Fluctuations and Search for the QCD Critical Point

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1. Integrated multiplicity fluctuations
2. Differential multiplicity fluctuations
3. K to π and p to π fluctuations
4. Test of v2 scaling at low transverse kinetic energy

The QCD Critical Point Workshop
11 Aug, 2008 in INT, Univ. of Washington, U.S.A.

Analysis on integrated multiplicity fluctuations

Quark Number Susceptibility

Consider quark number susceptibility, χ_q at the critical point.

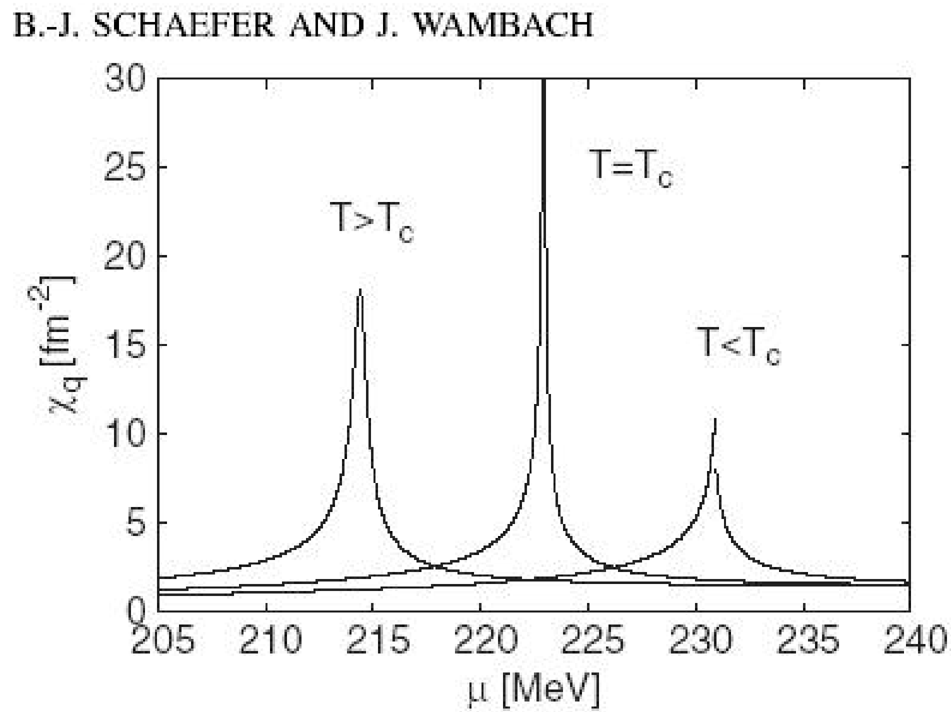
$\chi_q = \langle q^\dagger q \rangle = \partial n(T,\mu)/\partial \mu$

This is related to the isothermal compressibility:

$k_T = \chi_q(T,\mu)/n^2(T,\mu)$

In a continuous phase transition, k_T diverges at the critical point…

$$k_T \propto \left(\frac{T - T_c}{T_C}\right)^{-\gamma}$$



B.-J. Schaefer and J. Wambach, Phys. Rev. D75 (2007) 085015.

Integrated Multiplicity Fluctuations

Multiplicity fluctuations may be sensitive to divergences in the compressibility of the system near the critical point.

Grand Canonical Ensemble

$$\left(\sigma^2/\mu\right) = \omega_N = \mu/k_{NBD} + 1 = k_B T \left(\frac{\mu}{V}\right) k_T$$

ω_N → "Scaled Variance"

- Multiplicity fluctuations have been measured in the following systems:
  - 200 GeV Au+Au
  - 62.4 GeV Au+Au
  - 200 GeV Cu+Cu
  - 62.4 GeV Cu+Cu
  - 22.5 GeV Cu+Cu
  - 200 GeV p+p (baseline)
- Survey completed as a function of centrality, charge and p_T.

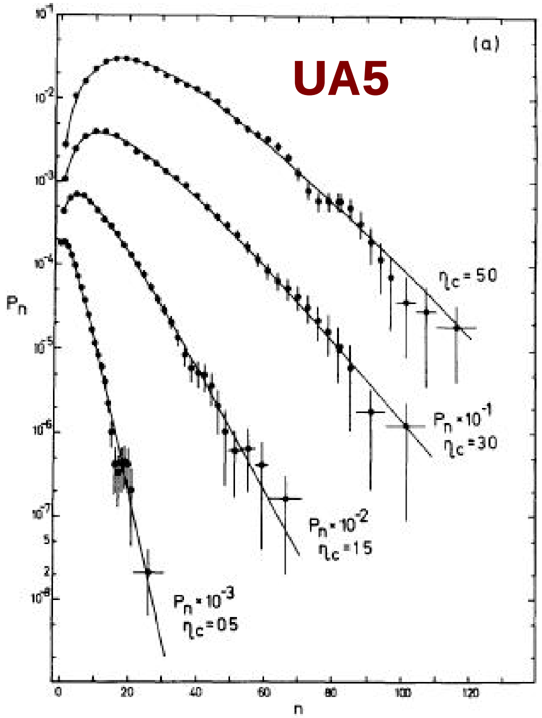
Multiplicity Fluctuations and Negative Binomial Distributions

Multiplicity distributions in hadronic and nuclear collisions can be well described by the Negative Binomial Distribution.

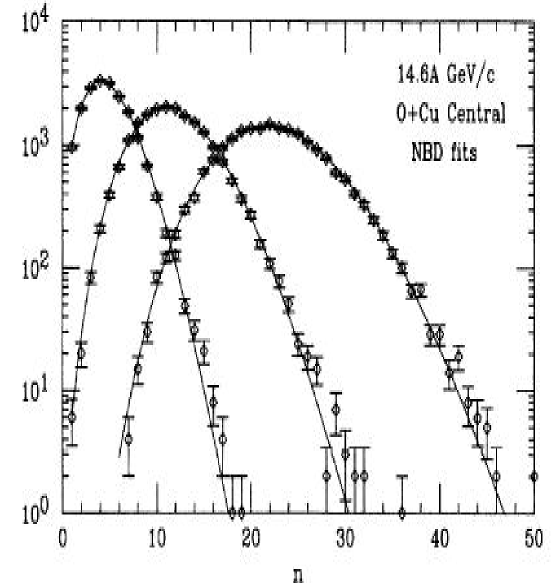
UA5: sqrt(s)=546 GeV p-pbar, Phys. Rep. 154 (1987) 247.

E802: 14.6A GeV/c O+Cu, Phys. Rev. C52 (1995) 2663.

UA5



E802



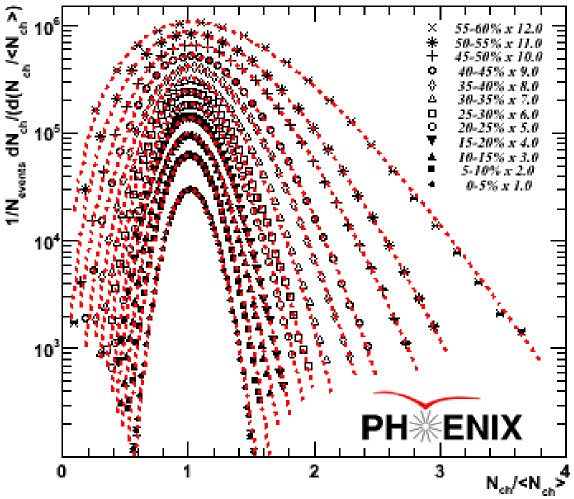
NBD $$P_n^{(k)} = \frac{\Gamma(n+k)}{\Gamma(n-1)\Gamma(k)}\left(\frac{\mu/k}{1+\mu/k}\right)^n \frac{1}{(1+\mu/k)^k}$$

$$\frac{\sigma^2}{\mu^2} = \frac{1}{\mu} + \boxed{\frac{1}{k}} \qquad \mu \equiv < n >$$

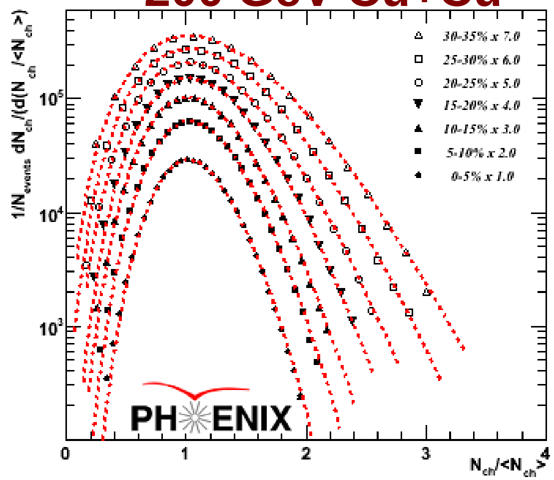
k=1 Bose-Einstein
k=∞ Poisson

Au+Au, Cu+Cu, p+p with NBD Distributions

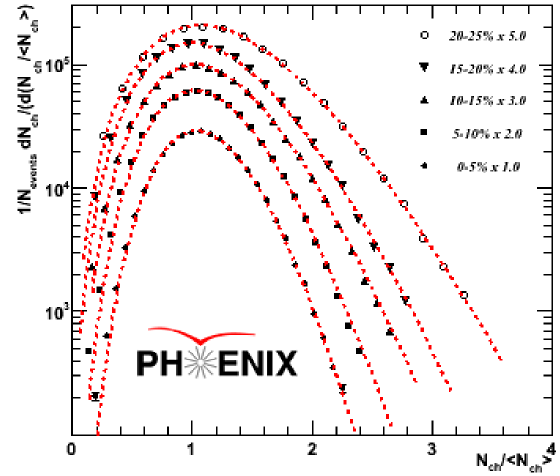
200 GeV Au+Au



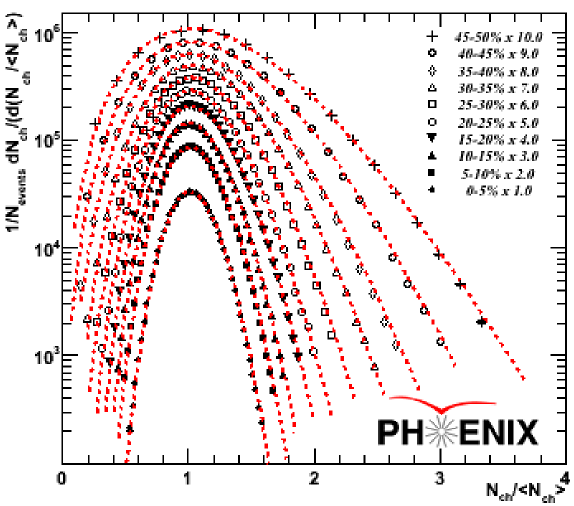
200 GeV Cu+Cu



62.4 GeV Cu+Cu

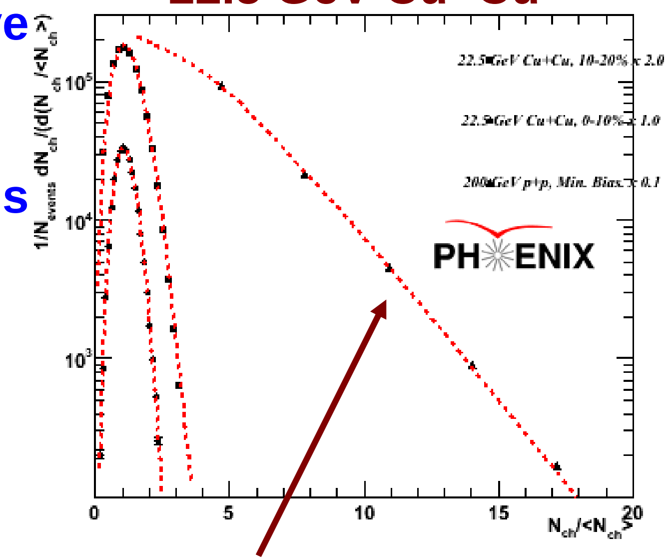


62.4 GeV Au+Au



Red lines represent the NBD fits. The distributions have been normalized to the mean and scaled for visualization. Distributions measured for $0.2<p_T<2.0$ GeV/c.

