INT 08-2b "The QCD Critical Point" 2008.Jul.-Aug.

The possible quasi-particle picture of the quark near Tc and its effect of the dilepton production rate

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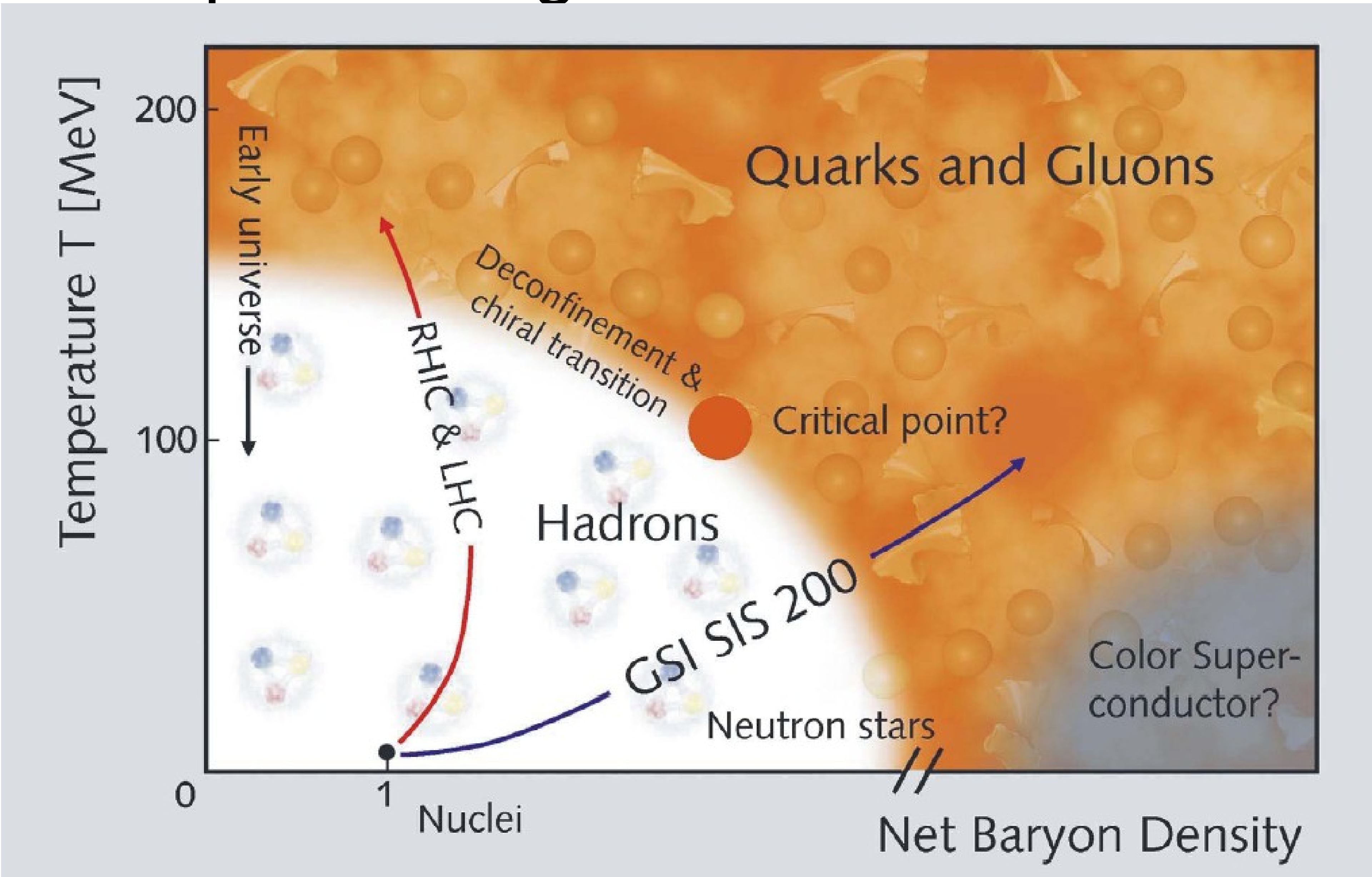
   - The possibility of the quasi-particle picuture near Tc,
2. Quark quasi-particle picture
   - Chiral soft mode and 3-peak structure, the effect of the finite quark mass
3. Dilepton production rate
   1. in the HTL scheme
   2. Our result

QCD phase diagram



QGP near Tc - sQGP

- Phenomenological side
  - Collective flow
  - Success of hydrodynamic model in the analysis of RHIC data
- Theoretical side
  - J/Psi and eta_c above Tc in lattice calculation
  - Hadronic excitation in model calculations

    (Hatsuda & Kunihiro, Shuryak &Zahed)

Interesting structure of vacuum. It must change the properties of fundamental degrees of freedom = quarks and gruons

There are only a few works on quark property near Tc while gluonic sector is studied by many people (potential, thermal mass…)

Quark near but above Tc

- [ph] quark number scaling of v_2 in RHIC
  - success of quark recombination models

2. [th] Success of 2-peak ansatz for LQCD result (at T/Tc = 1.5 ,3)
   - F.Karsch and M.Kitazawa (2007)
3. [th] Sharp peaks in the quark spectrum calculated by SD eq.
   - M.Harada, Y.Nemoto and S.Yoshimoto (2007); M.Harada, Y. Nemoto (2008)

PHENIX PRL 98, 162301

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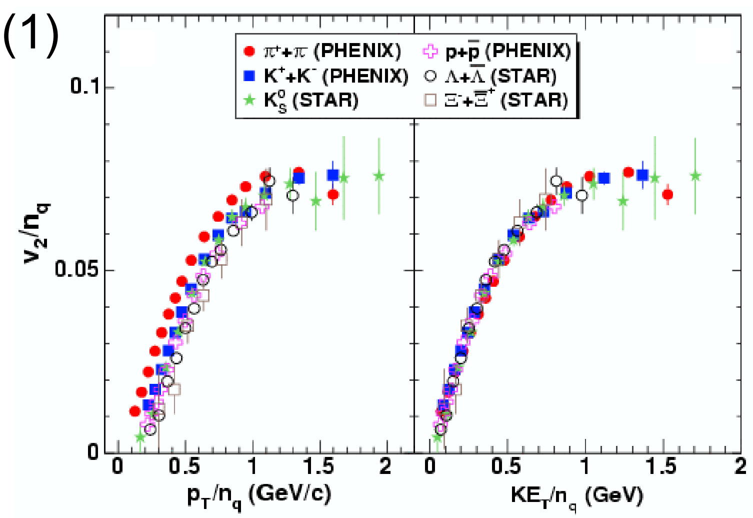

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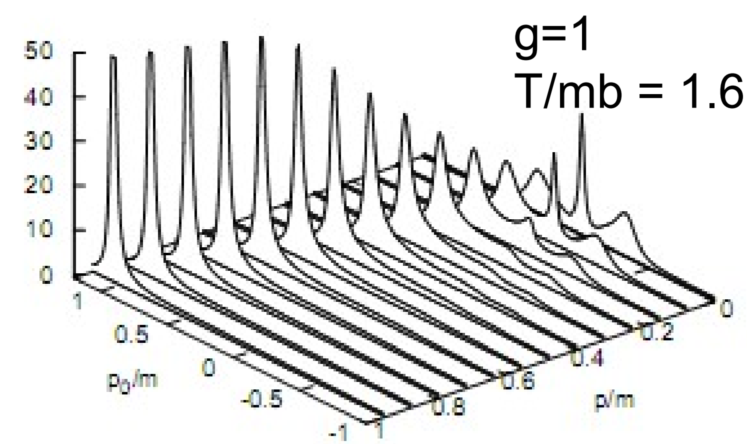
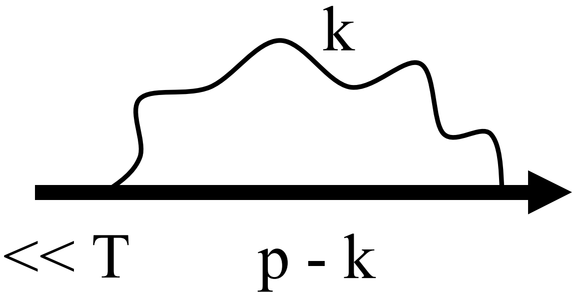


These result suggest the quark quasi-particle near Tc

HTL approximation

(E. Braaten and R. D. Pisarski 1990)

k ~ T is dominant in the loop integral

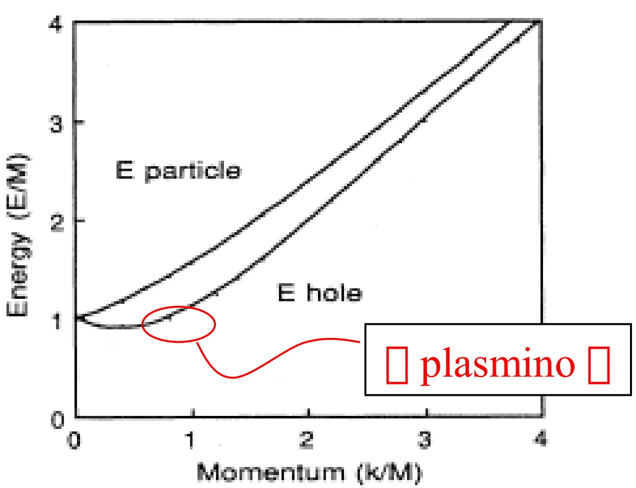


Available for $T >> m_f,\ p,\ \omega$

$$S^{\mathrm{HTL}}(\omega, \boldsymbol{p}) = [(\omega + i\eta)\gamma^0 - \boldsymbol{p}\cdot\boldsymbol{\gamma} - \Sigma^{\mathrm{HTL}}(\omega + i\eta, \boldsymbol{p})]^{-1}$$

$$\Sigma^{\mathrm{HTL}}(\omega, \boldsymbol{p}) = \frac{m_T^2}{p} Q_0\left(\frac{\omega}{p}\right)\gamma^0 + \frac{m_T^2}{p}\left(1 - \frac{\omega}{p} Q_0\left(\frac{\omega}{p}\right)\right)\boldsymbol{\gamma}\cdot\hat{\boldsymbol{p}}$$

$$Q_0 = (1/2)\ln(x+1)/(x-1) \qquad m_T^2 = (1/8)\, g^2 T^2 \ (\times C_F)$$



H.A.Weldon, PRD**40(1989)2410**

- Two branches of dispersion relation
- One have minimum : `plasmino'
- The mass is proportional to T : `Thermal mass'
- Chiral symmetric mass

