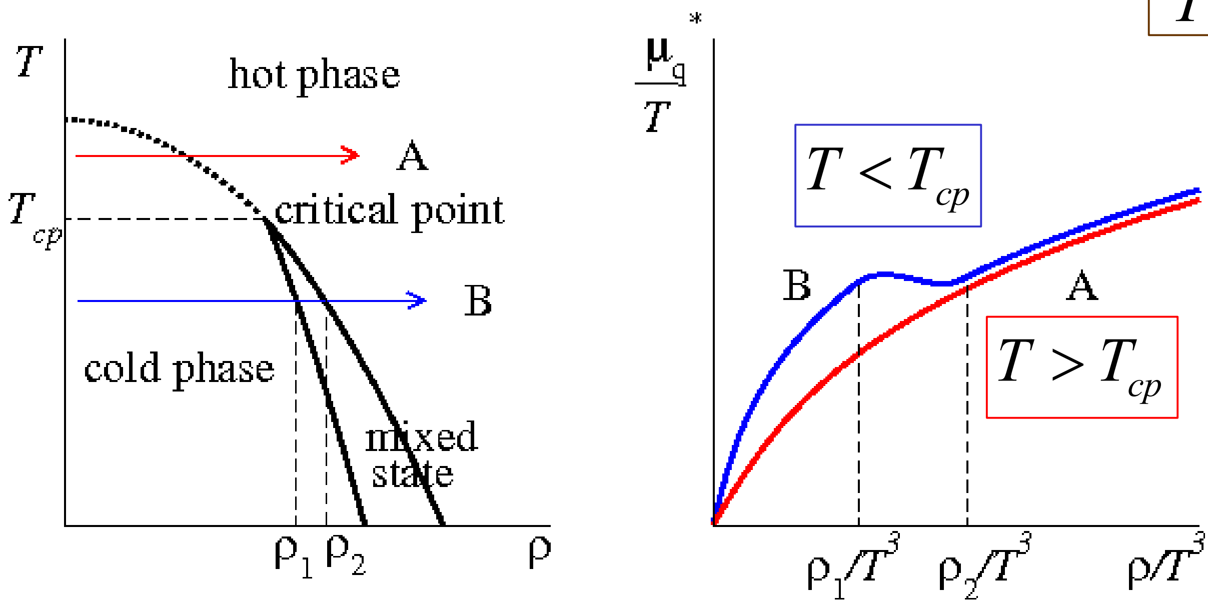
- First order phase transition: Two phases coexist.



First order phase transition line

In the thermodynamic limit, $\frac{\partial V_{\text{eff}}(N)}{\partial N} = 0,$ $\Rightarrow$ $\boxed{\frac{\mu^*}{T} = -\frac{\partial \ln Z_C(T,N)}{\partial N}}$

$$\frac{\mu^*}{T} \to \frac{\mu}{T} \quad \left(N_s^3 \to \infty\right)$$



- Mixed state ➡ First order transition
- Inverse Laplace transformation by Glasgow method

  Kratochvila, de Forcrand, PoS (LAT2005) 167 (2005)

  *N*f=4 staggered fermions, $6^3 \times 4$ lattice

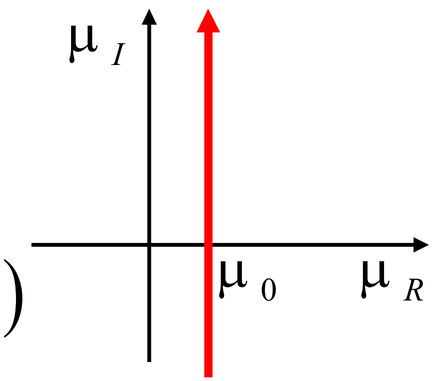
  – *N*f=4: First order for all ρ.

Canonical partition function

- Fugacity expansion (Laplace transformation)

$$Z_{GC}(T,\mu)=\sum_N \underline{\underline{Z_C(T,N)}}\exp(N\mu/T) \qquad \rho = N/V$$

canonical partition function



Integral

- Inverse Laplace transformation

$$Z_C(T,N)=\frac{3}{2\pi}\int_{-\pi/3}^{\pi/3} d(\mu_I/T)\, e^{-N(\mu_0/T+i\mu_I/T)} Z_{GC}(T,\mu_0+i\mu_I)$$

$$\frac{Z_{GC}(\mu)}{Z_{GC}(0)}=\frac{1}{Z_{GC}(0)}\int DU\, (\det M(\mu))^{N_f} e^{-S_g} = \left\langle \left(\frac{\det M(\mu)}{\det M(0)}\right)^{N_f} \right\rangle_{\mu=0}$$

Arbitrary μ_0

Integral path, e.g.
1, imaginary μ axis
2, Saddle point

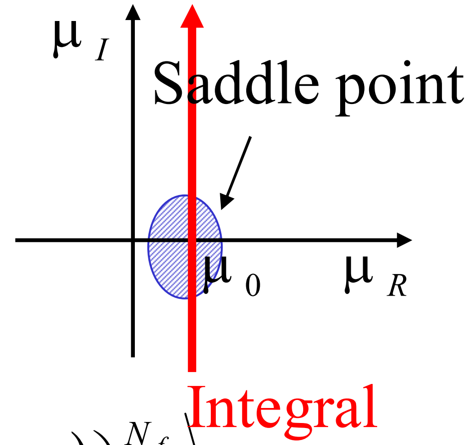
  – Note: periodicity $Z_{GC}(T,\mu+2\pi iT/3)=Z_{GC}(T,\mu)$

- Derivative of ln$Z$

$$\boxed{\frac{\mu^*}{T}\equiv-\frac{\partial \ln Z_C(T,N)}{\partial N}}$$

Saddle point approximation

(S.E., arXiv:0804.3227)



- Inverse Laplace transformation

$$Z_C(T,N) = \frac{3}{2\pi}\int_{-\pi/3}^{\pi/3} d(\mu_I/T)\, e^{-N(\mu_0/T + i\mu_I/T)} Z_{GC}(T, \mu_0 + i\mu_I)$$

$$= \frac{3Z_{GC}(0)}{2\pi}\left\langle \int_{-\pi/3}^{\pi/3} d(\mu_I/T)\, e^{-N(\mu_0/T + i\mu_I/T)} \left(\frac{\det M(\mu_0 + i\mu_I)}{\det M(0)}\right)^{N_f}\right\rangle$$

- Saddle point approximation (valid for large *V*, *1/V* expansion)
  - Taylor expansion at the saddle point. $\mu_0/T = z_0$ $\rho = N/V$