22.5 GeV Cu+Cu



200 GeV p+p

Simple base line: Participant Superposition Model

- **In a Participant Superposition Model, multiplicity fluctuations are given by:**

$$\omega_N = \omega_n + \langle N \rangle \omega_{Np}$$

where $\omega = \sigma^2/\mu$. ω_N = ***total fluctuation***, ω_n = ***fluctuation in each source (e.g. hadron-hadron collision),*** ω_{Np} = ***fluctuation in number of sources (participants),*** ***<N>=mean multiplicity per wounded nucleon.***

- **After correcting for fluctuations due to impact parameter,** $\omega_N = \omega_n$ **is independent of centrality.**

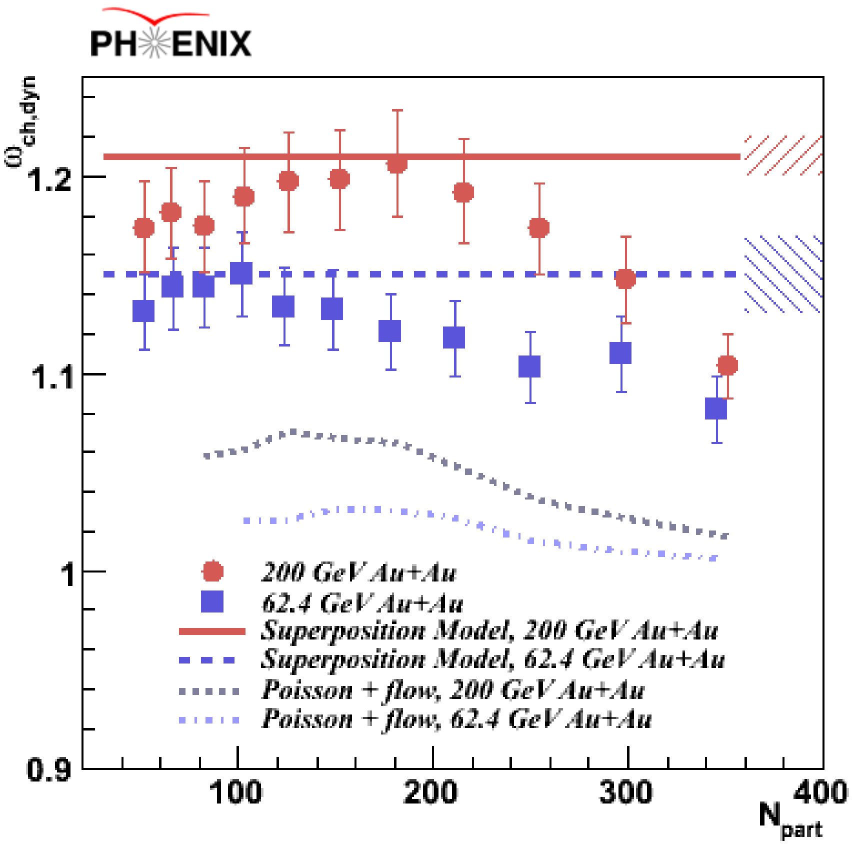
- **Multiplicity fluctuations are also dependent on acceptance:**

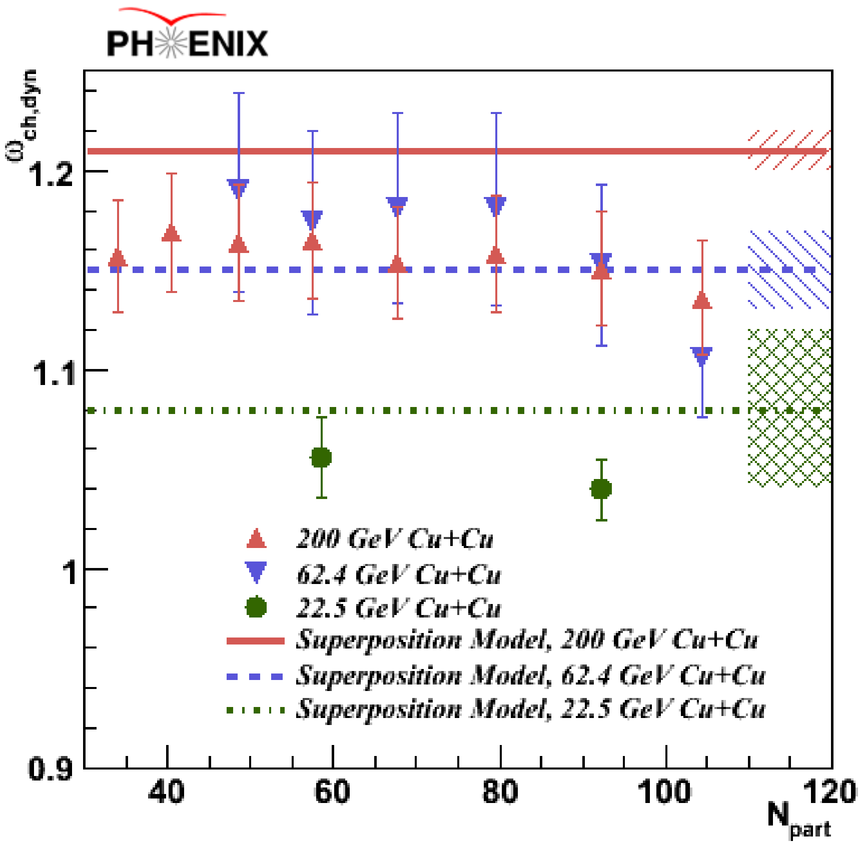
$$\omega_n = 1 + f(\omega_n - 1)$$

where $f = N_{accepted}/N_{total}$. ω_n **= fluctuations from each source in** 4π

Multiplicity Fluctuation Results

Bottom line: Near the critical point, the multiplicity fluctuations should exceed the superposition model expectation → No significant evidence for critical behavior is observed.





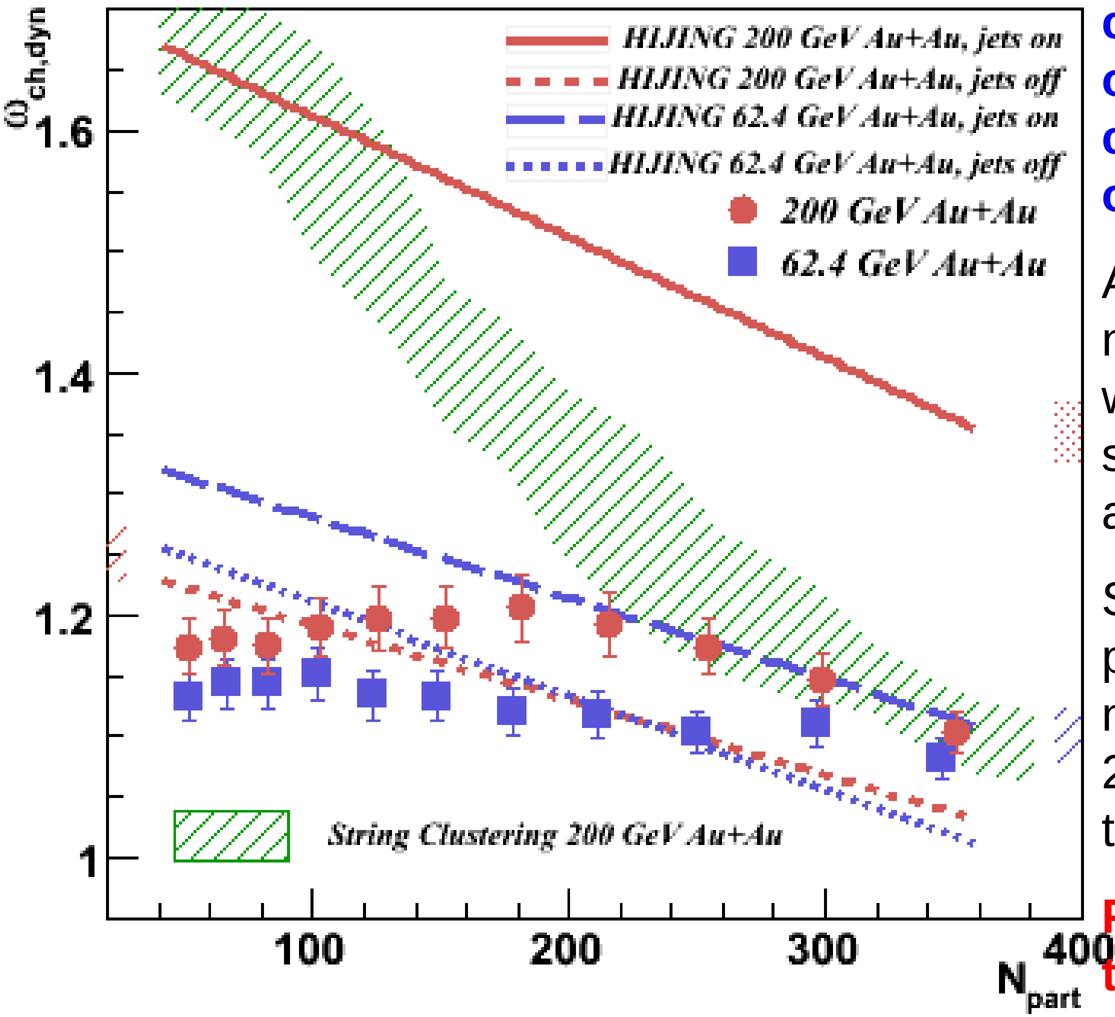
Centrality dependence is dominated by elliptic flow

Superposition model at 200 GeV taken from PHENIX measurements of 200 GeV p+p. The results agree with UA5 measurements in PHENIX's pseudorapidity window.

Superposition model at 22 GeV taken from NA22 measurements in PHENIX's pseudorapidity window.

Superposition model at 62 GeV taken from interpolation of UA5 results in PHENIX's pseudorapidity window.

String Percolation Model



String percolation: strings form clusters of geometrically overlapping strings and each cluster emits particles depending on the number strings.

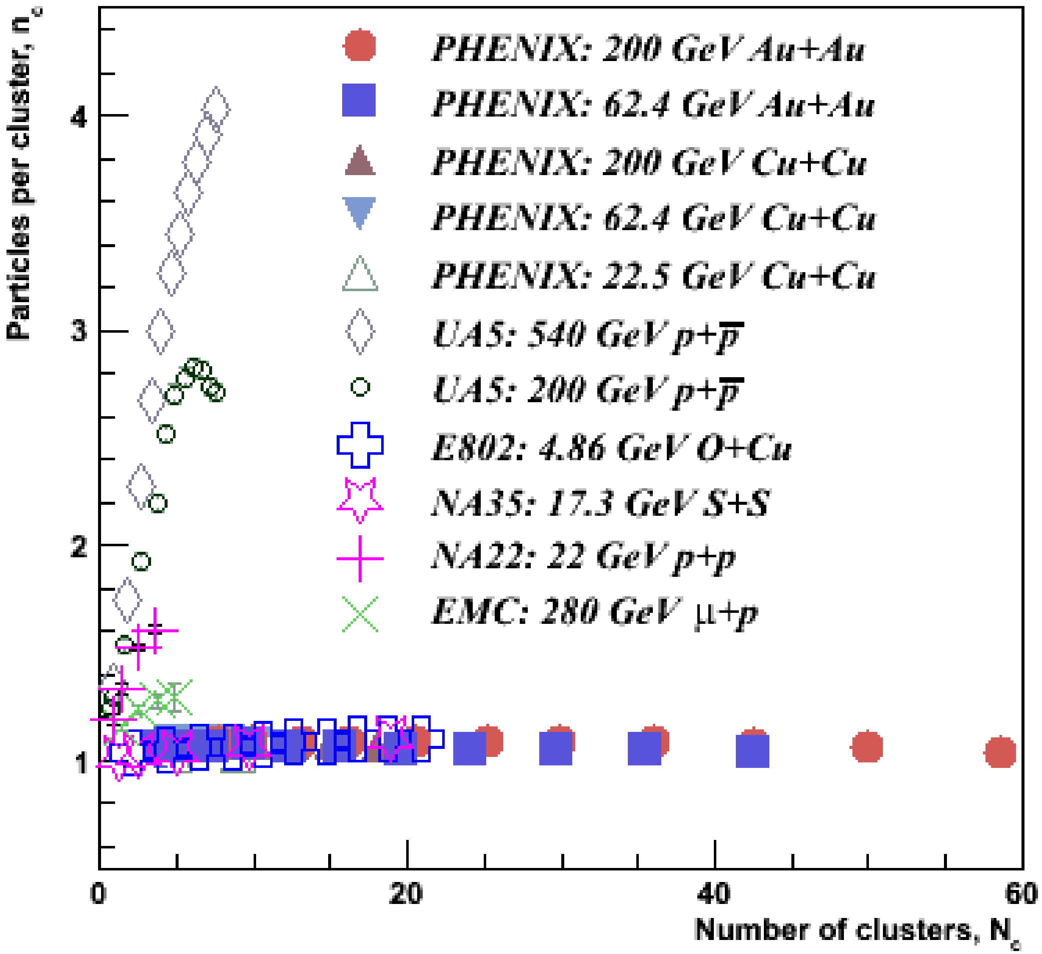
As the centrality increases, the number of clusters decreases along with the variance of the number of strings per cluster, which results in a decrease of scaled variance.

Shown in green are the direct predictions of the string percolation model (PRC72,024907(2005)) for 200 GeV Au+Au, scaled down to the PHENIX acceptance.

Percolation still does not explain the plateau in the most peripheral Au+Au collisions.