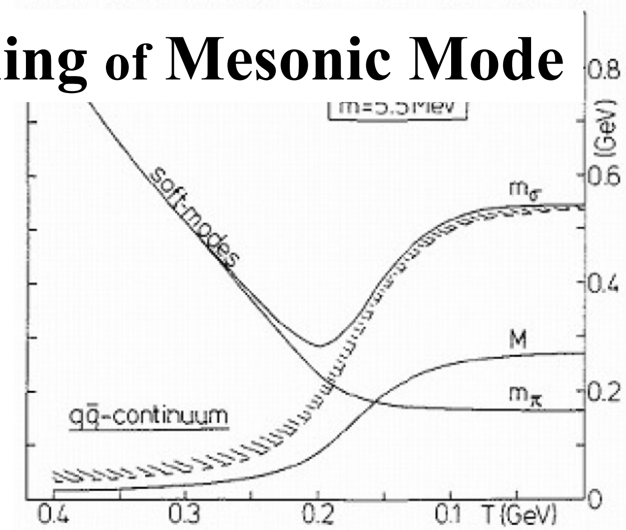
(first appear in V. V. Klimov, Yad. Fiz **33**, 1734 (1981))

•Chiral Soft Mode

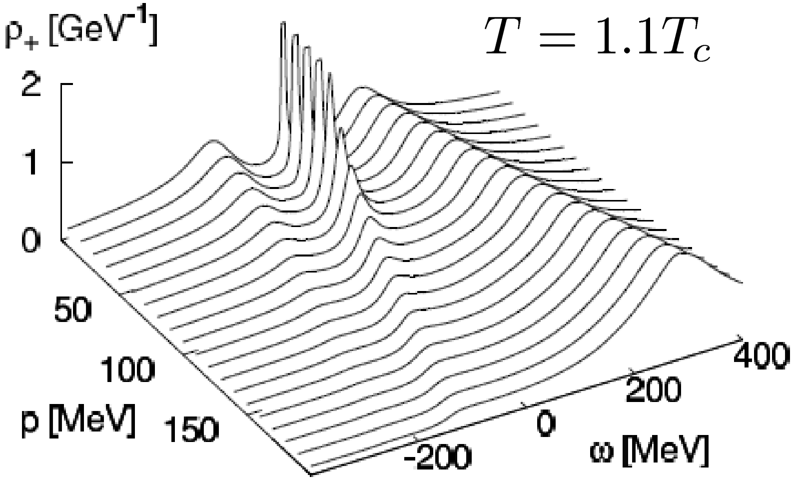
What is important near Tc is fluctuation of the order parameter, i.e., chiral soft mode, for the 2nd or nearly 2nd order P. T.

 T.Hatsuda and T.Kunihiro, PRL55, 158('85)

•Softening of Mesonic Mode



•Quark Spectrum Near And Above Tc



3-peak structure

→ Understood form Level Mixing

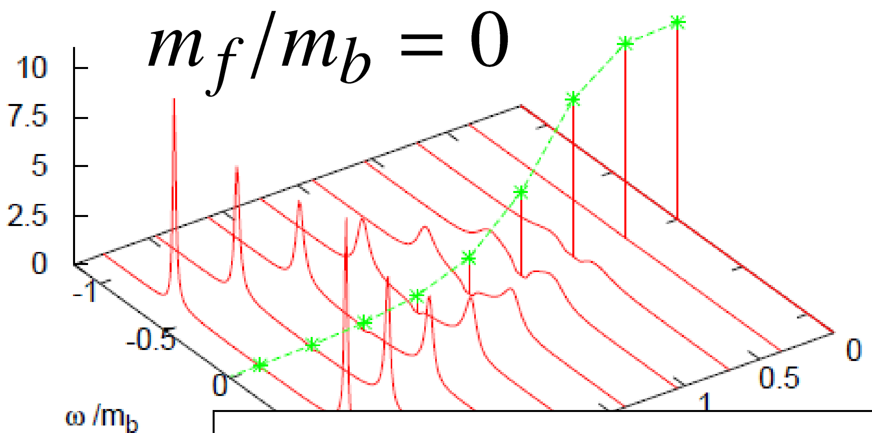
In NJL and Yukawa Model
$m_q = 0$ (chiral limit)

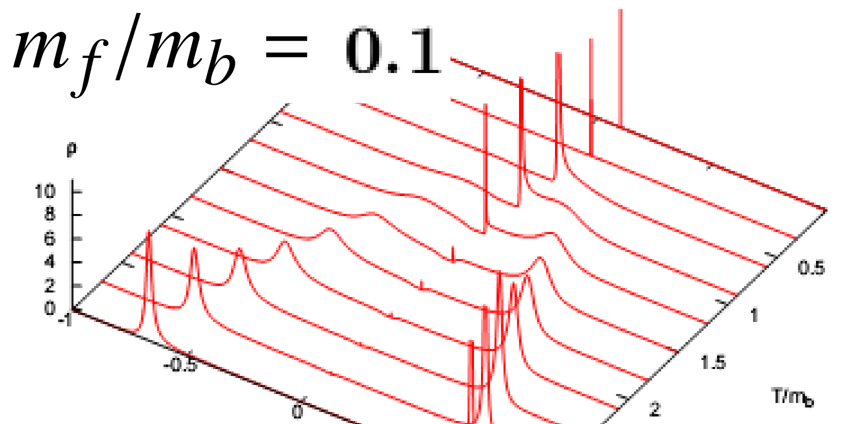
M.Kitazawa, T. Kunihiro and Y. Nemoto. Phys.Lett. **B633** (2006) 269; Prog.Theor.Phys.**117** (2007)103,

- But real quarks have finite masses
  - (constituent quark mass, thermal mass, current mass)

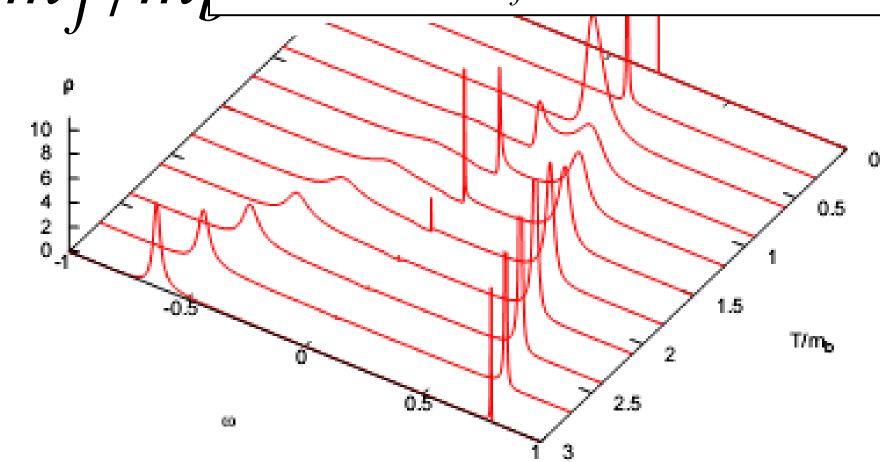
We investigate the effect of mass in a Yukawa model

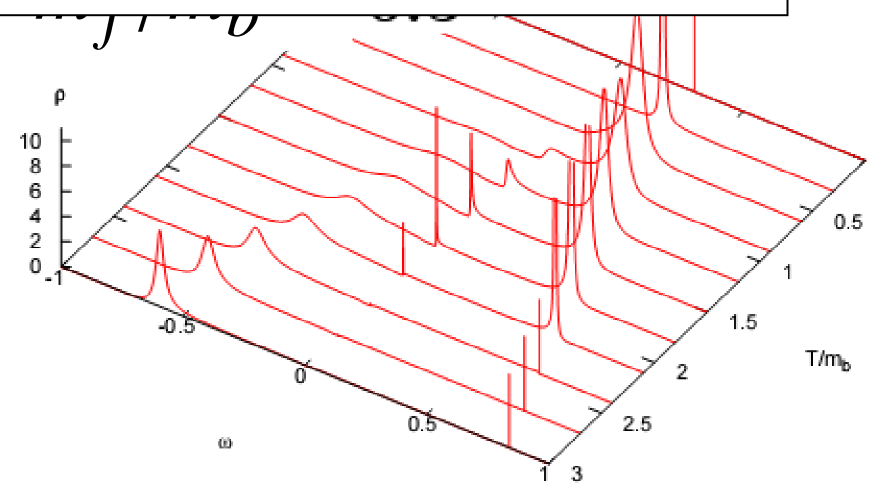
Spectral functions with finite m_f





With finite m_f, 3-peak structure gradually ceases because the negative energy excitation is suppressed for finite m_f.