Saddle point: z_0 $\left[\frac{N_f}{V}\frac{\partial(\ln\det M)}{\partial(\mu/T)} - \rho\right]_{\frac{\mu}{T}=z_0} = 0$ $V \equiv N_s^3$

- At low density: The saddle point and the Taylor expansion coefficients can be estimated from data of Taylor expansion around μ=0.

$$N_f \ln\det M(\mu) = N_f \sum_{n=0}^{\infty}\left[\frac{1}{n!}\left(\frac{\mu}{T}\right)^n \frac{d^n \ln\det M}{d(\mu/T)^n}\right] \equiv V N_f N_t \sum_{n=0}^{\infty}\left[D_n\left(\frac{\mu}{T}\right)^n\right]$$

Saddle point approximation

- Canonical partition function in a saddle point approximation

$$\frac{Z_C(T,\rho)}{Z_{GC}(T,0)} = \frac{3}{\sqrt{2\pi}} \left\langle \exp\left[N_f \ln\left(\frac{\det M(z_0)}{\det M(0)} \right) - V\rho z_0 \right] e^{-i\alpha/2} \sqrt{\frac{1}{V\left|R''(z_0)\right|}} \right\rangle_{(T,\mu=0)}$$

$$\equiv \frac{3}{\sqrt{2\pi}} \left\langle \exp\left(F + i\theta\right) \right\rangle_{(T,\mu=0)}$$

Saddle point: z_0 $\qquad R''\left(\frac{\mu}{T}\right) = \frac{N_f}{V} \frac{\partial^2 (\ln \det M)}{\partial (\mu/T)^2} \equiv \left|R''\right| e^{i\alpha}$

- Chemical potential

$$\frac{\mu^*(\rho)}{T} \equiv \frac{-1}{V} \frac{\partial \ln Z_C(T,\rho)}{\partial \rho} \approx \frac{\left\langle \underline{z_0}\ \underline{\exp\left(F + i\theta\right)} \right\rangle_{(T,\mu=0)}}{\left\langle \underline{\exp\left(F + i\theta\right)} \right\rangle_{(T,\mu=0)}}$$

saddle point $\qquad$ reweighting factor

$\Rightarrow$ Similar to the reweighting method

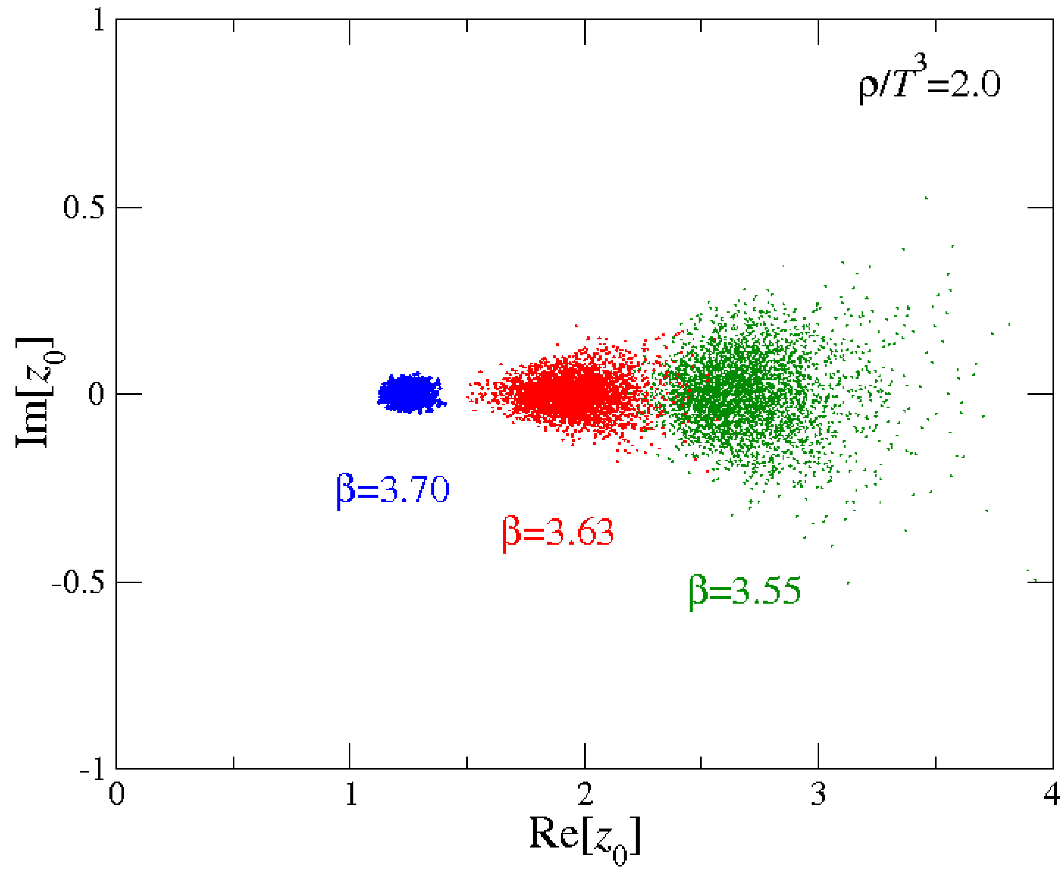
(sign problem & overlap problem)

Saddle point in complex μ/T plane

- Find a saddle point z_0 numerically for each conf.

$$\left[\frac{N_f}{V}\frac{\partial(\ln\det M)}{\partial(\mu/T)}-\rho\right]_{\frac{\mu}{T}=z_0}=0$$

- Two problems
  - Sign problem
  - Overlap problem



Technical problem 1: Sign problem

- Complex phase of det$M$ $\quad (\text{phase}) = N_f \, \mathrm{Im}[\ln \det M(\mu)]$
  - Taylor expansion (Bielefeld-Swansea, PRD66, 014507 (2002))

$$\theta = \mathrm{Im}\left[V \left(N_f N_t \sum_{n=1}^{\infty} D_n z_0 - \rho z_0 \right) \right] - \frac{\alpha}{2} \quad \Rightarrow \quad \theta\text{: NOT in the range } [-\pi, \pi]$$

- $|\theta| > \pi/2$: Sign problem happens.