CLAN Model

A. Giovannini et al., Z. Phys. C30 (1986) 391.



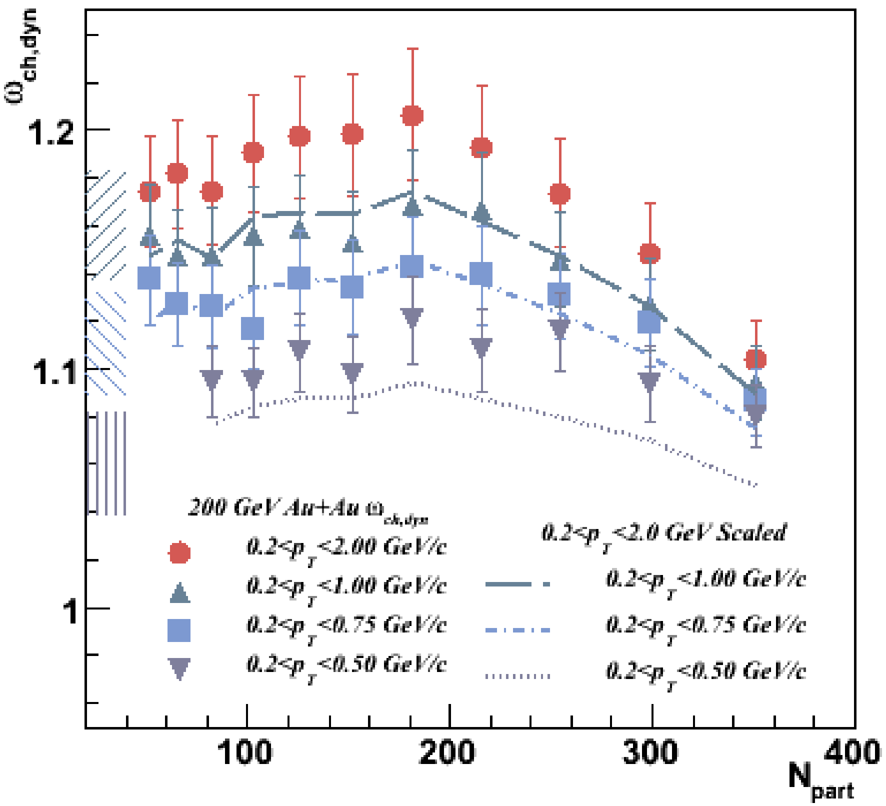
The CLAN model was developed to attempt to explain the reason that p+p multiplicities are described by NBD rather than Poisson distributions.

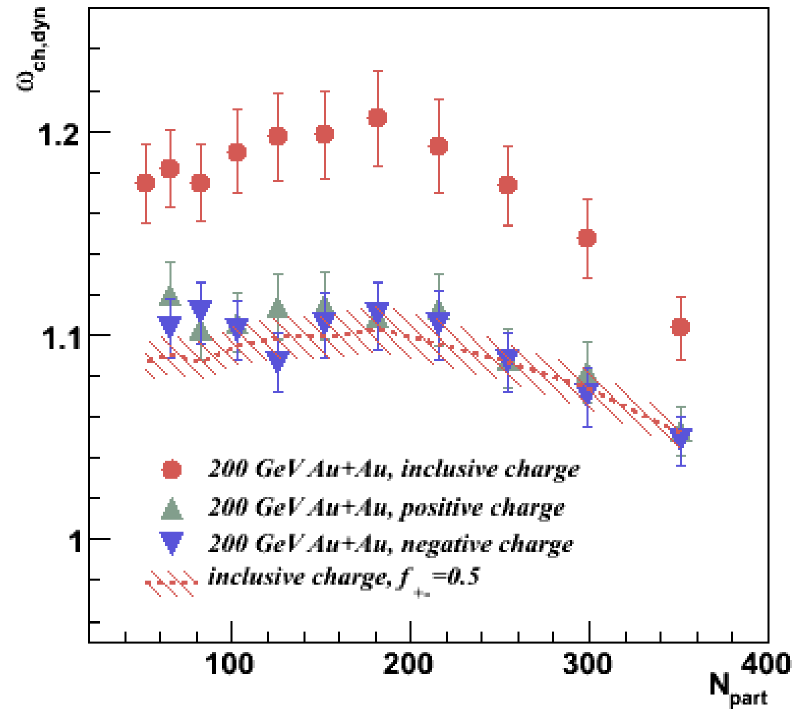
Hadron production is modeled as independent emission of a number of hadron clusters, N_c, each with a mean number of hadrons, n_c. These parameters can be related to the NBD parameters:

$N_c = k_{NBD} \log(1 + \mu_{ch}/k_{NBD})$ and
$\langle n_c \rangle = (\mu_{ch}/k_{NBD})/\log(1 + \mu_{ch}/k_{NBD})$.

A+A collsions exhibit weak clustering characteristics, independent of collision energy.

Charge and p_T-Dependence





$\omega_{+-} = 1 + f(\omega_{inclusive} - 1)$

where f=0.5.

If pT-dependence is random, the scaled variance should scale with <N> in the same manner as acceptance:

$\omega_{pT} = 1 + f(\omega_{pT,max} - 1)$

Within errors, no charge dependence of the fluctuations is seen for 200 GeV Au+Au.

Analysis on differential multiplicity fluctuations

Density-density correlation in longitudinal space

Longitudinal space coordinate z can be transformed into rapidity coordinate in each proper frame of sub element characterized by a formation time τ at which dominant density fluctuations are embedded.

$$z = \tau \sinh(y)$$

$$t = \tau \cosh(y)$$

$$dz = \tau \cosh(y) dy$$

Due to relatively rapid & symmetric expansion in y, analysis in y would have an advantage to extract initial fluctuations compared to analysis in transverse plane in high energy collision.

$$g(T,\phi,h) - g_0 = \int_{\delta y} dy \cancel{\int_{S\perp} d^2 x_\perp}$$

$$\left[\frac{1}{2\tau^2 \cosh(y)} \left(\frac{\partial \phi}{\partial y} \right)^2 + \cosh(y) \left(\cancel{\frac{1}{2} (\nabla_\perp \phi)^2} + U(\phi) \right) \right]$$

In narrow midrapidity region like PHENIX, cosh(y)~1 and y~η.

Longitudinal multiplicity density fluctuation from the mean density can be an order parameter:

$$\phi(\eta) = \rho(\eta) - \langle \rho \rangle$$

Direct observable for Tc determination

GL free energy density *g* with $\phi \sim 0$ from high temperature side is insensitive to transition order, but it can still be sensitive to Tc

$$g(T,\phi,h) = g_0 - \underbrace{\frac{1}{2}A(T)(\nabla\phi)^2}_{\text{spatial correlation}} + \underbrace{\frac{1}{2}a(T)\phi^2}_{\phi \text{ disappears at Tc}} + \cancel{\frac{1}{4}b\phi^4 + \frac{1}{6}c\phi^6} \cdots - h\phi$$

spatial correlation φ disappears at Tc → $a(T) = a_0(T - T_c)$

Fourier analysis on

$G_2(y)=\langle\phi(0)\phi(y)\rangle$

$$\left\langle |\phi_k|^2 \right\rangle = Y\int G_2(y)e^{-ik(y)}dy$$

$$\left\langle |\phi_k|^2 \right\rangle = \frac{NT}{Y}\frac{1}{a(T)+A(T)k^2}$$

Susceptibility

$$\chi_k = \frac{\partial\phi_k}{\partial h} \propto \left(\frac{\partial^2(g-g_0)}{\partial\phi^2{}_k}\right)^{-1} = \frac{1}{a_0(T-T_c)(1+k^2\xi^2)}$$

1-D two point correlation function

$$G_2(y) = \frac{NT}{2Y^2A(T)}\xi(T)e^{-|y|/\xi(T)}$$

Susceptibility in long wavelength limit

$$\chi_{k=0} = \frac{1}{a_0(T-T_c)} \propto \frac{\xi}{T}G_2(0)$$

Correlation length

$$\xi(T)^2 \equiv \frac{A(T)}{a_0(T-T_c)}$$

Product between correlation length and amplitude can also be a good indicator for T~Tc

Intuitive observable: blob intensity α x blob size ξ

Order parameter

$\phi(\eta)=\rho(\eta)-\langle\rho(\eta)\rangle$

$\phi \ll 1$ in $T \gg T_c$,

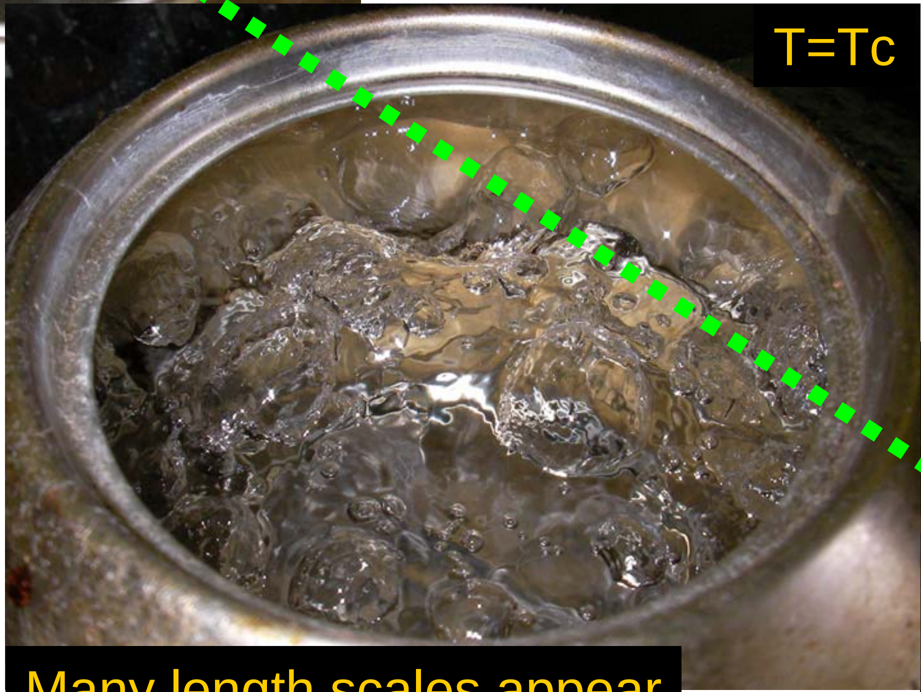
Ginzburg-Landau(GL) free energy up to 2nd order term

Two point correlation $\langle\phi(\eta_1)\phi(\eta_2)\rangle$ in 1-D longitudinal space

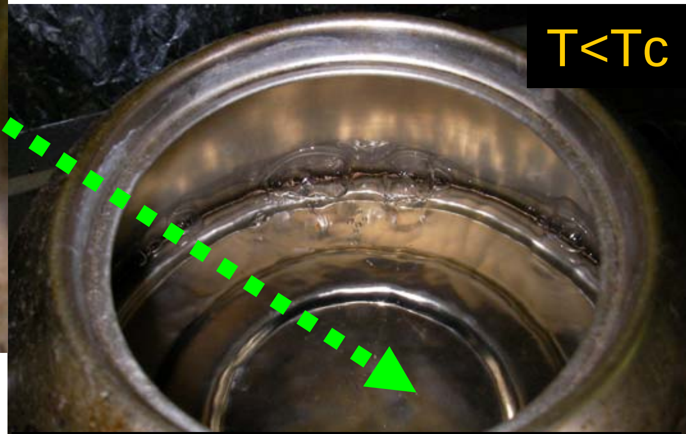
$$C_2 \propto \alpha \exp(-|\eta_1-\eta_2|/\xi)$$

$$\alpha\xi \propto \chi_{k=0} T < \rho >^{-2} \propto < \rho >^{-2} \frac{1}{1-T_c/T}$$

Non monotonic increase of αξ indicates T~Tc w.r.t. monotonically decreasing baseline as mean density <ρ> increases.