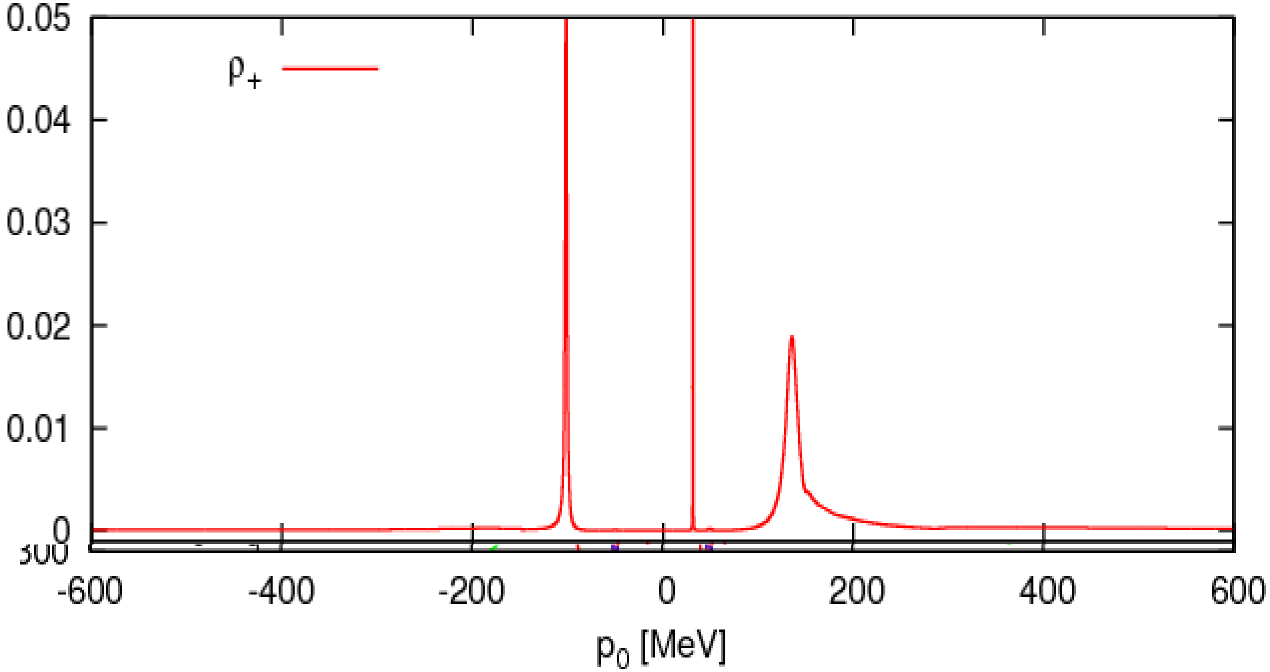




M. Kitazawa, T. Kunihiro, KM and Y. NemotoPhys.Rev.D77:045034,2008

Finite m_f NJL model (Y. Nemoto, M. Kitazawa, T. Kunihiro)

$$\mu = 0, T = T_{PC}$$



The pseudo-critical line is determined from a maximum of the spectral function for p=10 MeV (dynamic chiral susceptibility).

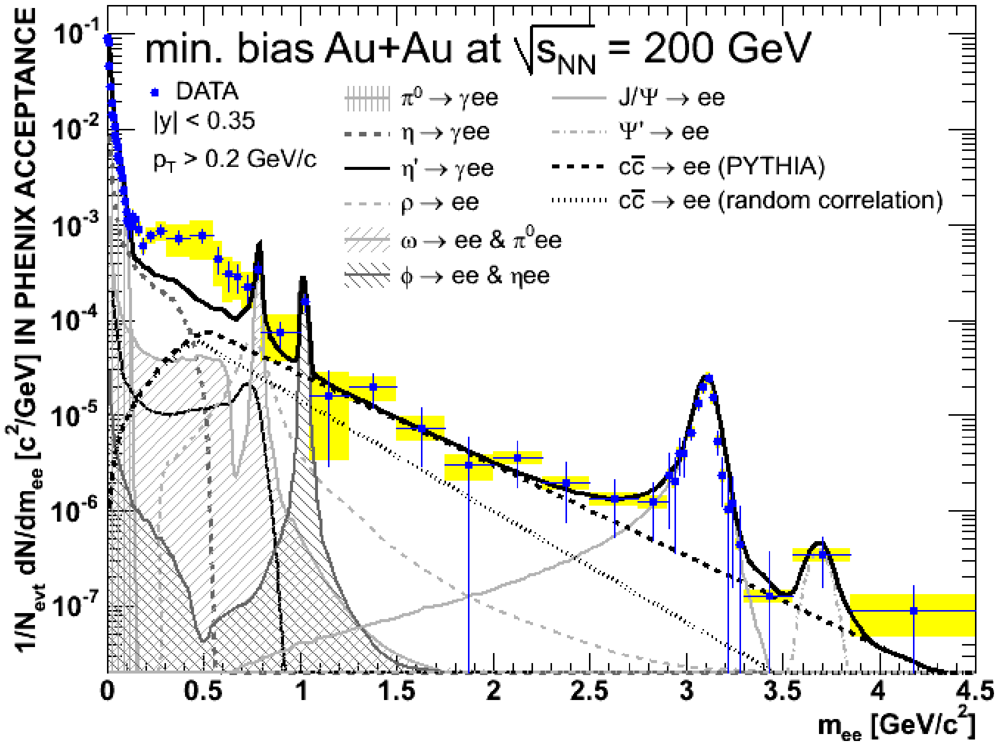
Although the finite quark mass tend to suppress the 3-peak structure
3-peak structure survive in the NJL model with the finite m_f for the van Hove
singularity caused by the change of the meson di

Can be observed ?

Dilepton production as a probe

- Clean from final-state interaction
- Recently experimental results at RHIC are available

PHENIX arXiv:0706.3034



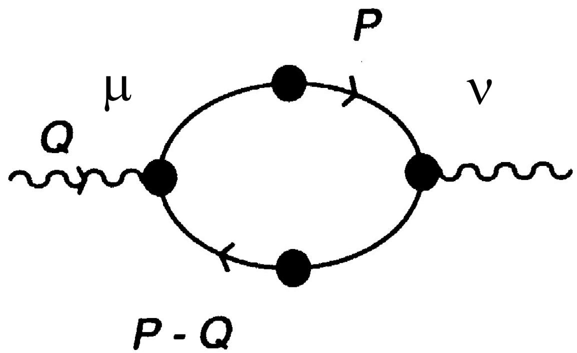
Dilepton production rate and quark quasi-particle in HTL approx.

E. Braaten, R. D. Pisarski and T. C. Yuan ('90)

$$\frac{d\Gamma}{dq_0 d^3q} = \frac{\alpha}{12\pi^4}\frac{1}{Q^2}\frac{1}{e^{\beta q_0}-1}\mathrm{Im}\Pi^{\mu}_{\mu}(q_0+i\eta,\vec{q})$$

$$\propto \qquad P_{++}(q_0) + P_{+-}(q_0) + P_{--}(q_0)$$

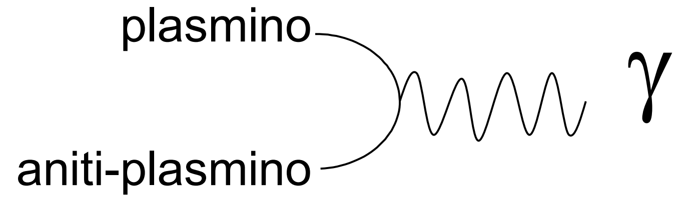
$$P_{ij}(q_0) = \int_0^\infty p^2 dp \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} d\omega' \delta(q_0-\omega-\omega')\Lambda_{ij}(\omega,\omega',p)\rho_i(\omega,p)\rho_j(\omega',p)$$

$\Pi^{\mu\nu}(Q)=$ 

$\rho = \rho_+\Lambda_+ + \rho_-\Lambda_-$

Λ: quark number projection

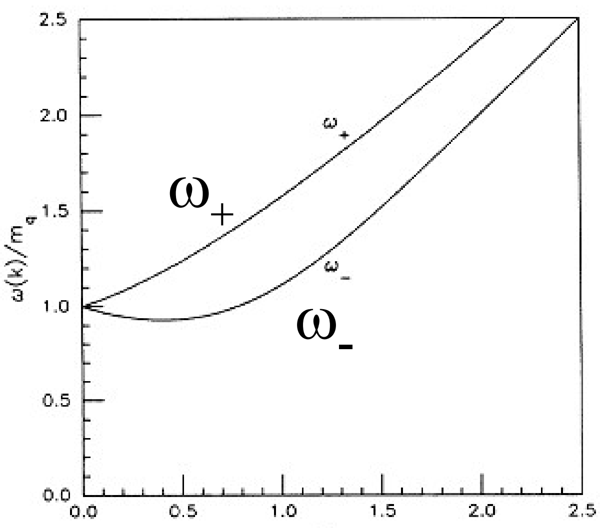
For example



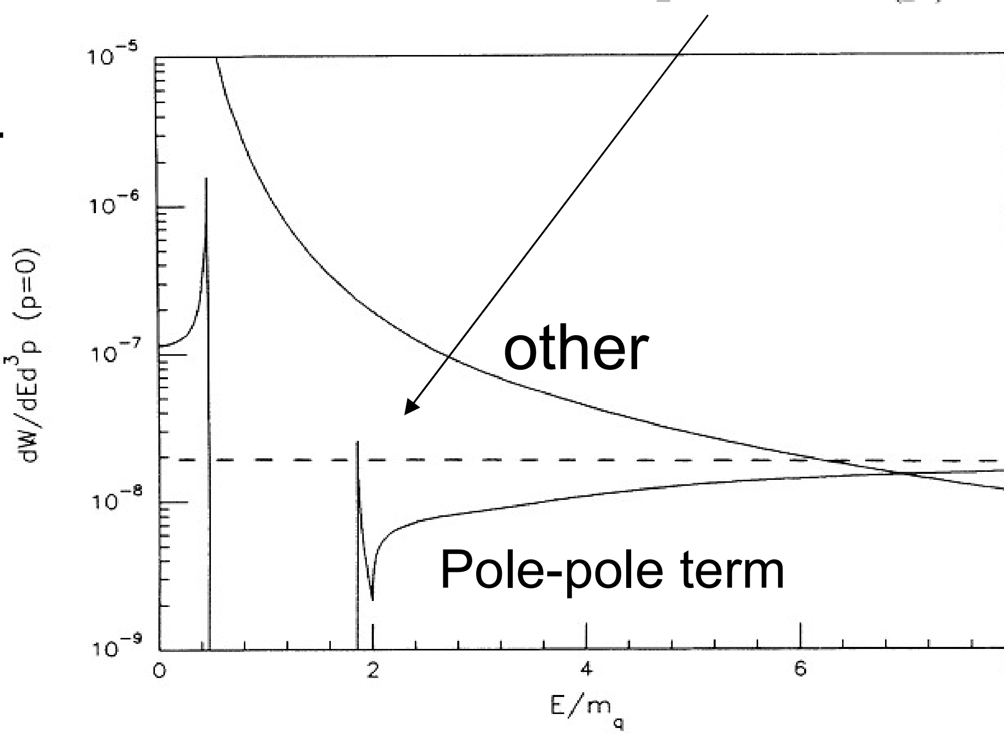
$$P_{--}(q_0) \sim \int_0^\infty p^2 dp \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} d\omega' \Lambda_{--}(\omega, \omega', p)\delta(\omega - \omega_-(p))\delta(\omega' - \omega_-(p))$$
$$= \int_0^\infty dp p^2 \Lambda(\omega_-(p), \omega_-(p), p)\delta(q_0 - 2\omega_-(p))$$
$$= \int_0^\infty dp p^2 \Lambda(\omega_-(p), \omega_-(p), p)\frac{1}{2\,|\partial_p \omega_-(p)|}\delta(p - \tilde{p}(q_0))$$