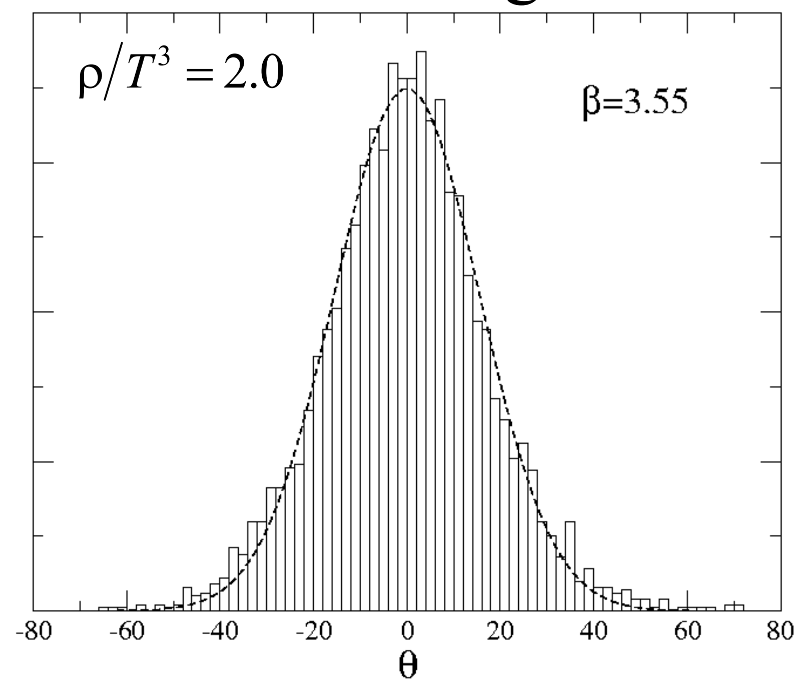
$\Rightarrow$ $e^{i\theta}$ changes its sign.

- Gaussian distribution
  - Results for p4-improved staggered
  - Taylor expansion up to $O(\mu^5)$
  - Dashed line: fit by a Gaussian function

Well approximated

$$\boxed{W(\theta) \approx \sqrt{\frac{\alpha}{\pi}} e^{-\alpha\theta^2}}$$

histogram of θ

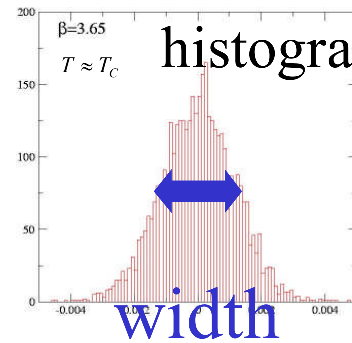


Sign problem (S.E., Phys.Rev.D77, 014508(2008))

Sign problem happens when exp($i\theta$) changes its sign frequently.

$$\Rightarrow \quad \underline{\left\langle e^{i\theta} e^{F} \right\rangle << \text{(statistical error)}}$$

Assume: Gaussian distribution $\Rightarrow$ Sign problem is avoided.



$$\sqrt{\left\langle \theta^2 \right\rangle_F}$$

- Gaussian integral: $$\boxed{W(F,\theta) \approx \sqrt{\frac{\alpha}{\pi}} e^{-\alpha\theta^2} W'(F)}$$

$$\left\langle e^{i\theta} e^{F} \right\rangle = \int dF \int d\theta \; e^{i\theta} e^{F} W(F,\theta) \approx \int dF \; \exp\left(-\frac{1}{4\alpha(F)} \right) e^{F} W'(F)$$

$$\Rightarrow \boxed{\left\langle e^{i\theta} e^{F} \right\rangle \approx \left\langle e^{-\left\langle \theta^2 \right\rangle_F / 2} e^{F} \right\rangle}$$

real and positive (No sign problem)

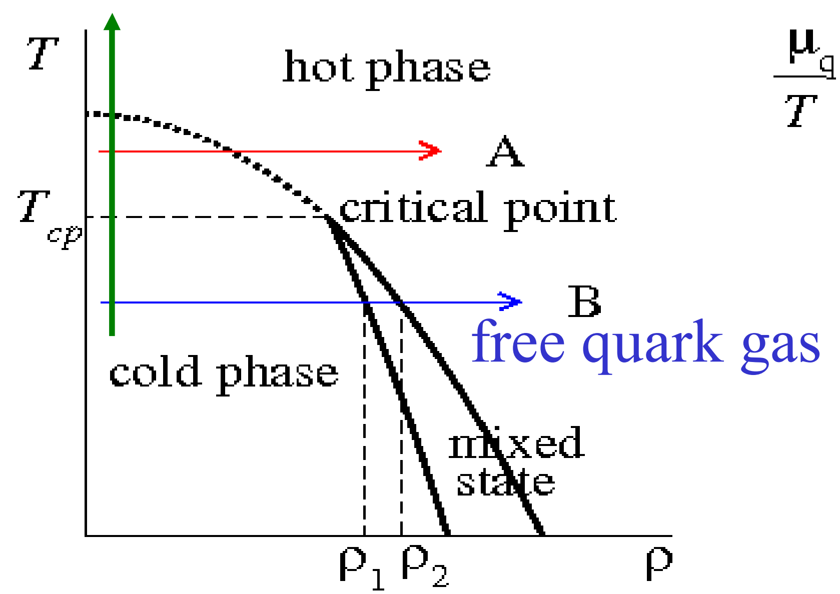
Technical problem 2: Overlap problem

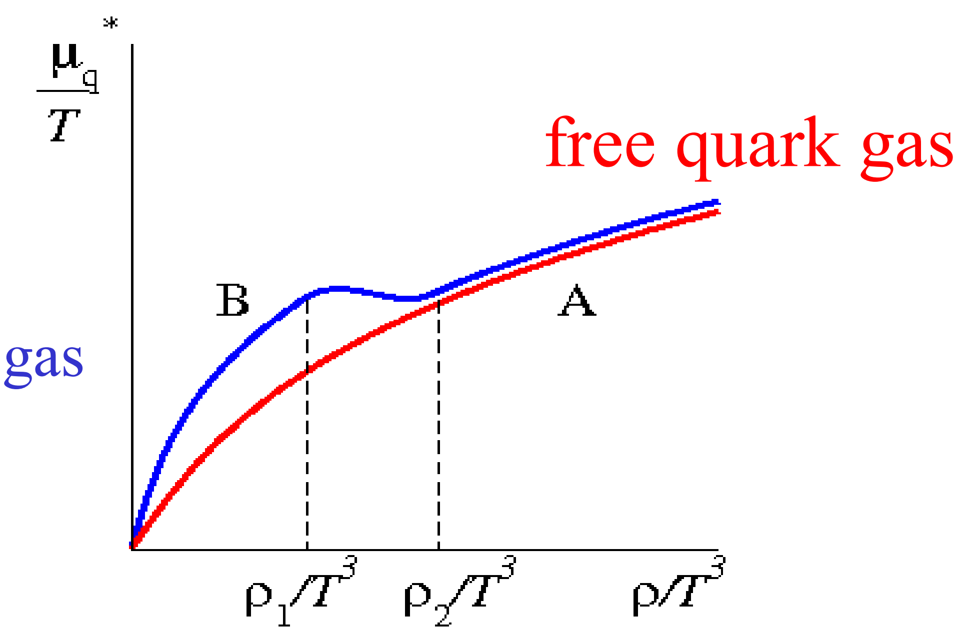
Role of the weight factor exp(F+$i\theta$)