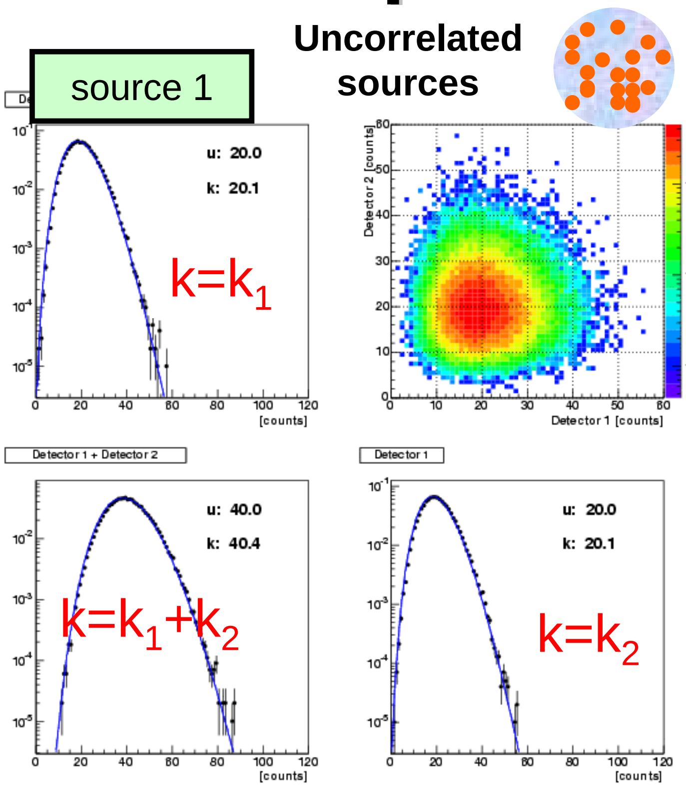


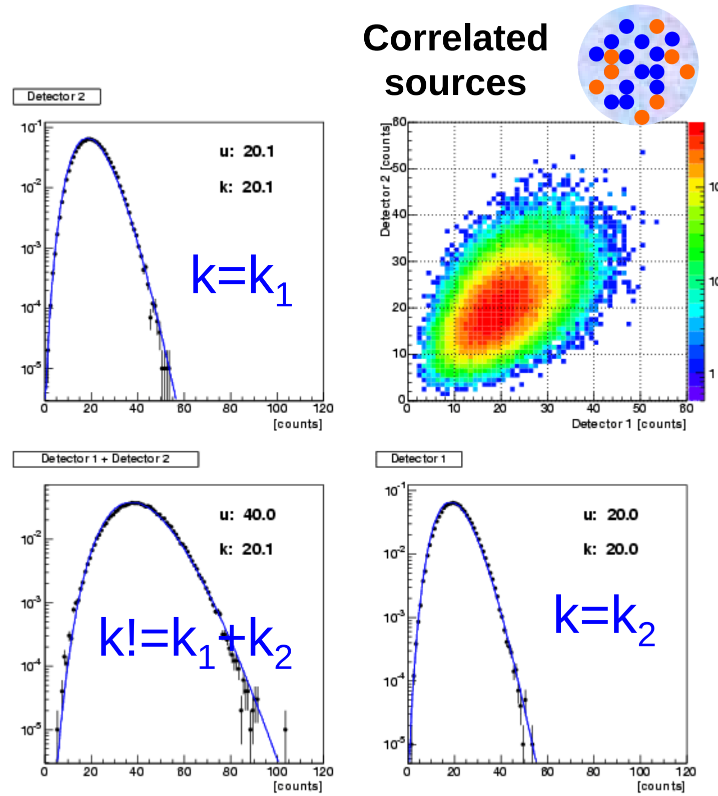
Many length scales appear (a typical ϕ_k disappears)



GL with higher order terms

Two point correlation via NBD





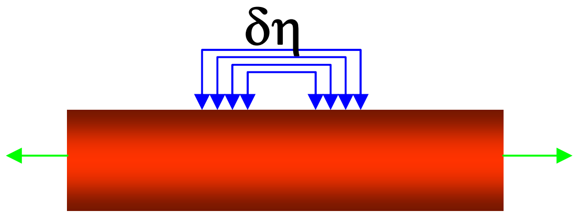
NBD $$P_n^{(k)} = \frac{\Gamma(n+k)}{\Gamma(n-1)\Gamma(k)} \left(\frac{\mu/k}{1+\mu/k} \right)^n \frac{1}{(1+\mu/k)^k}$$

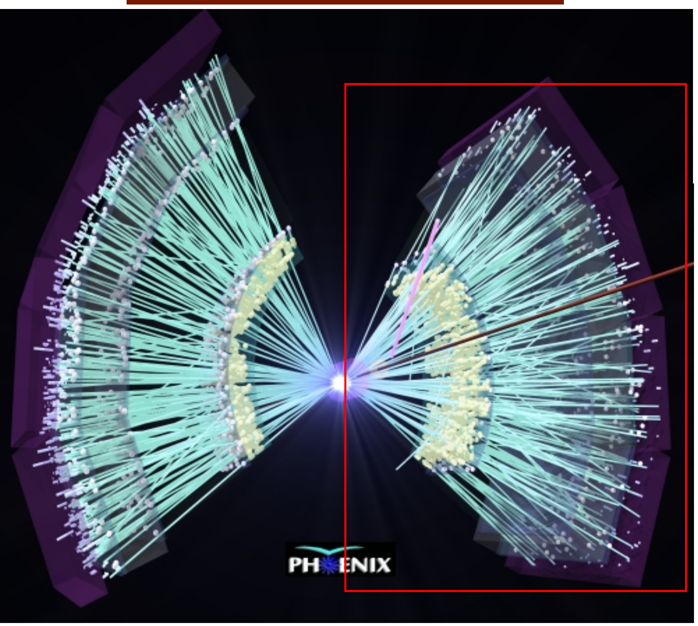
k=1 Bose-Einstein
k=∞ Poisson

$$\frac{\sigma^2}{\mu^2} = \frac{1}{\mu} + \boxed{\frac{1}{k}} \qquad \mu \equiv \langle n \rangle$$

1/k corresponds to integral of two point correlation

Differential multiplicity measurements

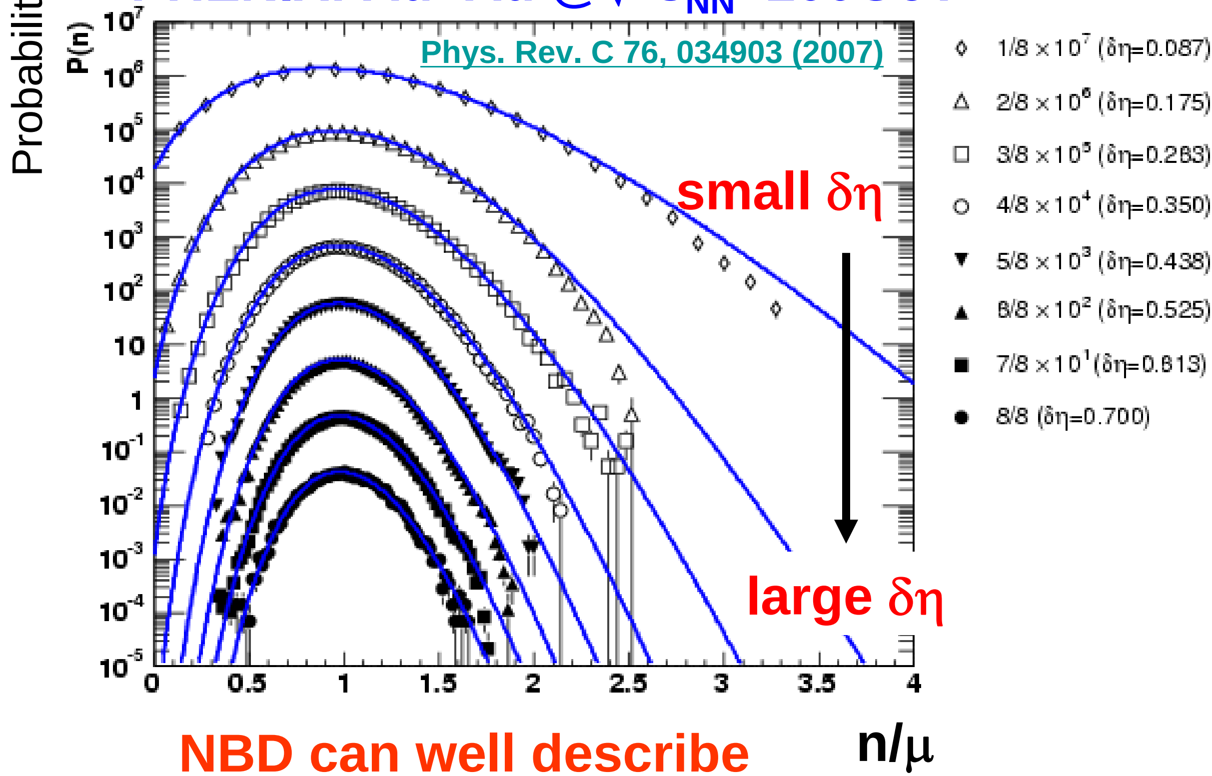




Zero magnetic field to enhance low pt statistics per collision event.

$\Delta\eta < 0.7$ integrated over $\Delta\phi < \pi/2$

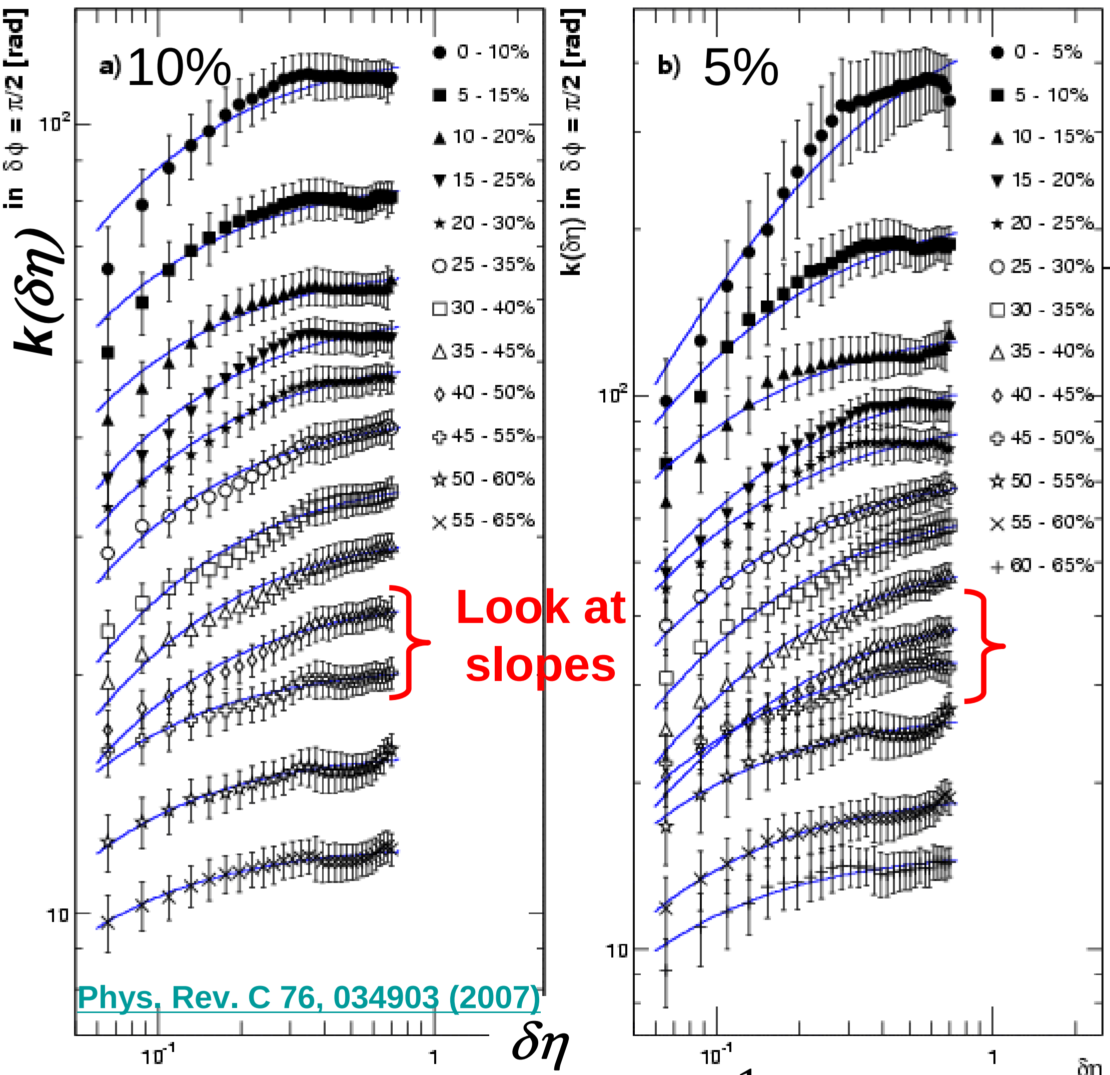
PHENIX: Au+Au @$\sqrt{s_{NN}}$=200GeV



NBD can well describe differential distribution too.