Divergence of the DoS

$$D(\omega) \propto \frac{dp}{d\omega}$$



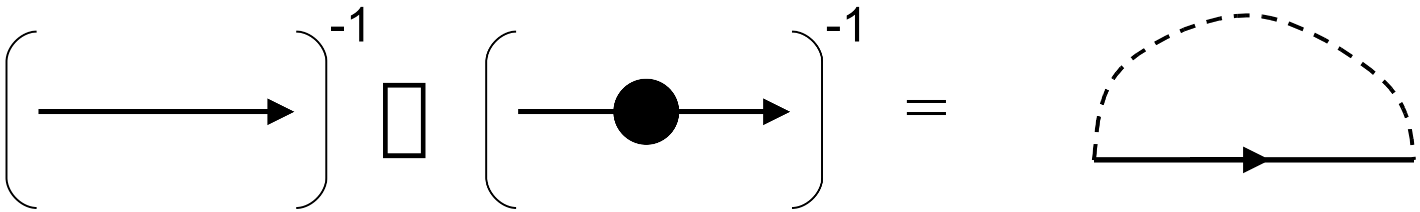
$q_0 - 2\omega_-(\tilde{p}) = 0$



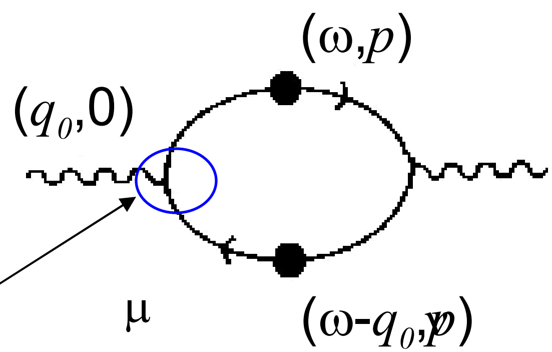
Dilepton Production rate in a Yukawa model

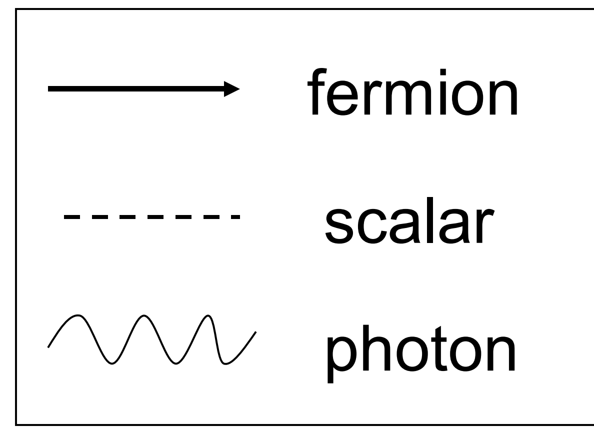
In collaboration with M. Kitazawa (Osaka) and T. Kunihiro (Kyoto)

mf = 0 , mb ≠ 0, fermion is coupled with scalar and photon field



$$\Pi^{\mu\nu}(q_0, q=0) =$$





- though we must use effective vertex to keep gauge invariance, we adopt bare one for numerical simplicity here.

Dilepton production rate

$$\frac{d\Gamma}{dq_0 d^3 q} = \frac{\alpha}{12\pi^4} \frac{1}{Q^2} \frac{1}{e^{\beta q_0} - 1} \mathrm{Im}\Pi^{\mu}_{\mu}(q_0 + i\eta, \vec{q})$$

Approx. for the spectral function

Numerical calculation is heavy if calculating naively

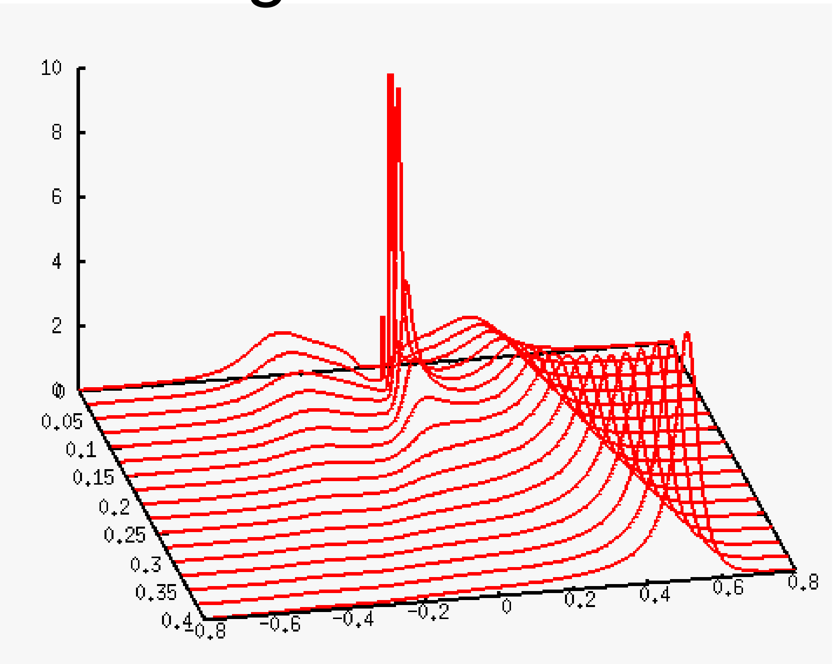
Breit-Wigner type approximation

$$\rho_{\text{approx.}}(\omega, p) = \sum_{i=1,2,3} \frac{1}{\pi} \frac{Z_i(p)\Gamma_i(p)}{(\omega - E_i(p))^2 + \Gamma_i^2}$$

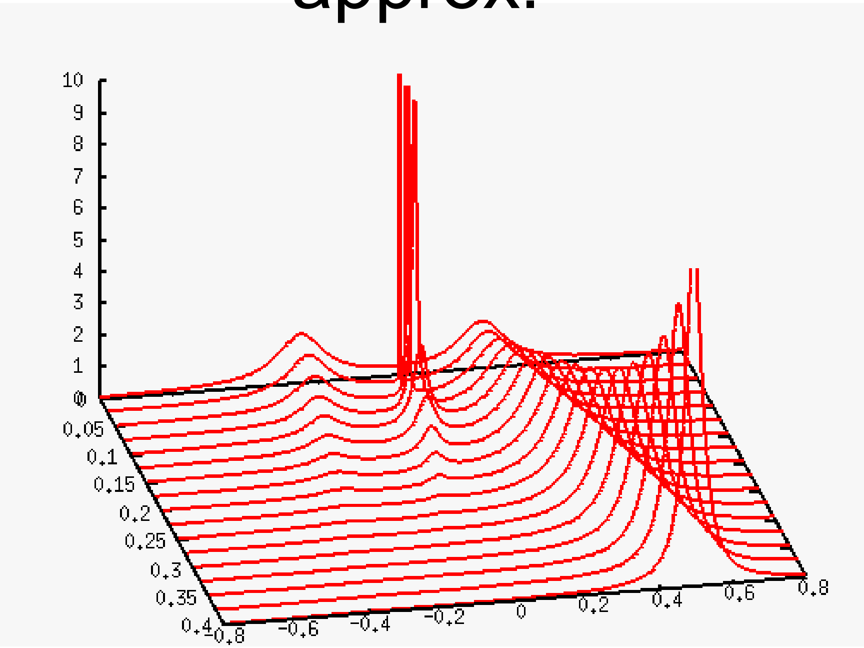
- For each p fit the parameters (E_i, G_i, Z_i) to $\rho(\omega,p)$
- Assume an analytic form for each function ($E_i(p)$ etc), and fit that to the result got in step 1

Approx. spectral function ($T/m_b = 1.13$)