- The weight factor has the same effect as when β (T) increased.
- μ^*/T approaches the free quark gas value in the high density limit for all temperature.

$$\boxed{\frac{\rho}{T^3} = N_{\mathrm{f}}\left[\frac{\mu}{T} + \frac{1}{\pi^2}\left(\frac{\mu}{T}\right)^3\right]}$$

free quark gas





Technical problem 2: Overlap problem

- Density of state method

  $W(P)$: plaquette distribution

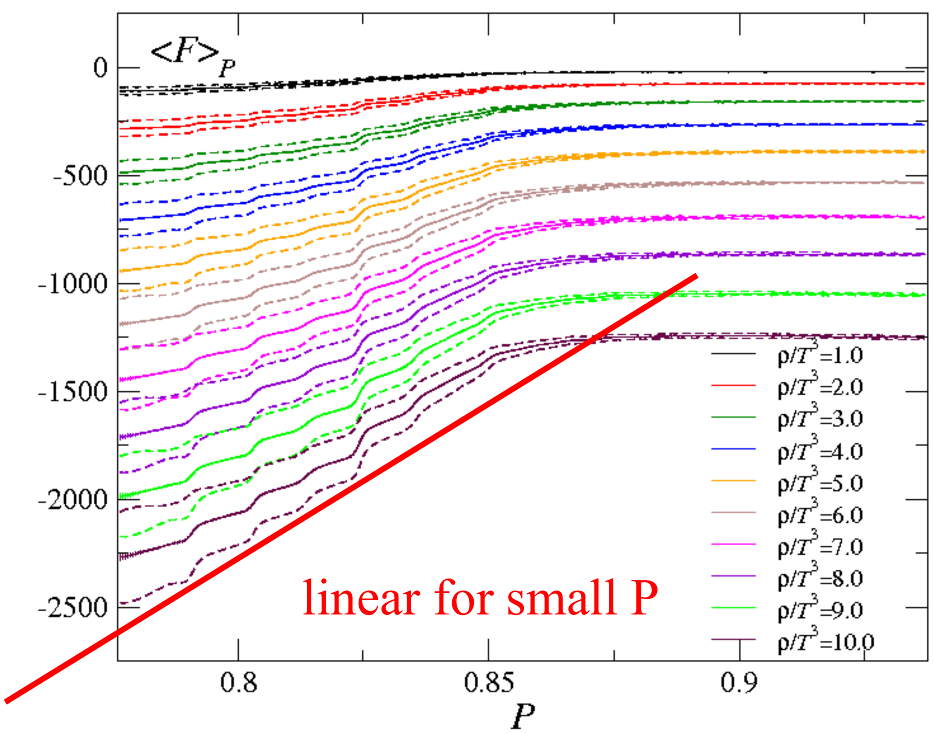
$$\frac{\mu^*(\rho)}{T} = \frac{\int \langle z_0 \exp(F + i\theta) \rangle_P W(P) dP}{\int \langle \exp(F + i\theta) \rangle_P W(P) dP}$$

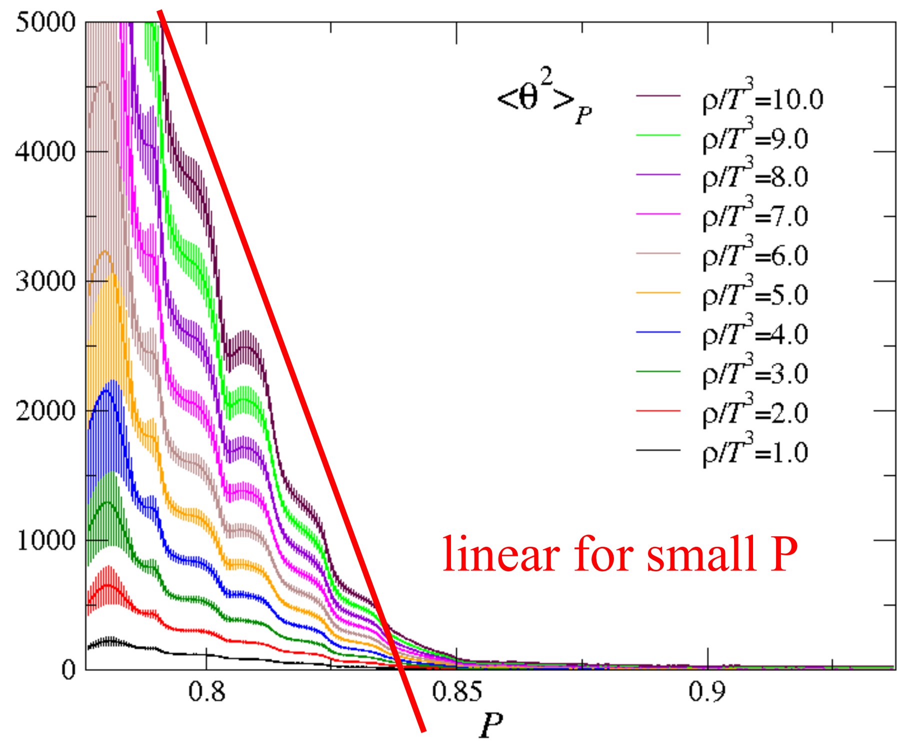
$$\langle \exp(F + i\theta) \rangle_P W(P) \approx \exp\left(\langle F \rangle_P - \langle \theta^2 \rangle_P / 2 + \cdots \right) W(P)$$

Same effect when β changes.