Extraction of αξ product

Fit with approximated functional form



Phys. Rev. C 76, 034903 (2007)

Approximated functional form

$$k(\delta\eta) = \frac{1}{2\alpha\xi/\delta\eta + \beta} \quad (\xi << \delta\eta)$$

Parametrization of two particle correlation

$$C_2(\eta_1,\eta_2) \equiv \rho_2(\eta_1,\eta_2) - \rho_1(\eta_1)\rho_1(\eta_2)$$

$$\frac{C_2(\eta_1,\eta_2)}{\bar{\rho}_1^2} = \alpha e^{-\delta\eta/\xi} + \beta$$

β absorbs rapidity independent bias: Npart fluctuation and reaction plane rotation and v2

Exact relation with NBD *k*

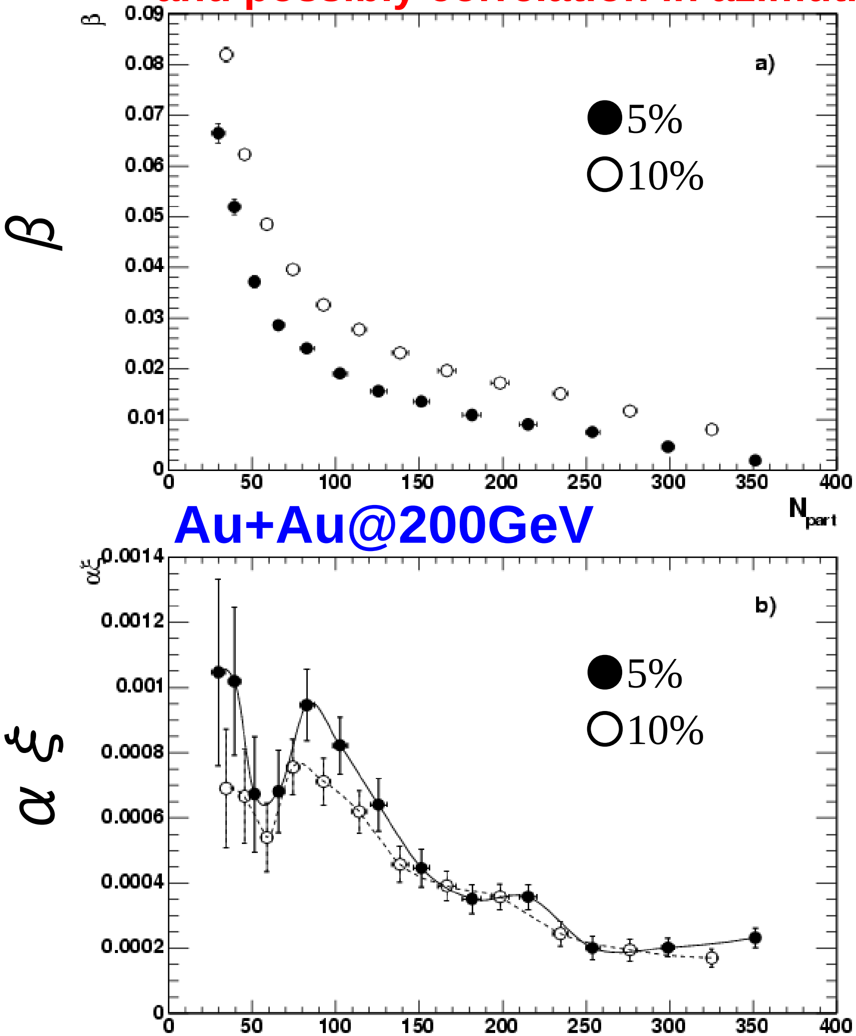
$$k^{-1}(\delta\eta) = \frac{\langle n(n-1)\rangle}{\langle n\rangle^2} - 1$$

$$= \frac{\int_0^{\delta\eta}\int_0^{\delta\eta} C_2(\eta_1,\eta_2)\,d\eta_1 d\eta_2}{\delta\eta^2 \bar{\rho}_1^2}$$

$$= \frac{2\alpha\xi^2(\delta\eta/\xi - 1 + e^{-\delta\eta/\xi})}{\delta\eta^2} + \beta$$

$\alpha\,\xi$, β vs. Npart

Dominantly Npart fluctuations and possibly correlation in azimuth



[Phys. Rev. C 76, 034903 (2007)](#) Npart

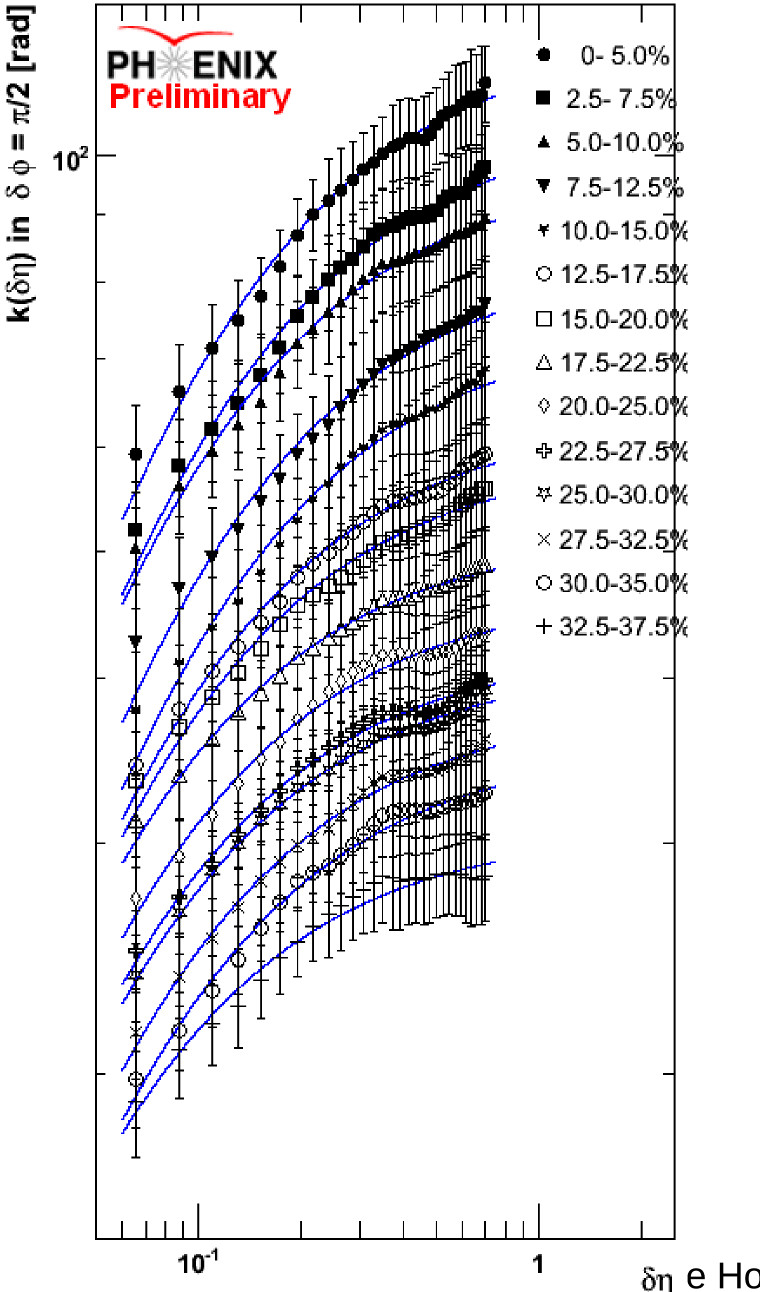
β is systematically shift to lower values as the centrality bin width becomes smaller from 10% to 5%. This is understood as fluctuations of Npart for given bin widths

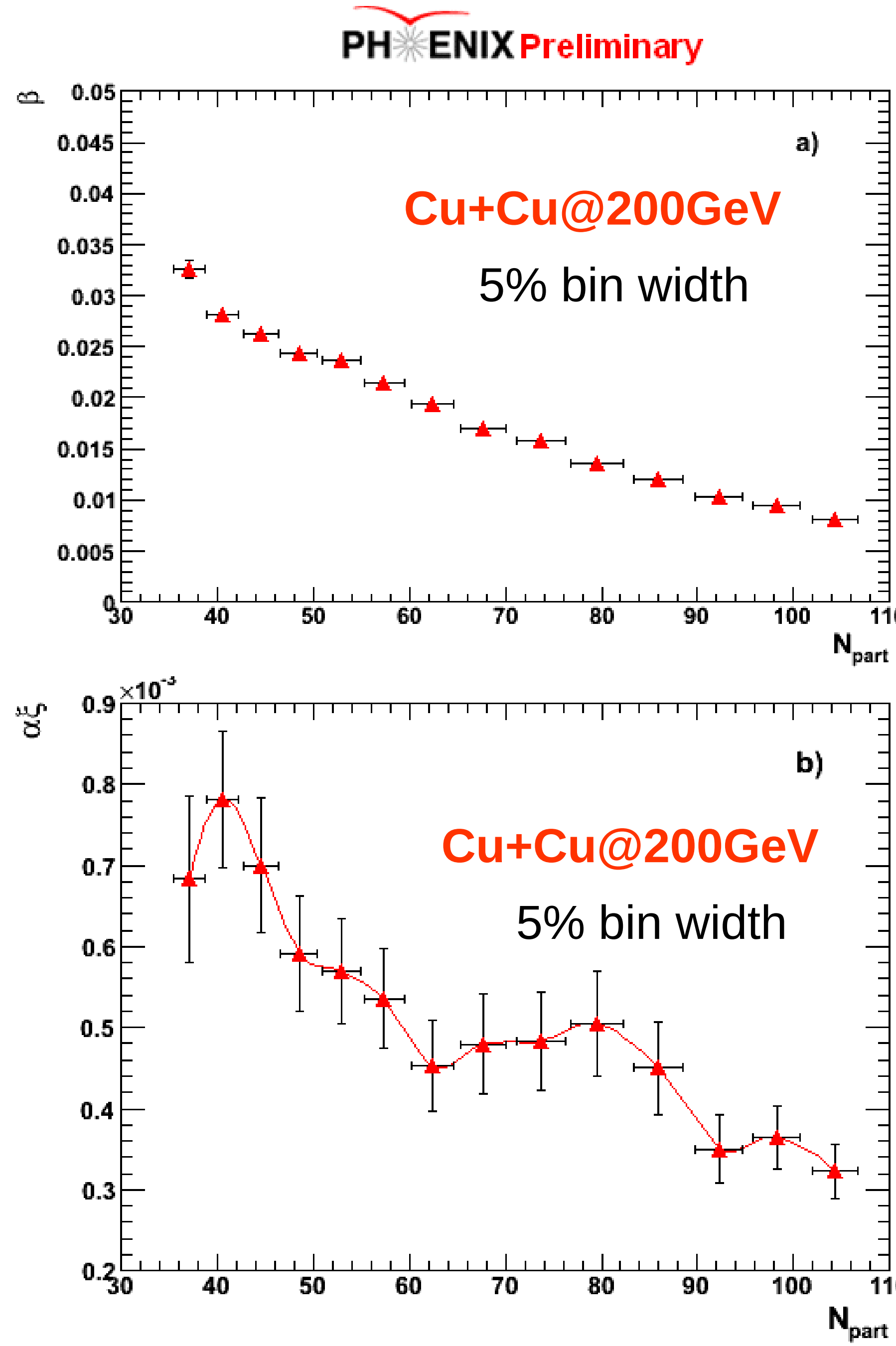
$\alpha\,\xi$ product, which is monotonically related with $\chi_{k=0}$ indicates the non-monotonic behavior around Npart ~ 90.

$$\alpha\xi = \chi_{k=0} T / \bar{\rho}_1^{\,2} \propto \bar{\rho}_1^{\,-2} \frac{T}{|T - T_C|}$$

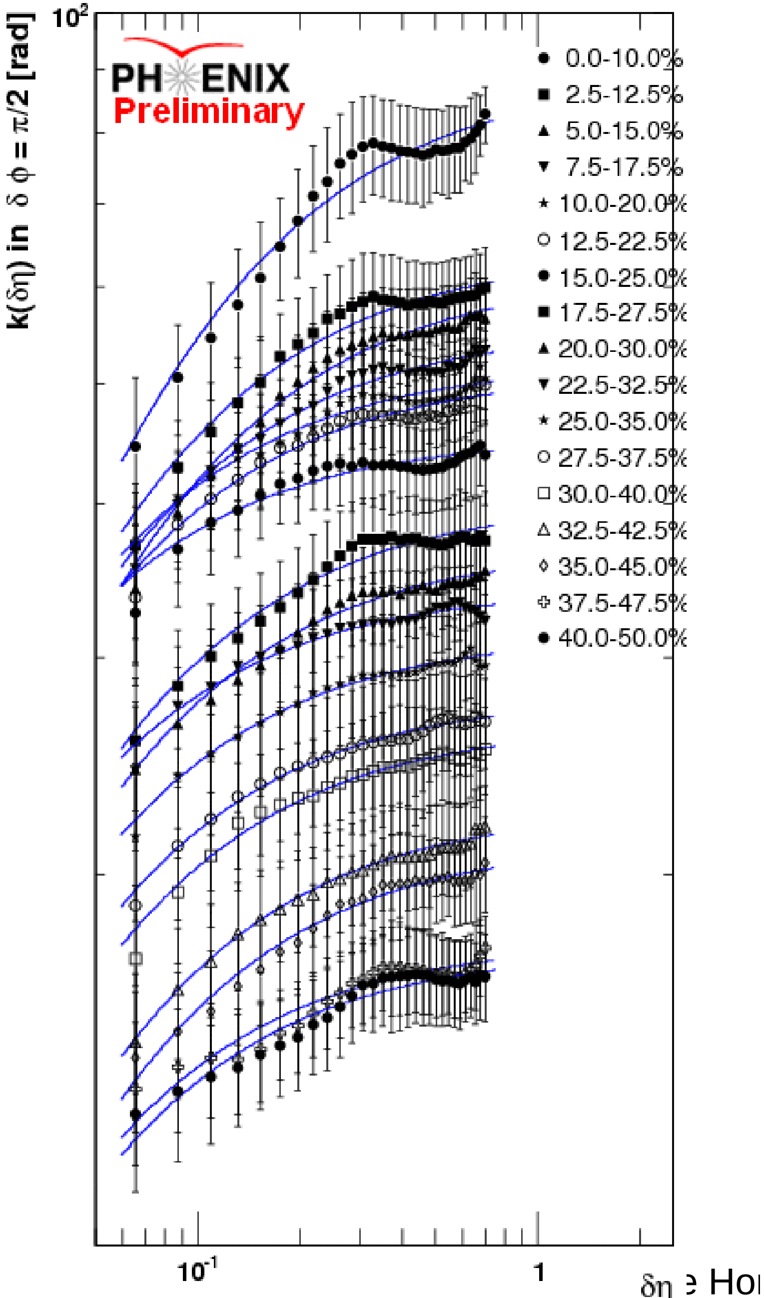
Significance with Power + Gaussian:
3.98 σ (5%), 3.21 σ (10%)
Significance with Line + Gaussian:
1.24 σ (5%), 1.69 σ (10%)