original

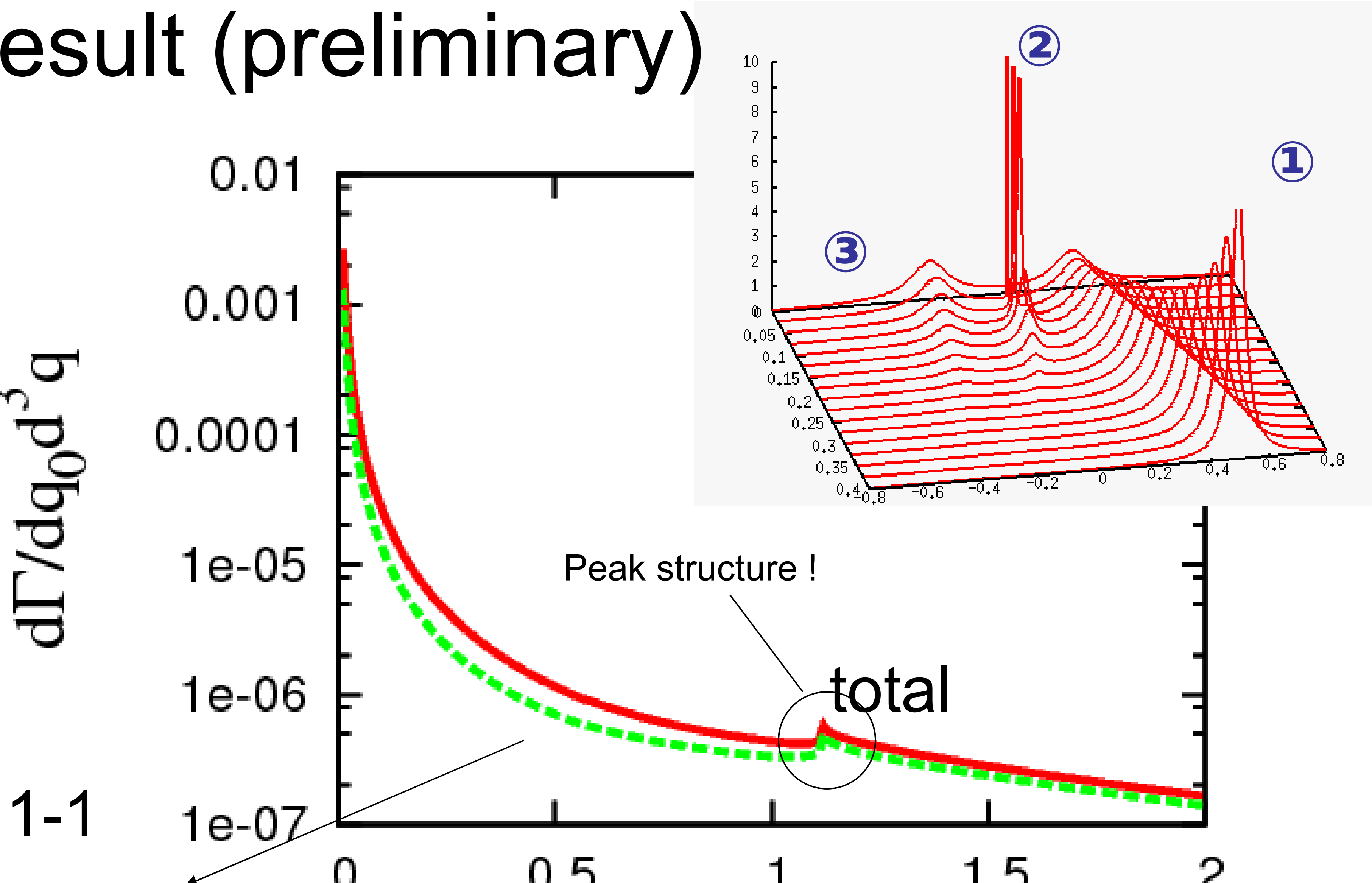


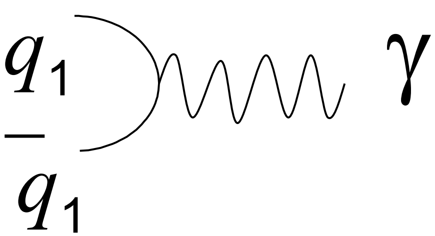
approx.



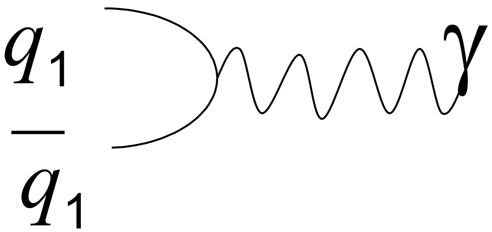
Approximated form of the spectral function well reproduce the qualitative features of the original spectral function

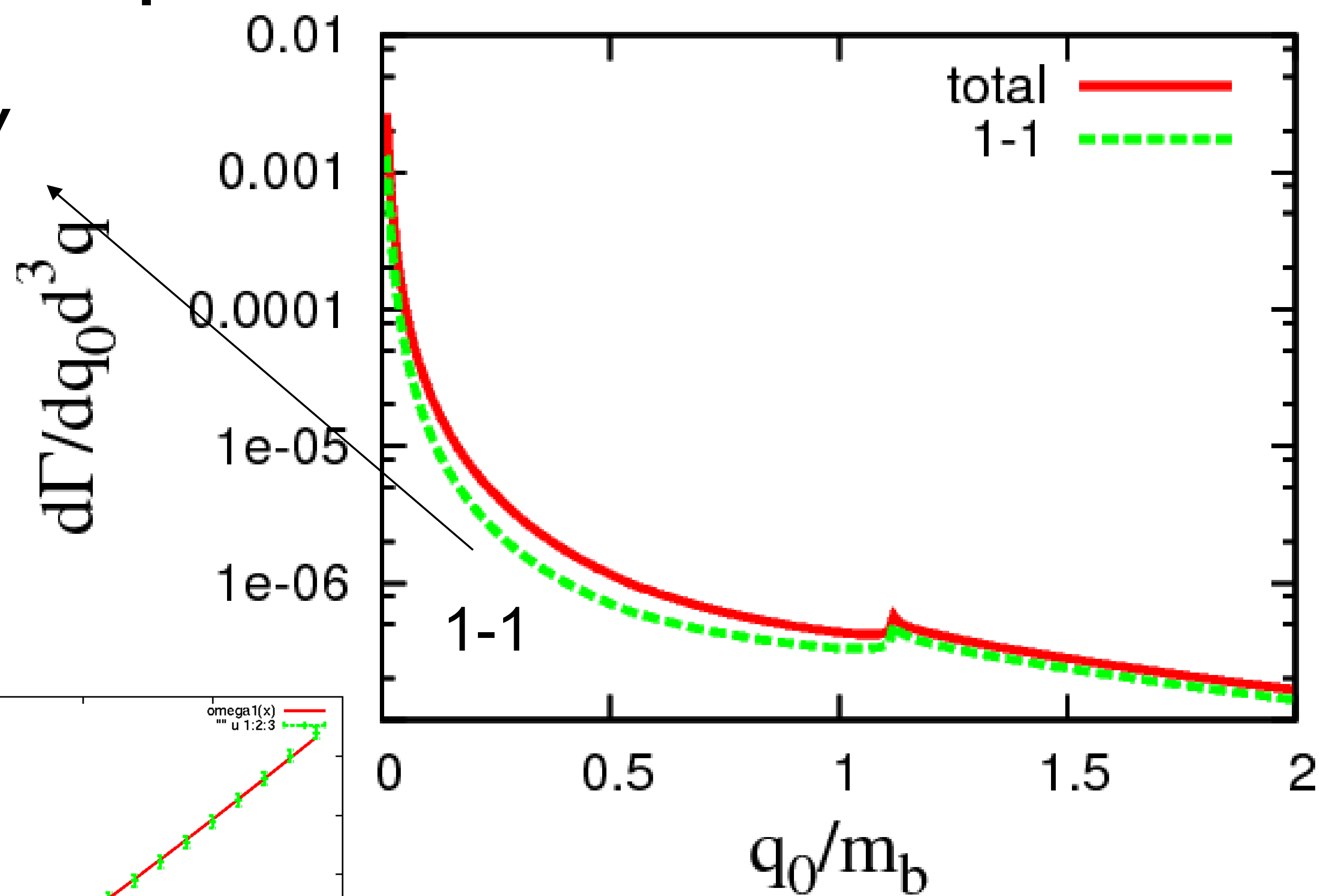
Result (preliminary)

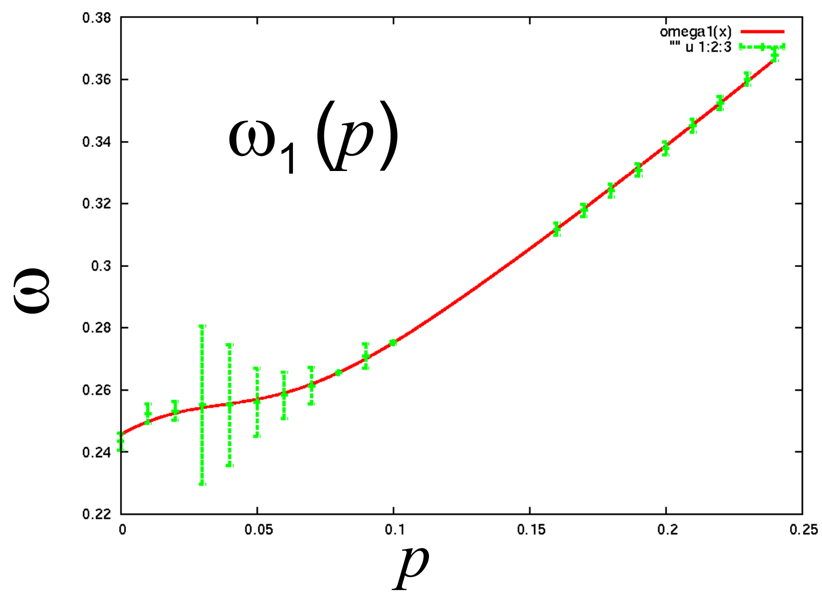




Physical interpretation







there is no minimum,
so it is not a van Hove singularity

A possibility : resonance

Summary

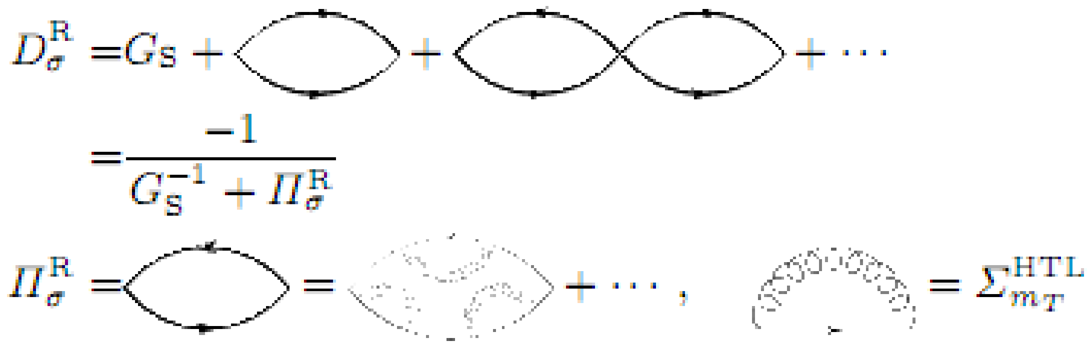
- 3-peak structure of quark may cause a `peak` in the photon decay rate.
  - Which indicate that fermion-scalar interaction cause a resonance of fermion-anti-fermion pair
- Result is generic : taking the density fluctuation as the soft mode, the same discussion can be near CP.