$$\propto \exp(\Delta\beta_{\text{eff}} P) W(P)$$

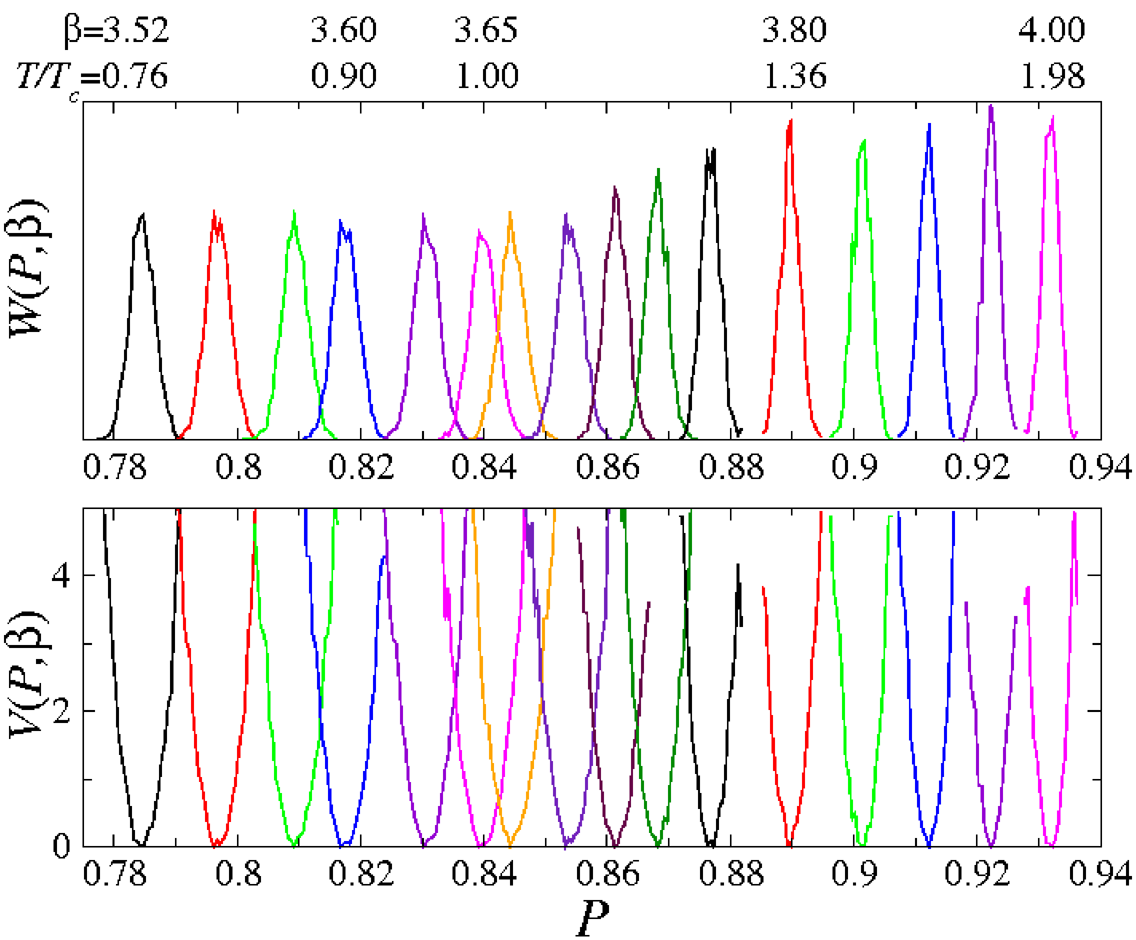
for small P





Reweighting for $\beta(T)=6g^{-2}$

(Data: N_f=2 p4-staggared, $m_\pi/m_\rho \approx 0.7$, $\mu=0$)



$$W(P',\beta)=\int DU\,(\det M)^{N_f}\, e^{-S_g(\beta)}\delta(P-P')$$

Change: $\beta_1(T) \rightarrow \beta_2(T)$

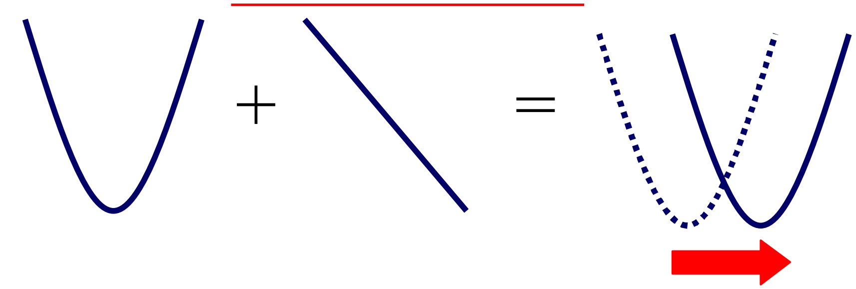
Distribution:

$$W(\beta_1)\Rightarrow W(\beta_2)=e^{-S_g(\beta_2)+S_g(\beta_1)}W(\beta_1)$$

$$S_g(\beta_2)-S_g(\beta_1)=-6N_{\mathrm{site}}(\beta_2-\beta_1)P$$

Potential:

$$-\ln W(\beta_1)\underline{-6N_{\mathrm{site}}(\beta_2-\beta_1)P}=-\ln W(\beta_2)$$



Effective β (temperature) for ρ≠0

$$\underline{\beta_{\mathrm{eff}}\equiv\beta+\left(\frac{d\langle F\rangle_P}{dP}-\frac{1}{2}\frac{d\langle\theta^2\rangle_P}{dP}\right)\frac{1}{N_{\mathrm{site}}}}$$

$\boxed{(\rho \text{ increases}) \approx (\beta\ (T) \text{ increases})}$

Overlap problem, Multi-β reweighting

Ferrenberg-Swendsen, PRL63,1195(1989)