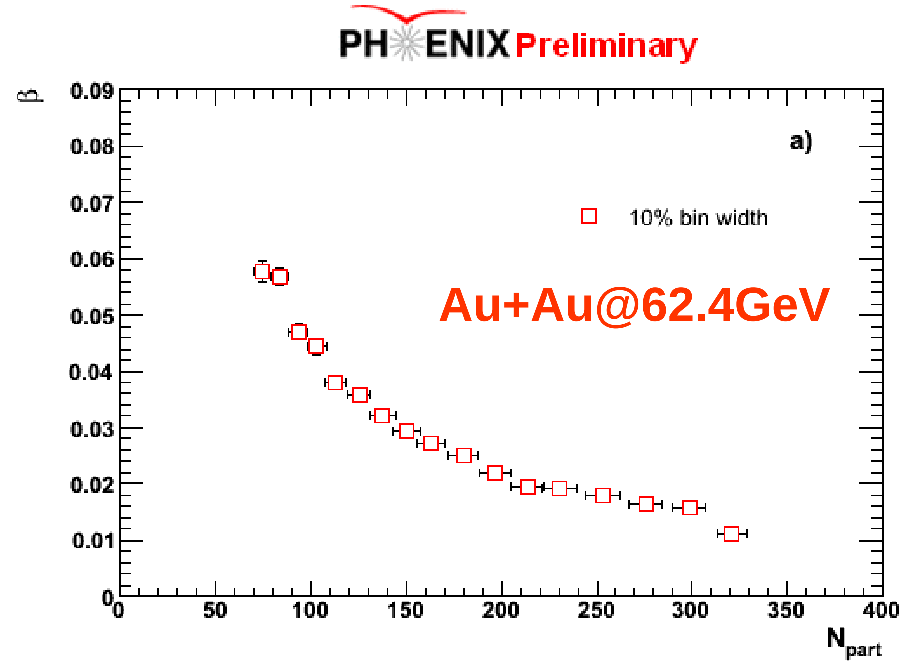
Analysis in smaller system: Cu+Cu@200GeV