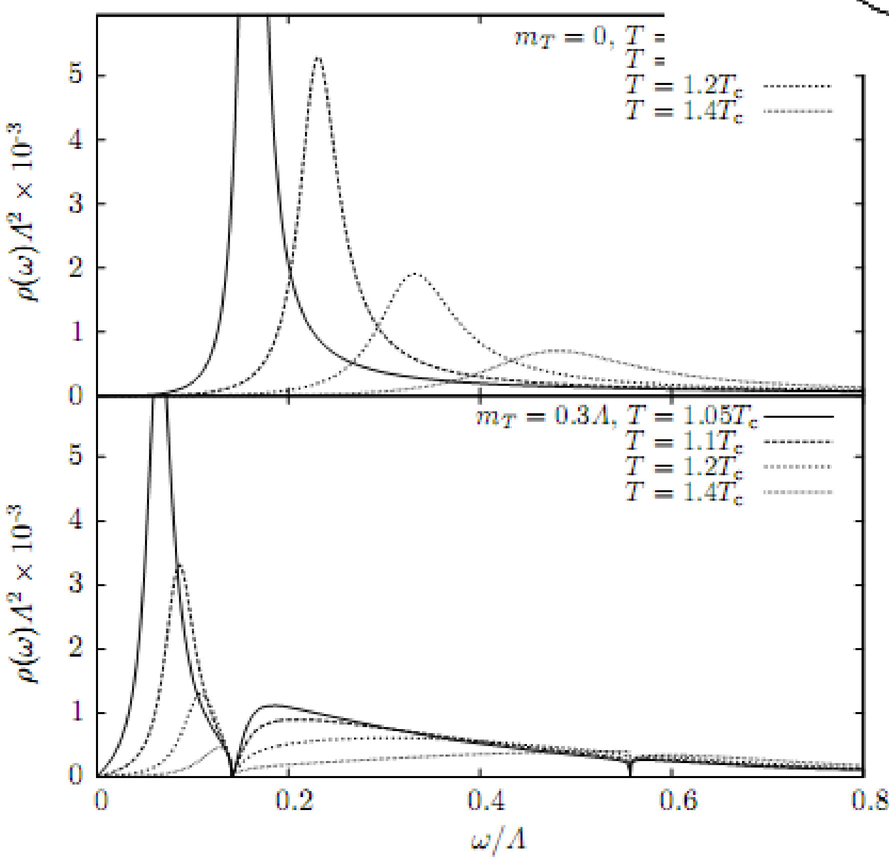
Future Work

- Physical interpretation
- Vertex correction

Back Up

meson by HTL quark pair





- the mesonic modes can exist even if $(p^{\mu})^2 < (2\, m_T)^2$

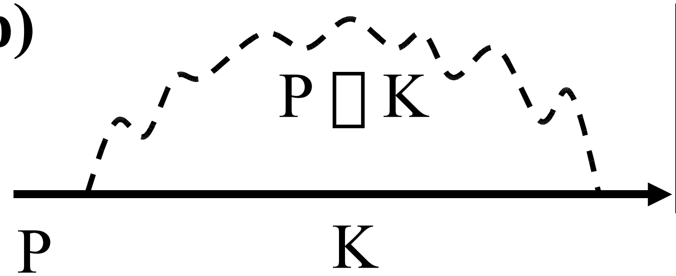
Model And Approximation

$$\mathcal{L} = \bar{\psi}(i\gamma\cdot\partial - m_f)\psi + \frac{1}{2}[(\partial_\mu\phi)^2 - m_b^2\phi^2] - g\phi\bar{\psi}\psi$$

Yukawa coupling of scalar filed and fermion

•**Self Energy (1-loop)**

$$\Sigma(\omega, \vec{p}) =$$



$\boxed{p = 0 \text{ only}}$

- to concentrate on the *T*-dependence
- plot on the w-*T* plane

Quark Green function

$$G^R(\omega) = \left[(\omega + i\eta) - m_f - \Sigma^R(\omega)\right]^{-1}$$

g = 1 : qualitative behavior is the same irrespective of g

The spectral function

$$\mathcal{A}(p_0,\vec{0}) = -\frac{1}{\pi}\mathrm{Im}\mathrm{G}^{\mathrm{R}}(p_0,\vec{0})$$

projection op. $\Lambda_{\pm}(\vec{0}) = \dfrac{1 \pm \gamma_0}{2}$

$$= [\rho_+\Lambda_+(\vec{0}) + \rho_-\Lambda_-(\vec{0})]\gamma_0$$

Deal with only positive number component
cf : parity property

$$\rho_+(\omega, p) = \rho_-(-\omega, p)$$

$$\rho_+(\omega) = -\frac{1}{\pi}\frac{\mathrm{Im}\Sigma_+^R(\omega)}{(\omega - m_f - \mathrm{Re}\Sigma_+^R(\omega))^2 + \mathrm{Im}\Sigma_+^R(\omega)^2}$$

$$\Sigma = [\Sigma_+\Lambda_+(\vec{0}) + \Sigma_-\Lambda_-(\vec{0})]\gamma_0$$

$$\mathrm{Re}\Sigma_+ = \omega - m_f$$

□□□ □ □ Im Σ_+ (ω) □□□□□□□□□□□□□□

Poles of the quark propagator

Poles are found by solving $$z - m_f - \Sigma_+^R(z) = 0$$

The residue at pole which indicate the strength of the excitation

$$Z = \left[1 - \frac{\partial \Sigma_+^R(z)}{\partial z}\right]\Bigg|_{z=z_+}$$

Pole approximation of the spectral function

$$\rho^{\text{pole}}(\omega) = \sum_{i=1}^{3} \left(-\frac{1}{\pi}\right) \text{Im}\left[\frac{Z_i}{\omega - z_i}\right]$$

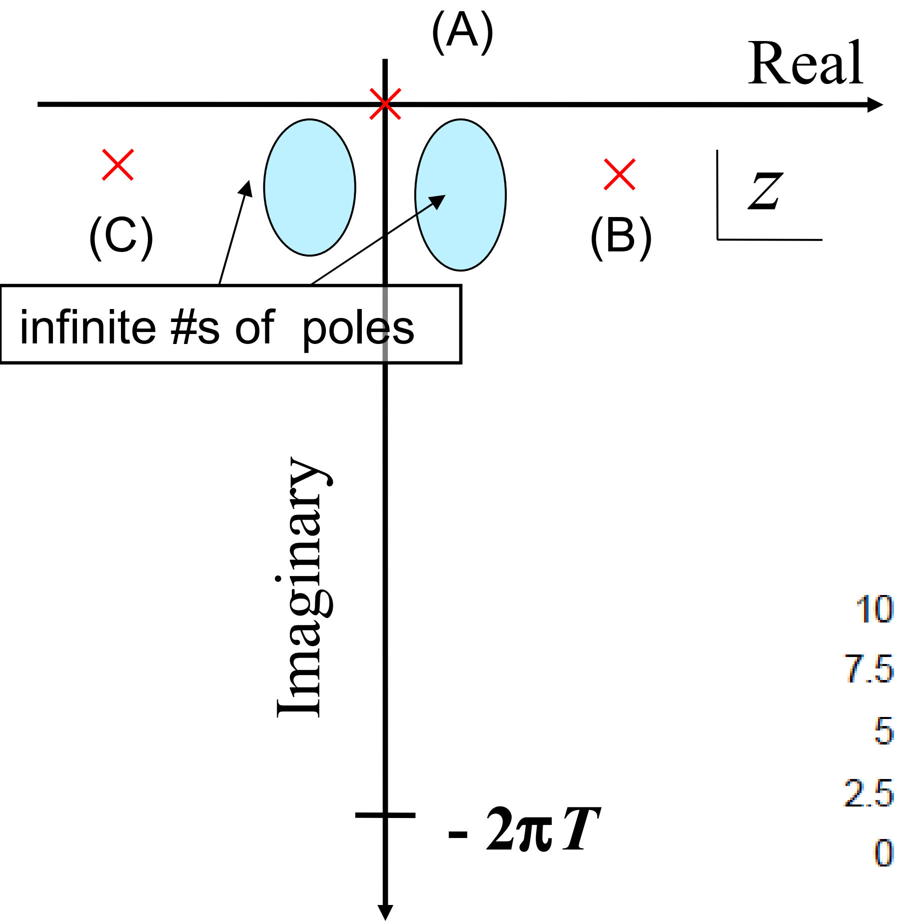
A sum rule for the fermion spectral function

$$\int_{-\infty}^{\infty} d\omega \rho_+(\omega) = 1$$

If pole approximation is good

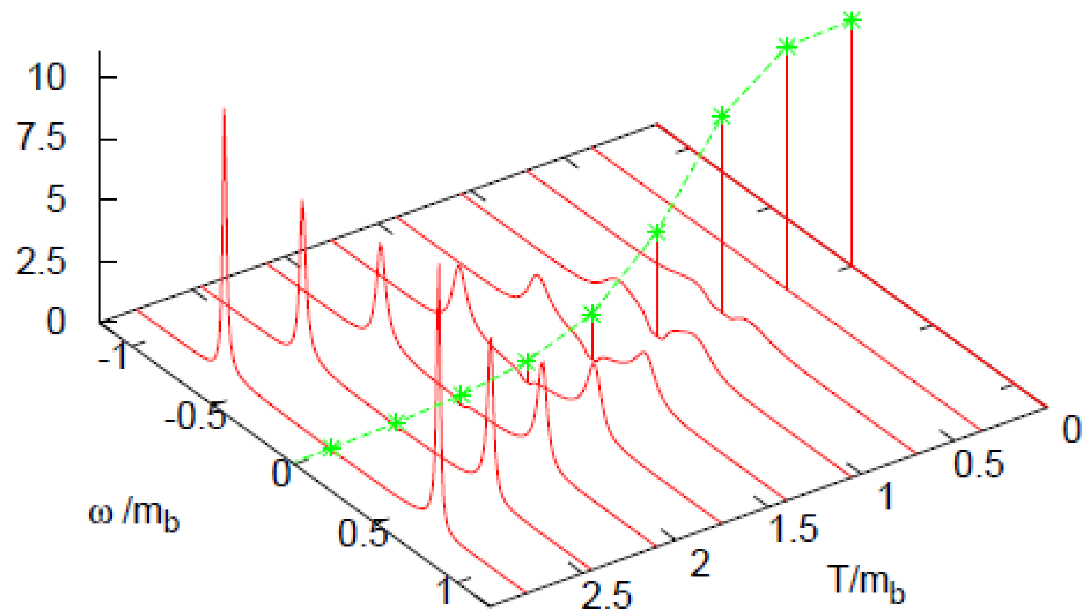
$$\Longrightarrow \quad \sum_i \text{Re} Z_i \simeq 1$$

Pole Structure Of Propagators ($m_f = 0$)