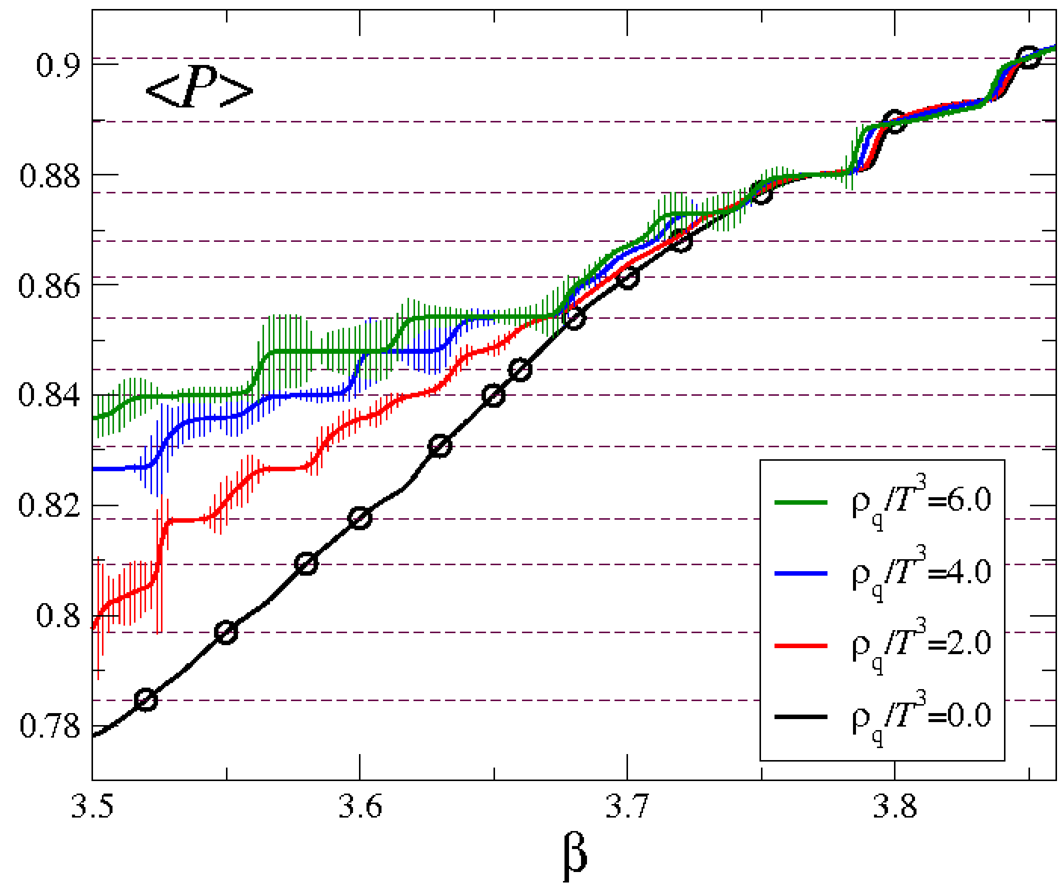
- When the density increases, the position of the importance sampling changes.
- Combine all data by multi-β reweighting

$$\langle P \rangle \approx \frac{\langle P \exp\left(F + i\theta\right)\rangle_{(T,\mu=0)}}{\langle \exp\left(F + i\theta\right)\rangle_{(T,\mu=0)}}$$

Plaquette value by multi-beta reweighting

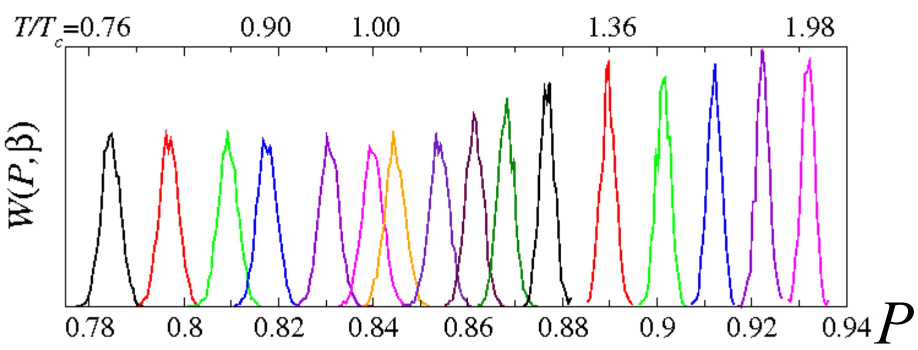
- - - peak position of the distribution

○ <*P*> at each β



Problem:

- Configurations do not cover all region of *P*.
- Calculate only when <*P*> is near the peaks of the distributions.



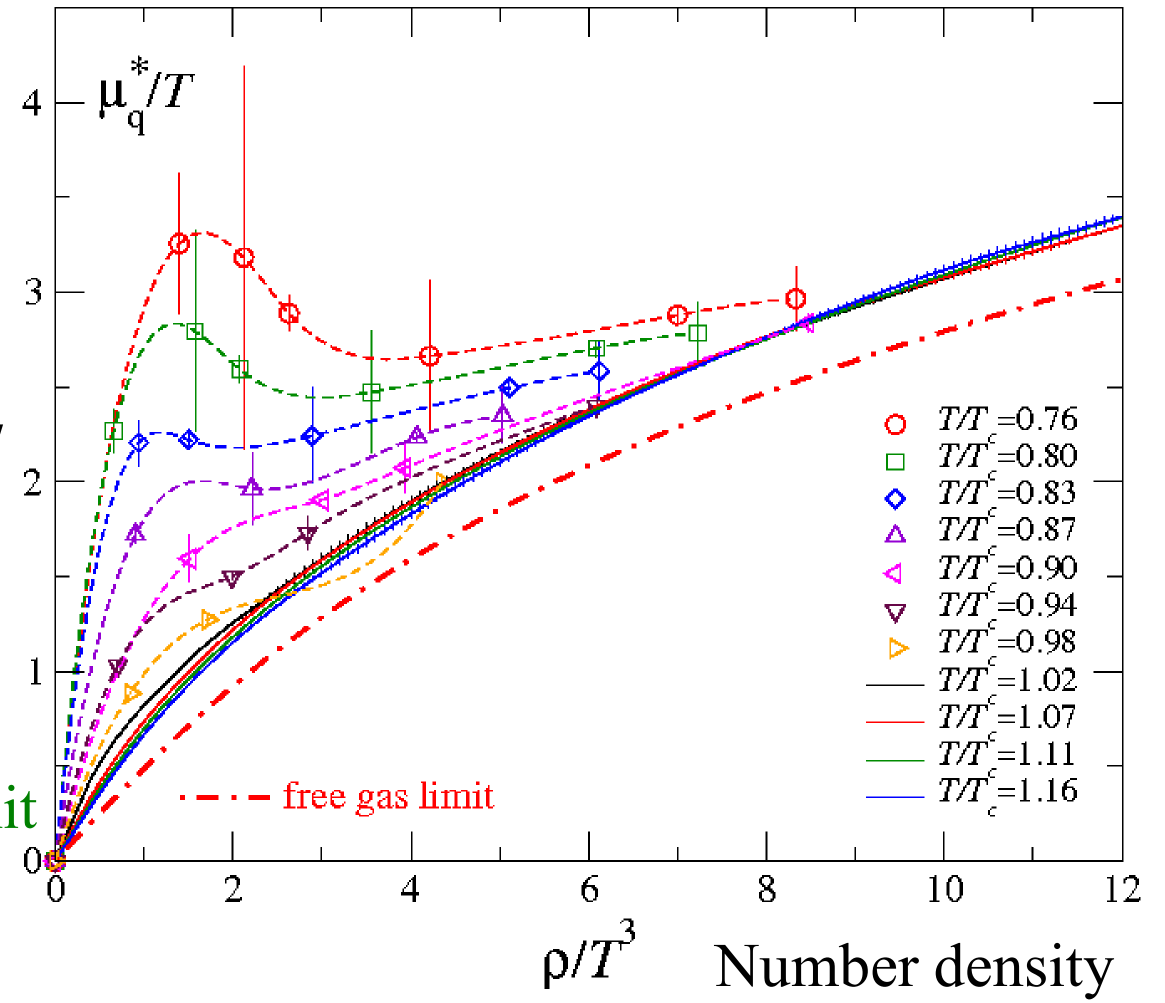
Chemical potential vs density

- Approximations:
  - – Taylor expansion: ln det M
  - – Gaussian distribution: θ
  - – Saddle point approximation