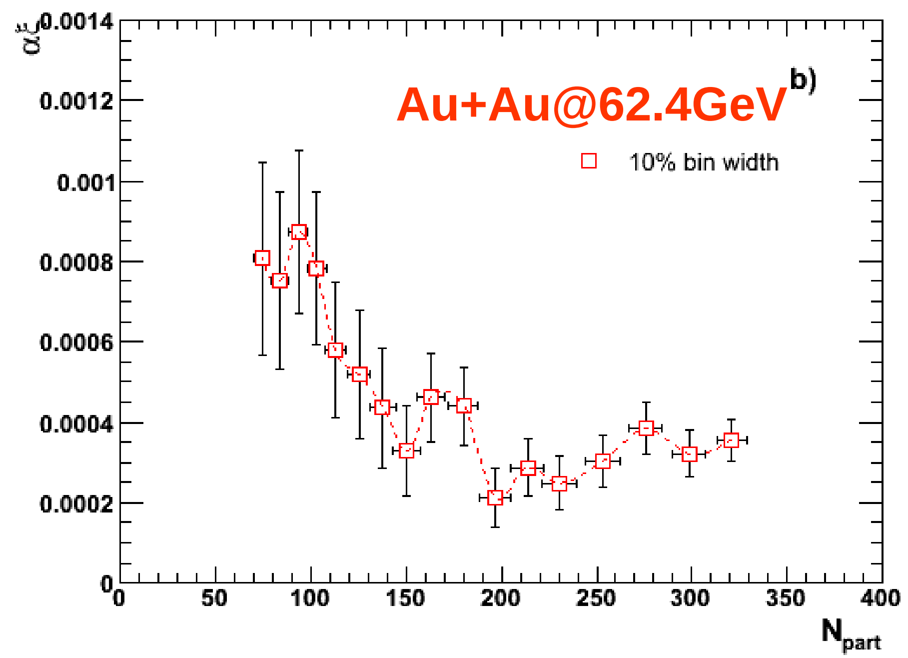




Analysis in lower energy: Au+Au@62.4GeV

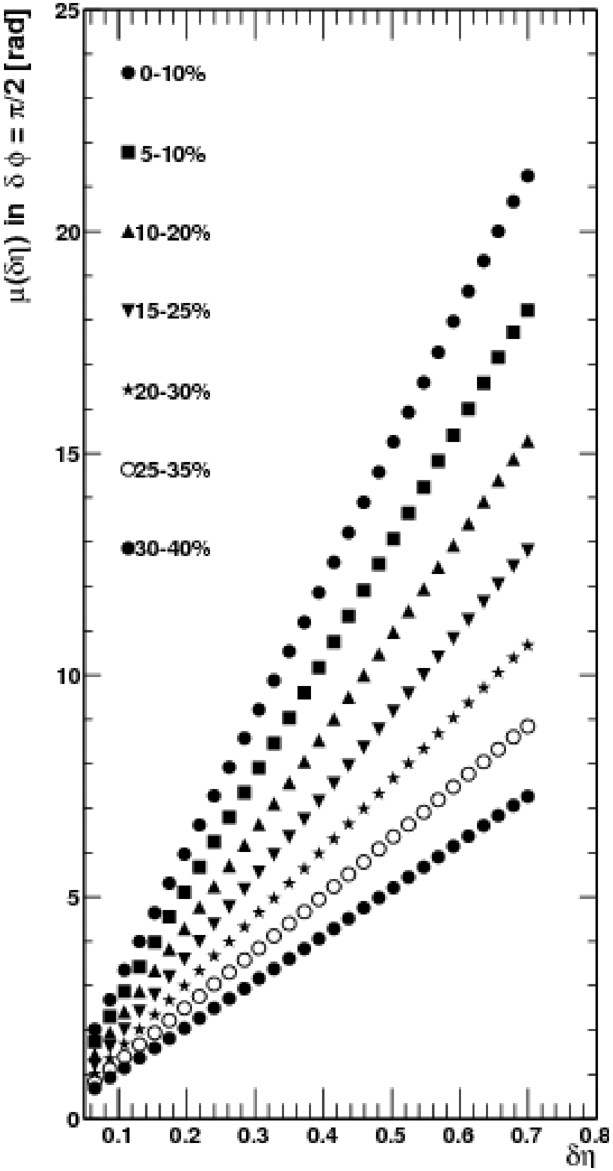




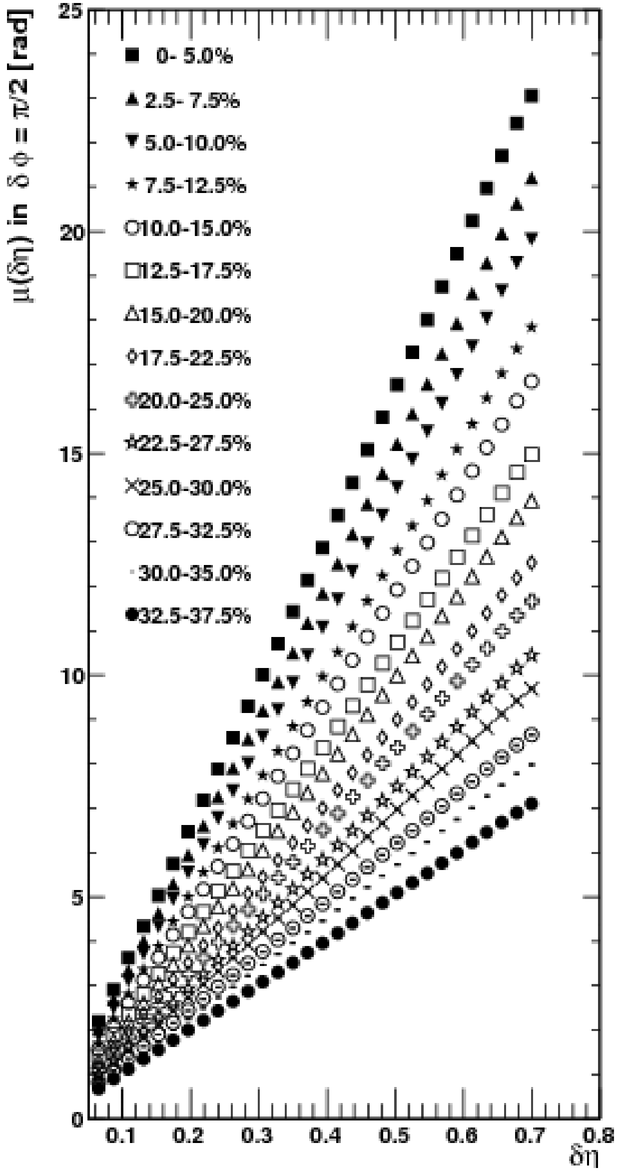


Corrected mean multiplicity <μ_c>

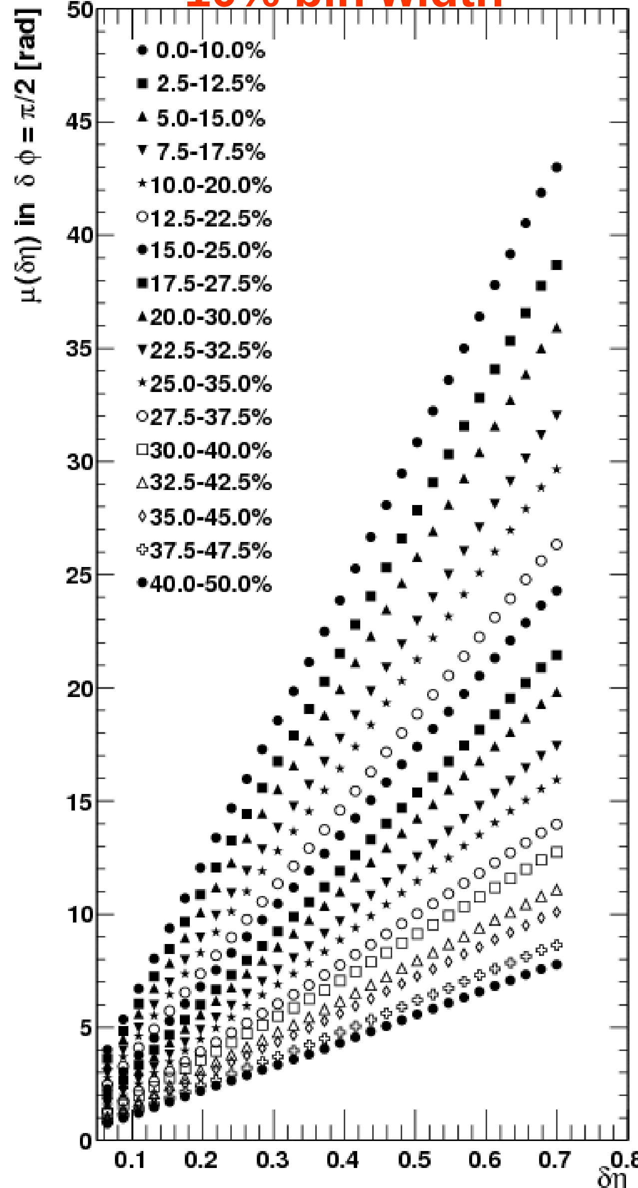
Cu+Cu@200GeV
10% bin width



Cu+Cu@200GeV
5% bin width



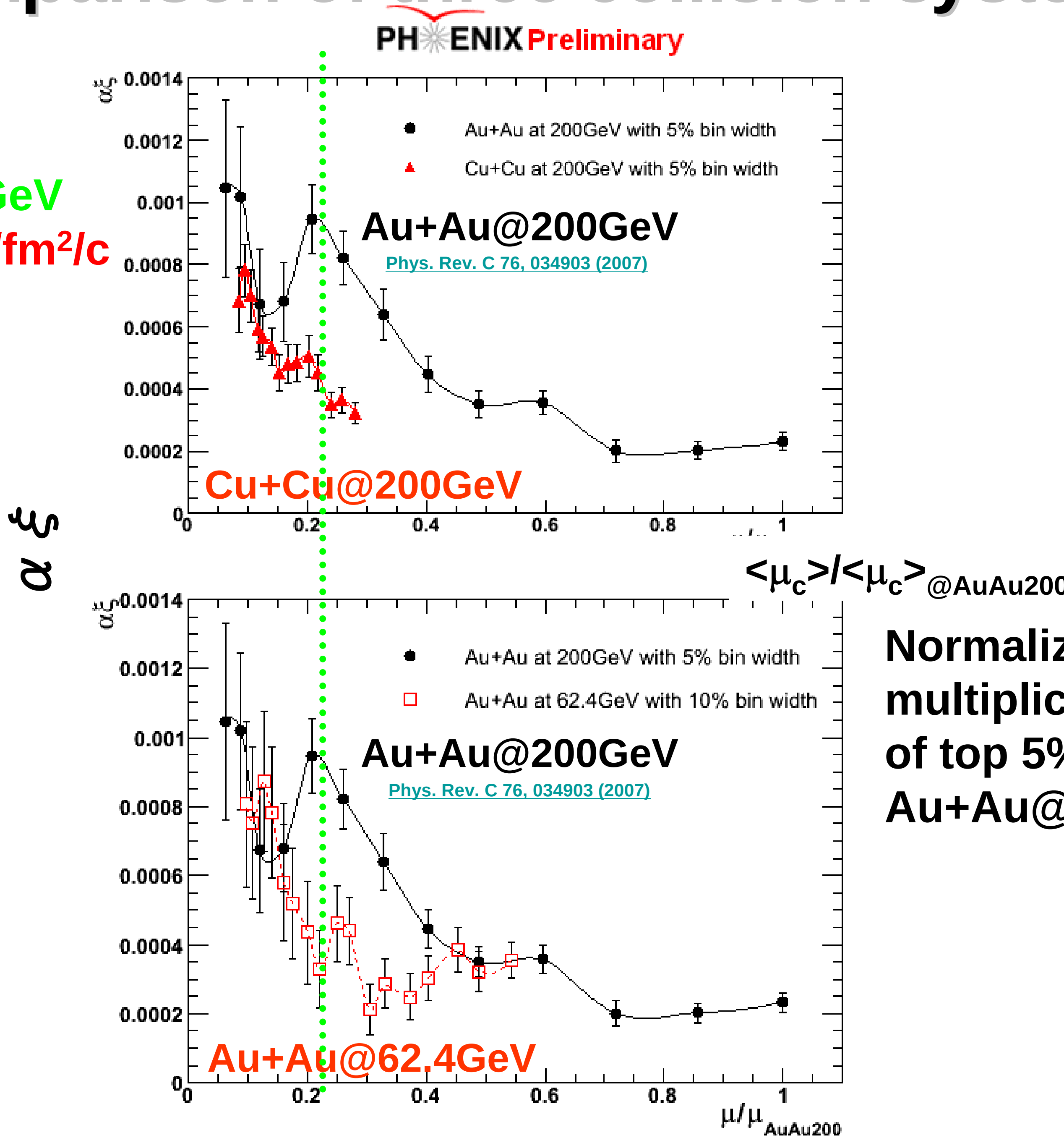
Au+Au@62.4GeV
10% bin width



Comparison of three collision systems

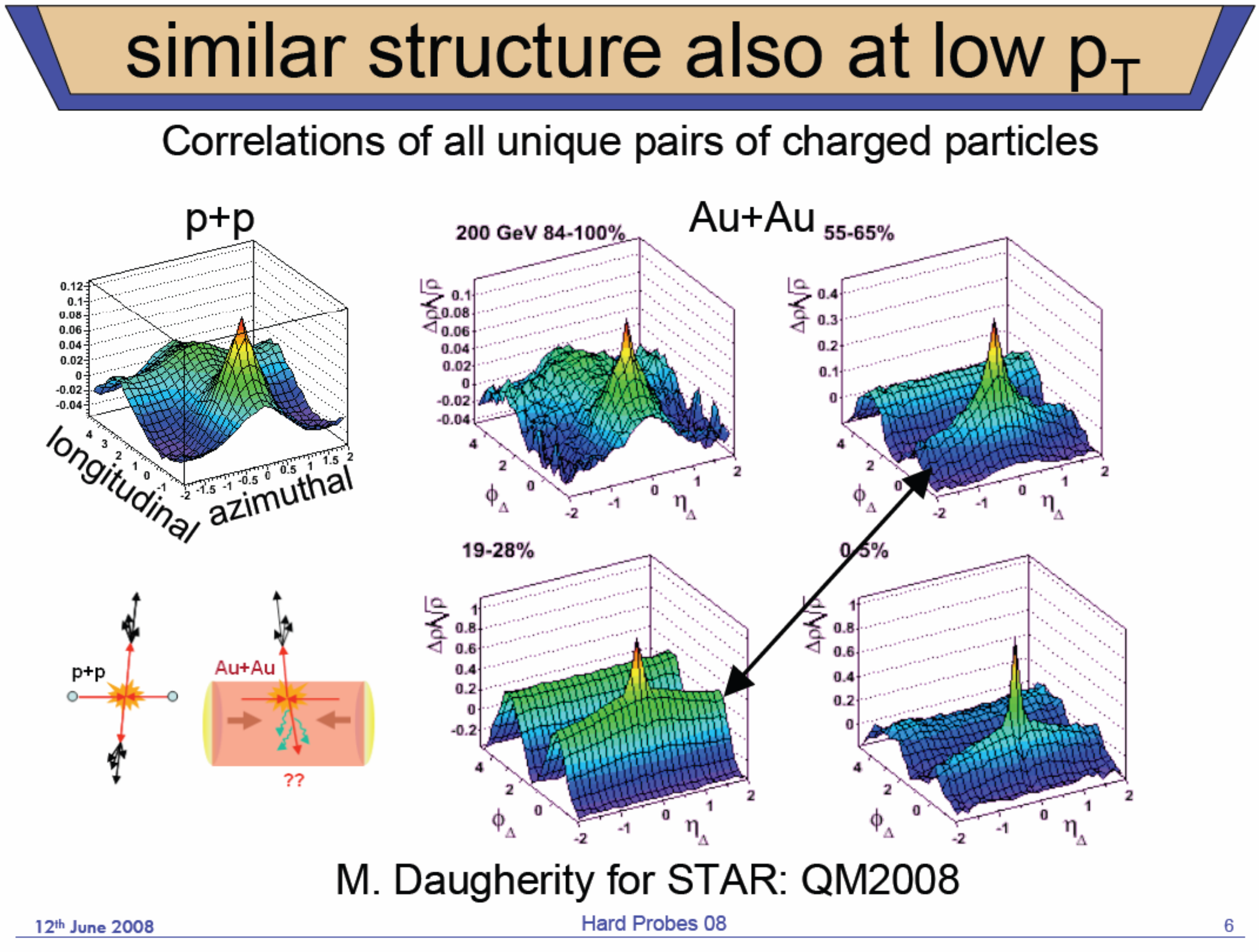
Npart~90 in AuAu@200GeV

$\varepsilon_{BJ}\tau$~2.4GeV/fm²/c

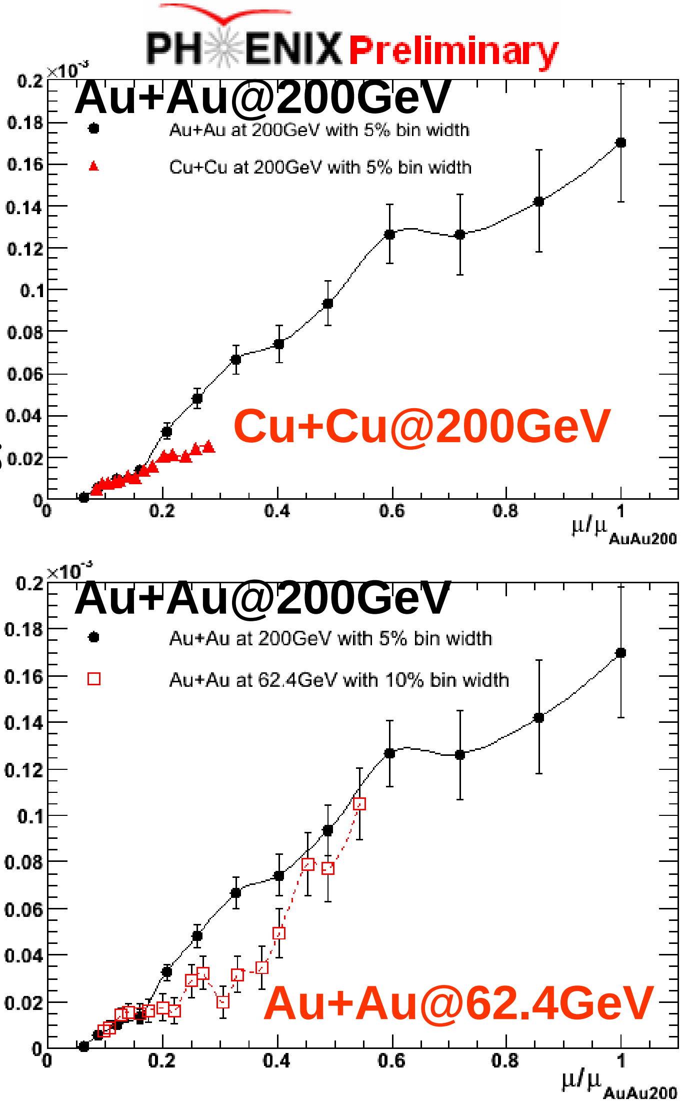


Normalized mean multiplicity to that of top 5% in Au+Au@200GeV

Slides from HP08 (P. Sorensen)

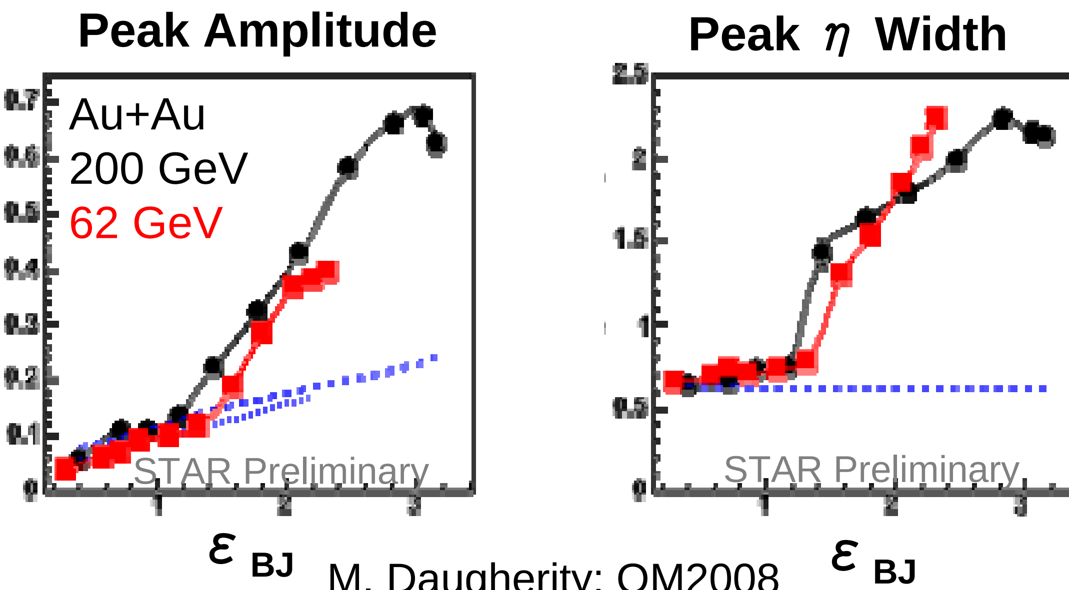


Similarity to STAR ridge results at low p_T



$\alpha\xi\mu^2$

$<\mu_c>/<\mu_c>_{@AuAu200}$



M. Daugherity: QM2008

Equivalent quantity;
$\chi T \propto \alpha\xi\mu^2 \propto$ amplitude x width
shows similar trends to what STAR sees.