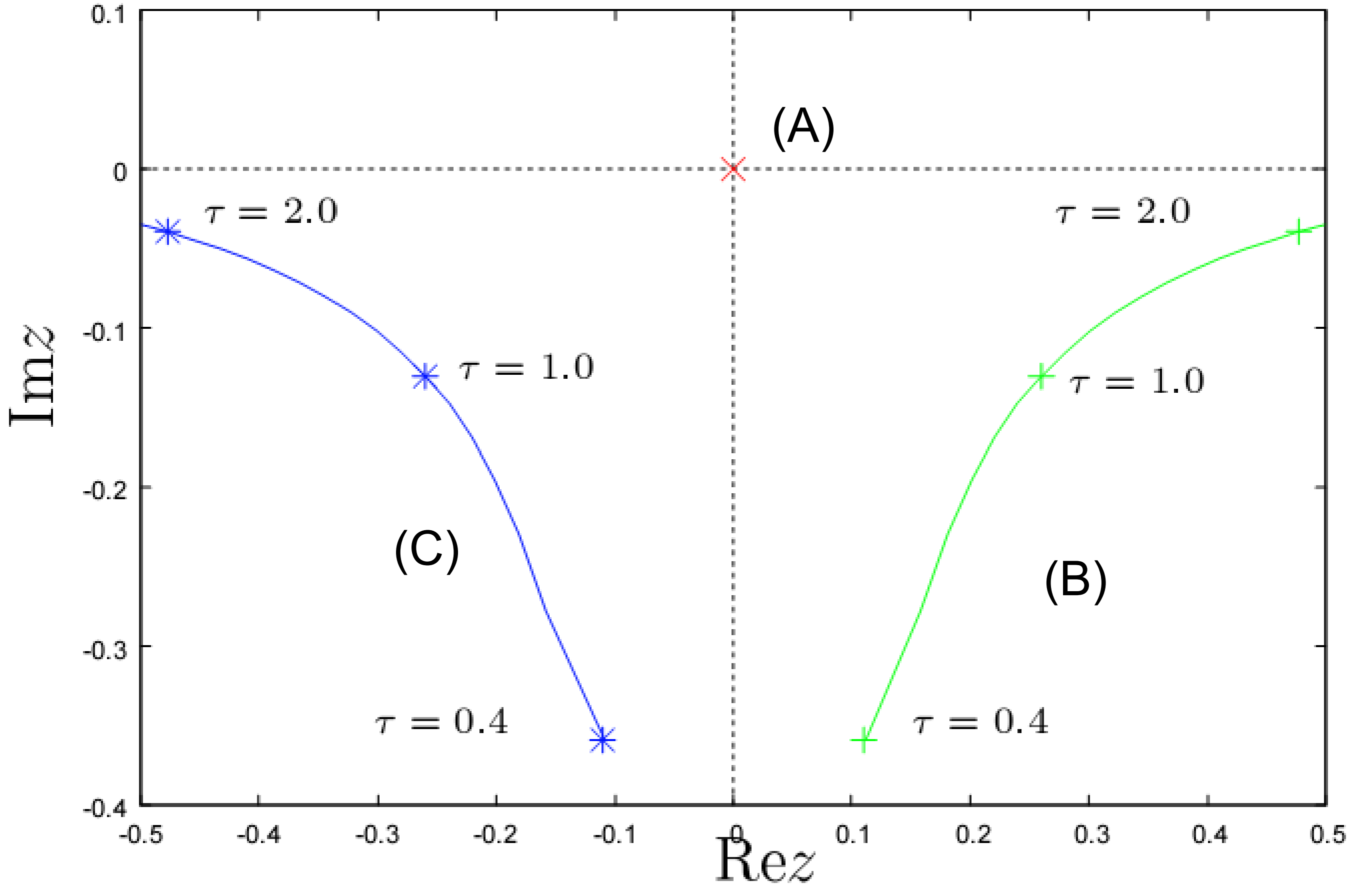


z: complex energy variable

- Poles of the retarded functions in the lower half plane

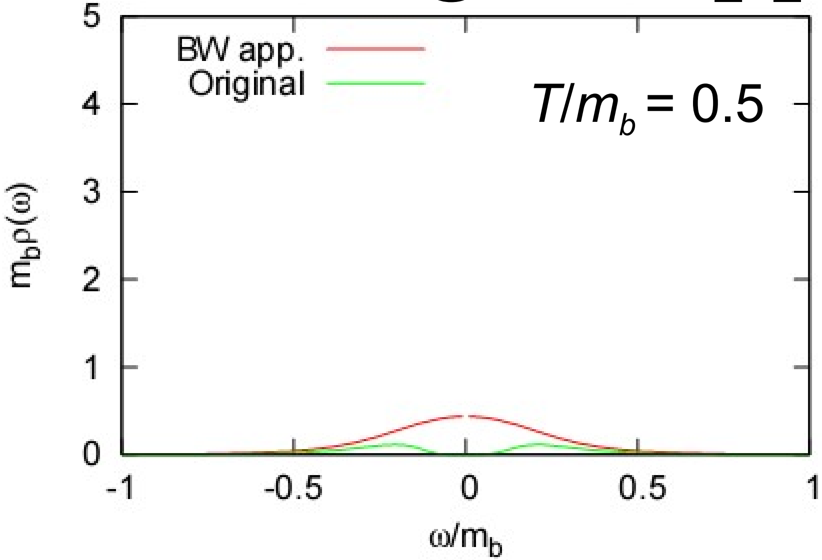


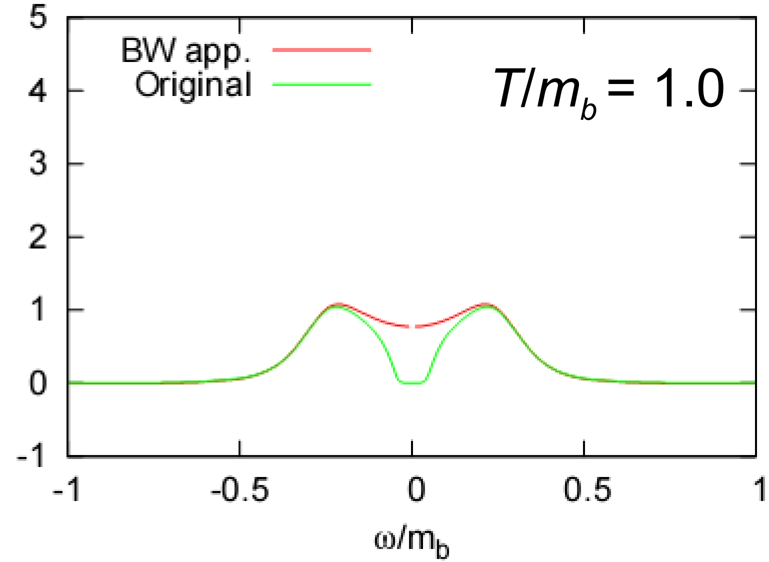
How the poles move

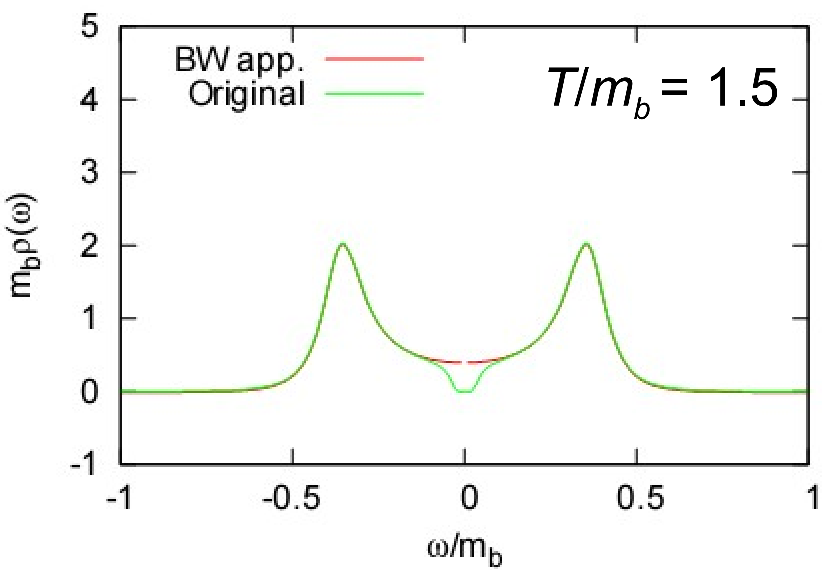


- pole (A) stay at the origin irrespective of *T*
- Imag. parts of the poles (B) and (C) become smaller as *T* is raised

Breit-Wigner approximations



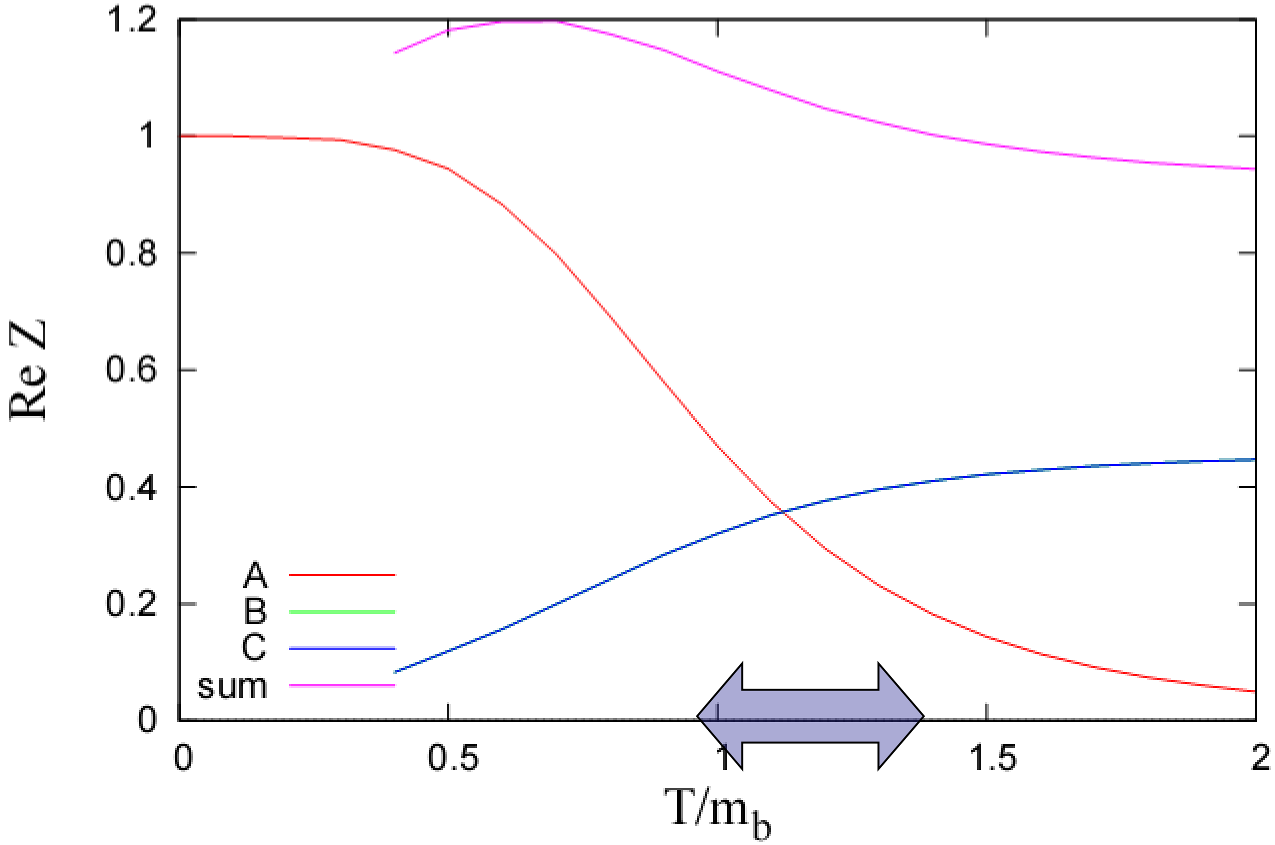




$$\rho^{\text{pole}}(\omega) = \sum_{i=1}^{3} \left(-\frac{1}{\pi}\right) \mathrm{Im}\left[\frac{Z_i}{\omega - z_i}\right]$$