- Two states at the same μ_q/T
  - – First order transition at $T/T_c < 0.83$, $\mu_q/T > 2.3$

- μ^*/T approaches the free quark gas value in the high density limit for all T.

N_f=2 p4-staggered, $16^3 \times 4$ lattice

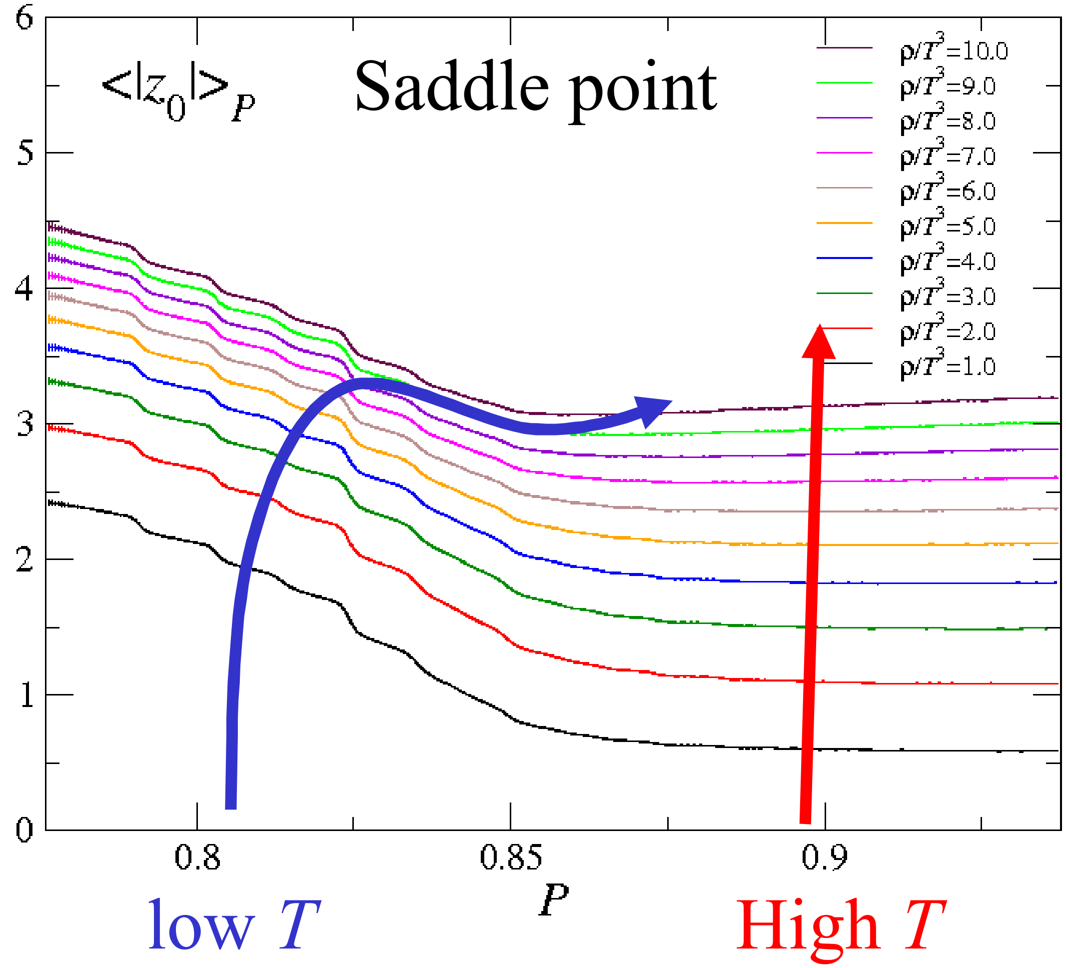


- Solid line: multi-b reweighting
- Dashed line: spline interpolation
- Dot-dashed line: the free gas limit

Chemical potential vs density

$$\frac{\mu^*(\rho)}{T} \sim \frac{\int \langle z_0 \rangle_P W(P, \beta_{\text{eff}}) dP}{\int W(P, \beta_{\text{eff}}) dP}$$

- Important configurations change for small P (or low T).
- High density limit
  ➡ High temperature limit
- Behavior of μ/T:
  non-trivial for low T.