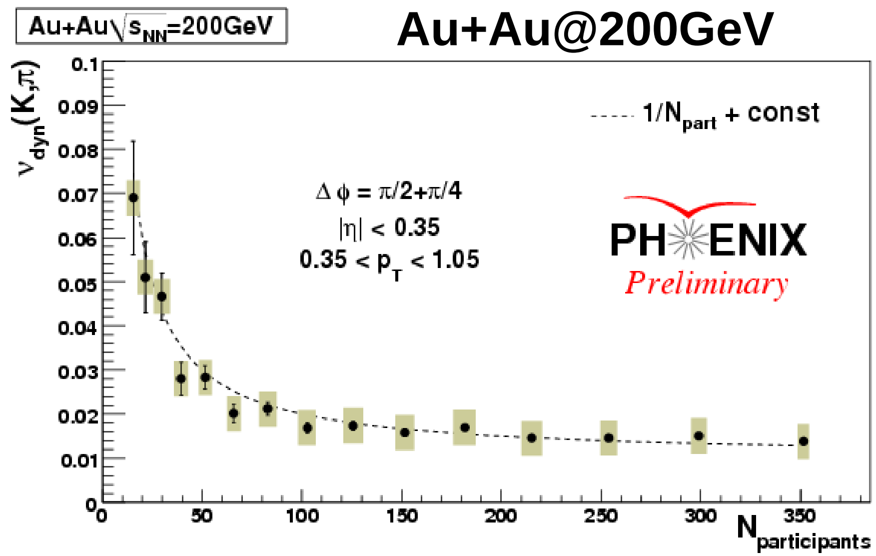
Do we see other peculiar centrality dependence?

- K to π and p to π fluctuations

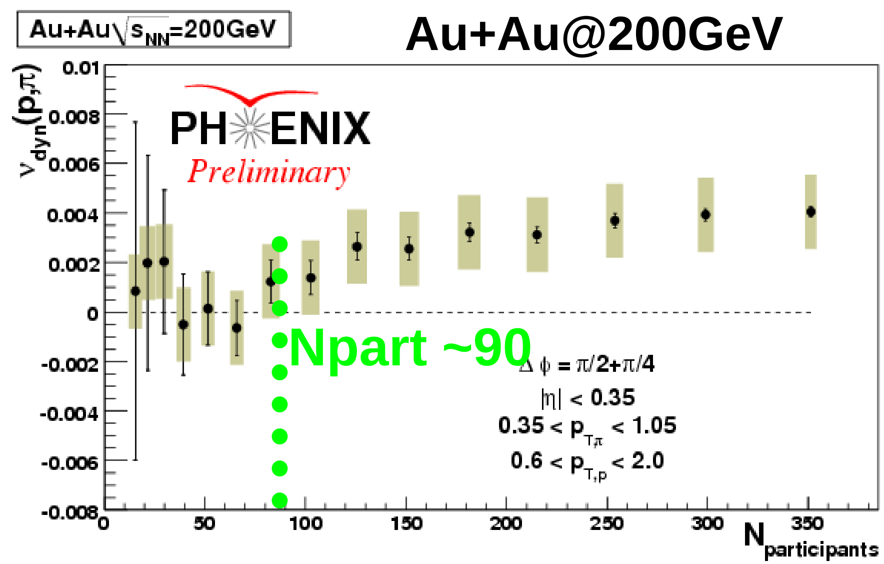
- Test of quark number scaling of v2 at low KE_T

Meson-meson and baryon-meson fluctuations

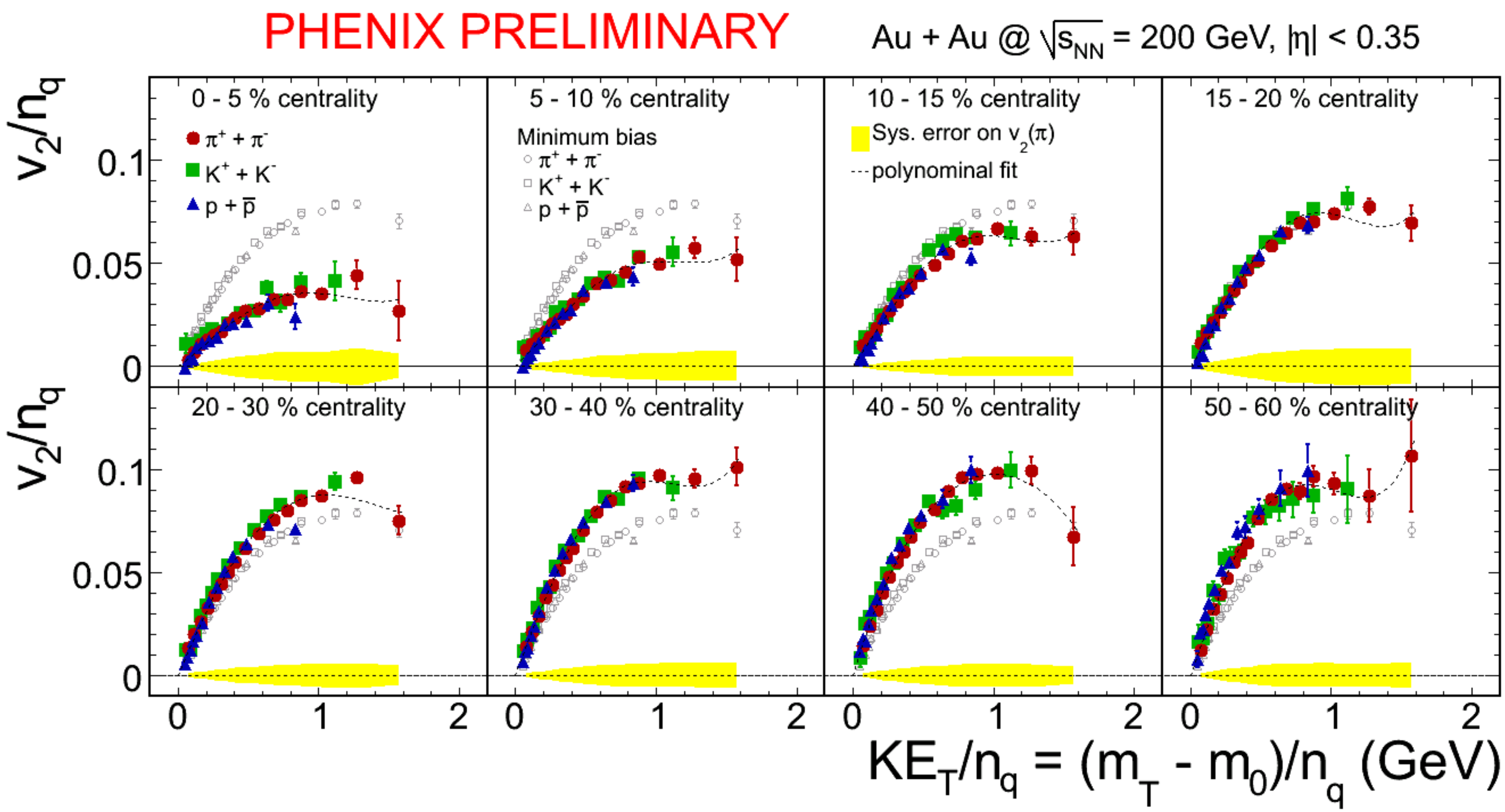
$$\nu_{dyn}(K,\pi) = \frac{\langle \pi(\pi-1)\rangle}{\langle \pi\rangle^2} + \frac{\langle K(K-1)\rangle}{\langle K\rangle^2} - 2\frac{\langle \pi\, K\rangle}{\langle \pi\rangle\langle K\rangle}$$



$$\nu_{dyn}(p,\pi) = \frac{\langle \pi(\pi-1)\rangle}{\langle \pi\rangle^2} + \frac{\langle p(p-1)\rangle}{\langle p\rangle^2} - 2\frac{\langle \pi\, p\rangle}{\langle \pi\rangle\langle p\rangle}$$

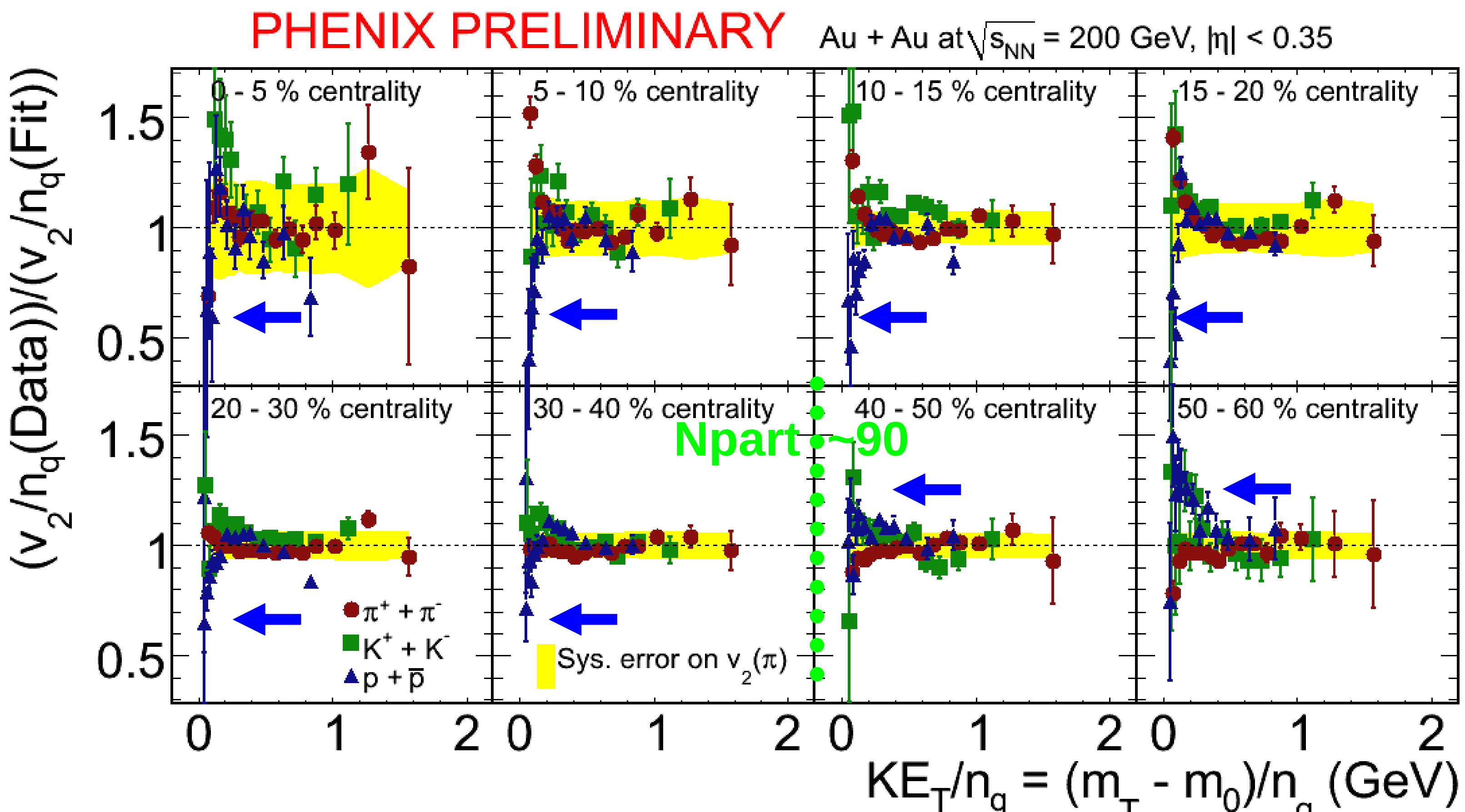


Number of constituent Quarks (NCQ) scaling of v2



- Scaling holds well for different centralitie

Deviation from scaling at low KE_T region ?



In lower KE_T, there seems to be different behaviors between baryon and mesons. The transition is at Npart~90.

Low mass sigma field may repulse pions and attract protons according to E.Shuryak hep-ph/0504048 .

Can this phenomenon be understood as such effects?

Summary

1. **Scaled variance shows no significant divergent behavior in all collisions systems achieved at RHIC so far.**
2. **The product between normalized amplitude and correlation length in two particle correlation function, $\alpha\xi$ as a function of Npart in Au+Au at 200GeV indicates a possible non monotonic increase at Npart~90. The trends of $\alpha\xi$ in smaller system in the same collision energy and in the same system size in lower collision energy as a function of mean multiplicity are similar to that of Au+Au at 200GeV except the region where the possible non monotonicity is seen. Since the exponential form is still valid to explain all k vs. $\delta\eta$, this implies that the system is not just on T=Tc yet. Transitions of the two point functional form from exp to power would be a measurable clear signature of the phase transition.**
3. **p to π fluctuation shows qualitatively different Npart dependence from K to π fluctuations.**
4. **At low KE_T below 0.3 GeV, the deviation of p from quark number scaled curve is opposite in sign to that of π and K. Interestingly the sign flips at around Npart~90.**