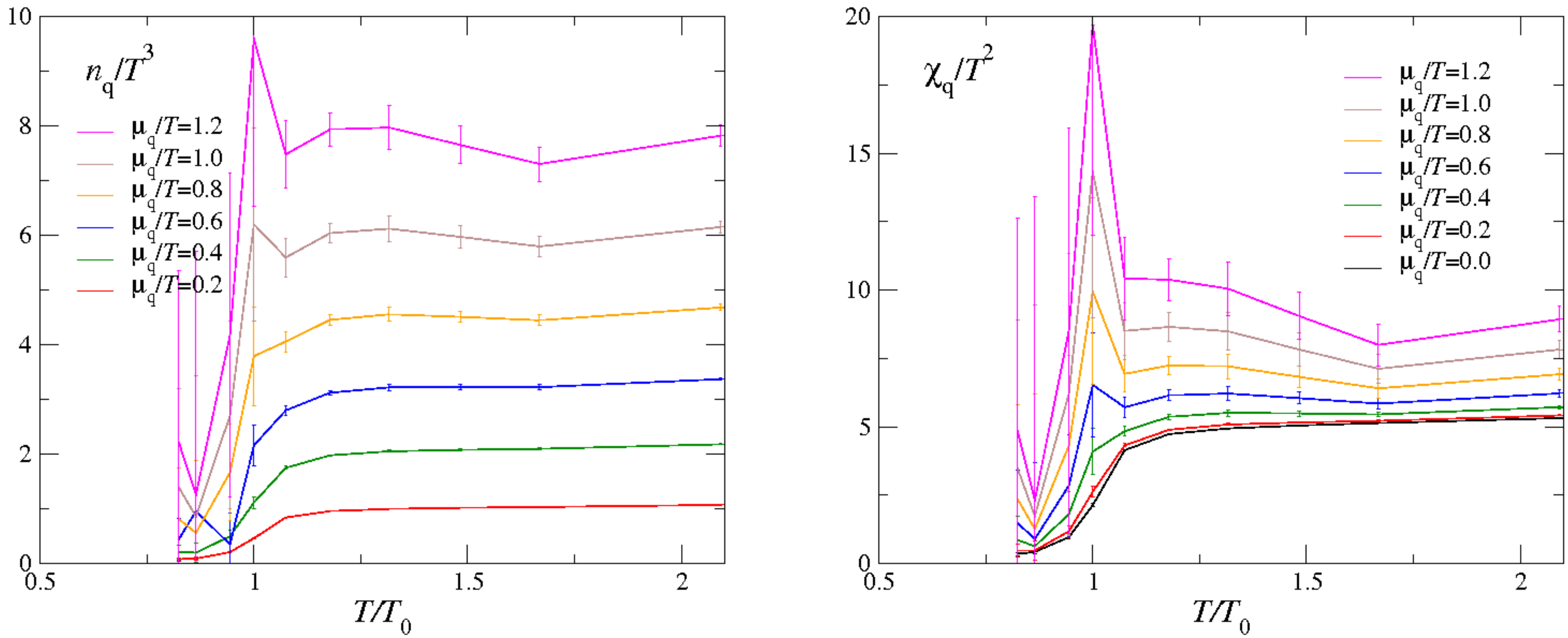


Equation of state by Wilson quark action

WHOT-QCD Collab., in preparation

RG gauge + 2-flavor Clover quark actions, $16^3 \times 4$ lattice, $m_\pi / m_\rho = 0.65$

Taylor expansion method up to $O(\mu_q^4)$



- Most of studies of QCD thermodynamics: staggared quark action.

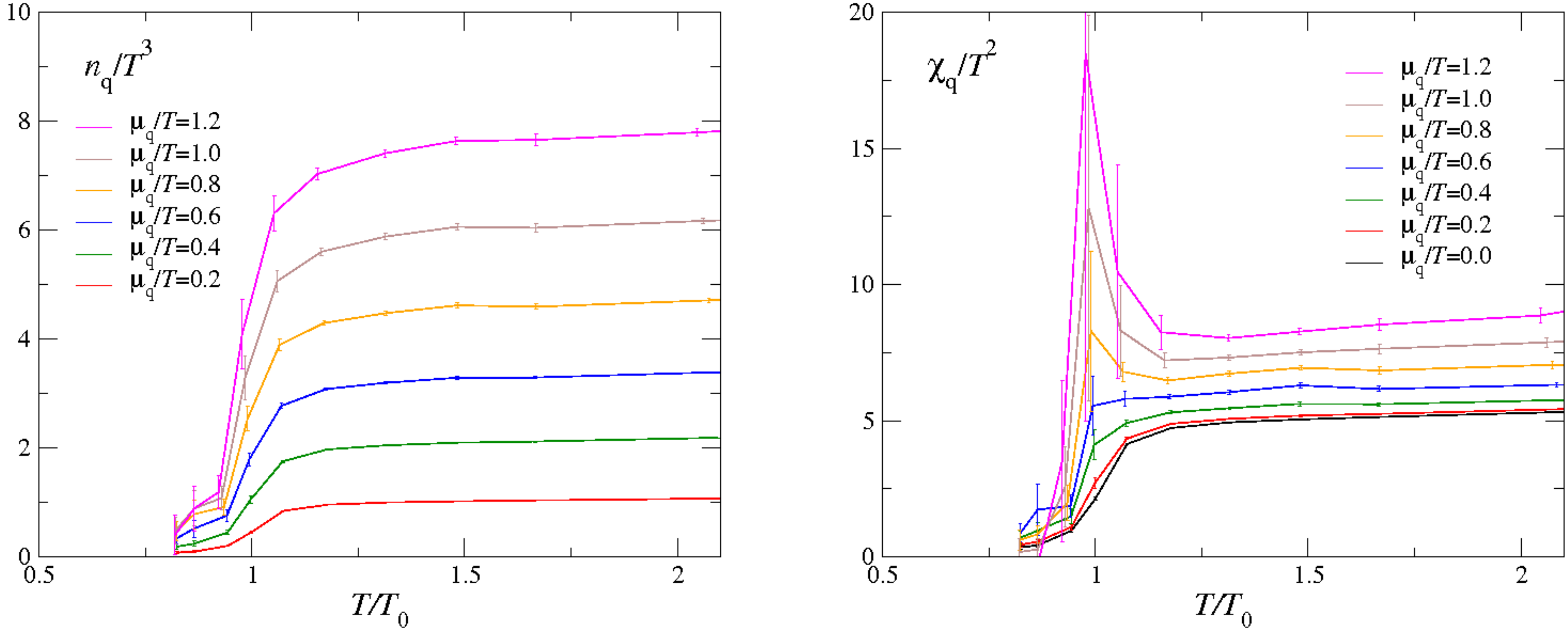
  ➡ Studies by Wilson quark: important.

- However, Wilson quark action: Large statistical errors.

Equation of state by Wilson quark action

RG gauge + 2-flavor Clover quark actions, $16^3 \times 4$ lattice, $m_\pi / m_\rho = 0.65$

Hybrid method of Reweighting and Taylor expansion up to $O(\mu_q^4)$



- Assumption: Complex phase: Gaussian distribution.
  - $O(\mu^6)$ error: (The level of the approximation does not change.)
- Large enhancement in the quark number fluctuations at high density. ➡ Critical point at finite μ ?

Summary

- Effective potentials as functions of the plaquette value and the quark number density are discussed.
- Approximation:
  - Taylor expansion of ln det M: up to $O(\mu^6)$
  - Distribution function of $\theta = N_f$ Im[ln det M] : Gaussian type.
  - Saddle point approximation ($1/V$ expansion)
- Simulations: 2-flavor p4-improved staggered quarks with $m_\pi/m_\rho \approx 0.7$ on 16^3x4 lattice
  - Existence of the critical point: suggested.
  - High ρ limit: μ/T approaches the free gas value for all T.
  - First order phase transition for $T/T_c < 0.83$, $\mu_q/T > 2.3$.
- Studies near physical quark mass: important.
  - Location of the critical point: sensitive to quark mass