The pole approximation describe the spectral function except near the origin
: the residues at many poles near the origin have negative real parts

The residues

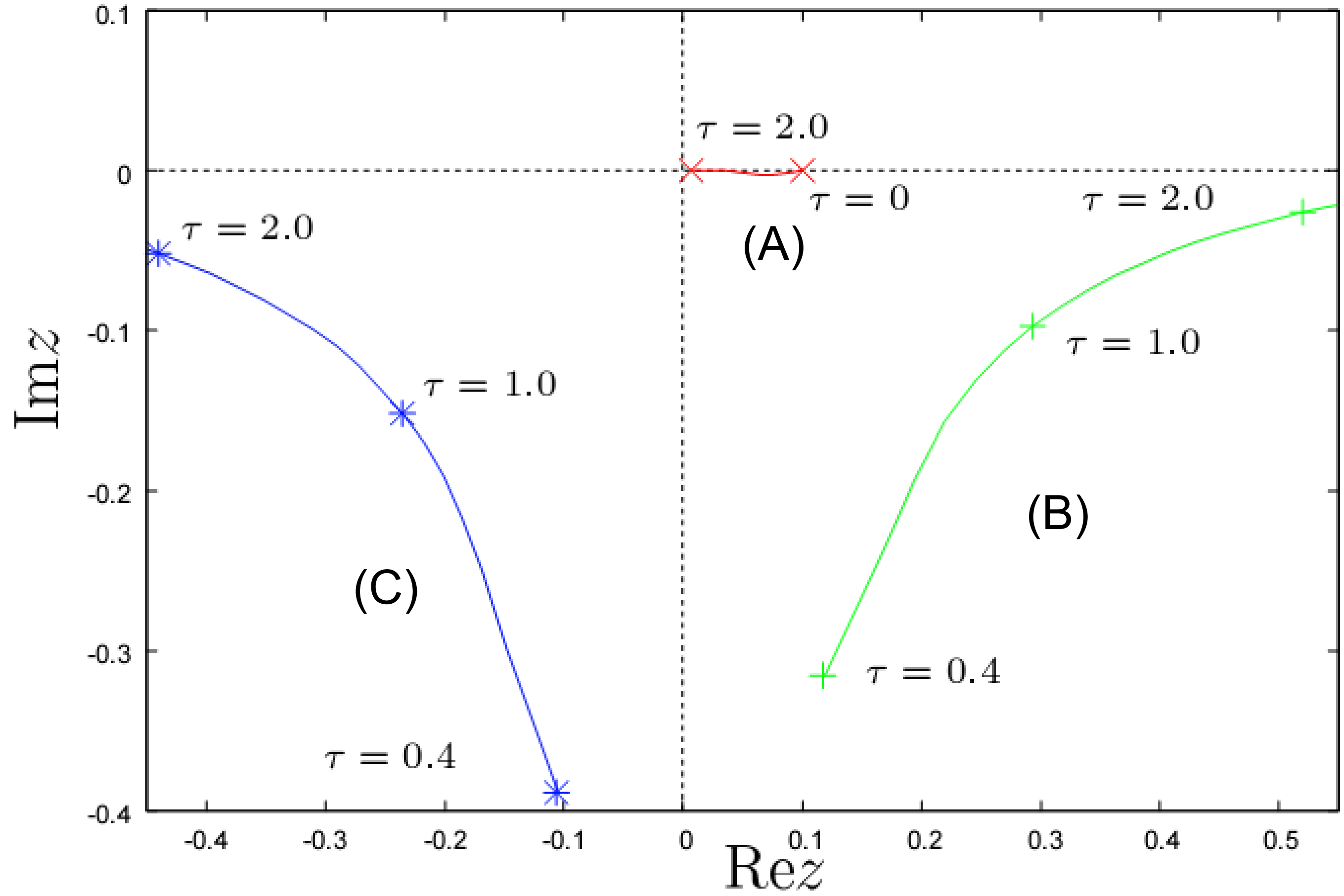


Sum rule is satisfied approximately

$$\sum_i \mathrm{Re} Z_i \simeq 1$$

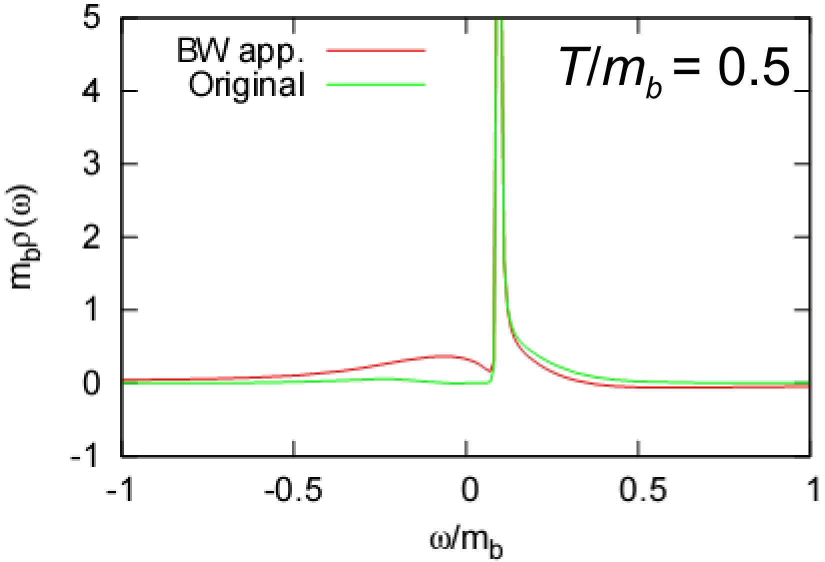
- Residues have similar values at $T \sim m_b$: consistent with 3-peak structure !

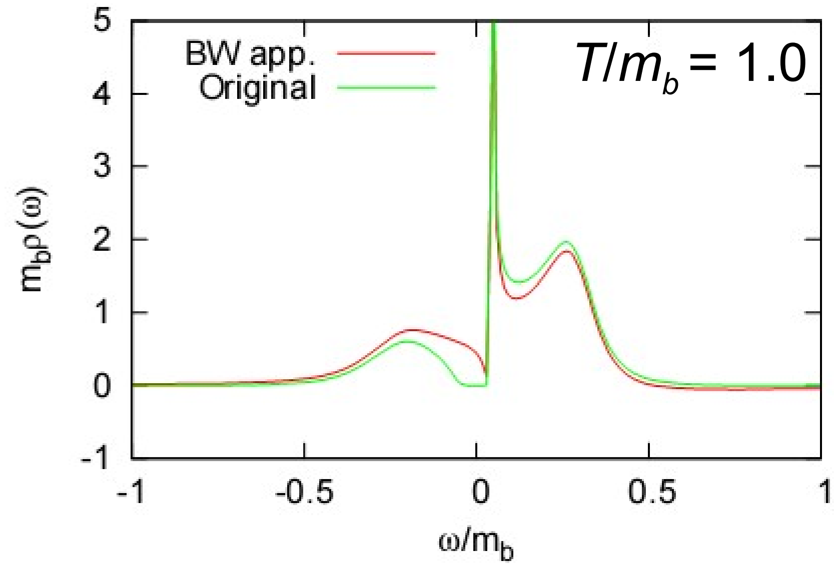
How the poles move (m_f/m_b=0.1)

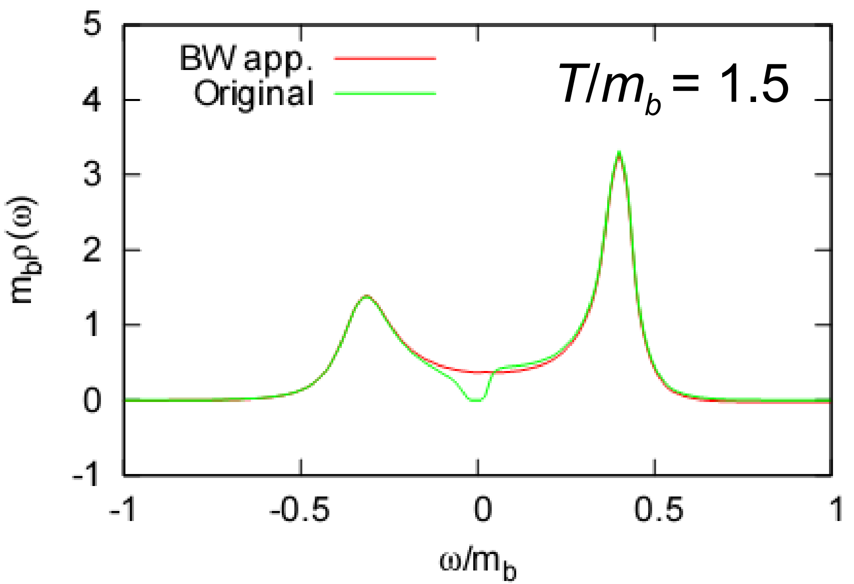


- Pole (A) moves toward the origin as T is raised
- Pole (C) have larger imaginary part than pole (B)

Breit-Wigner approximations

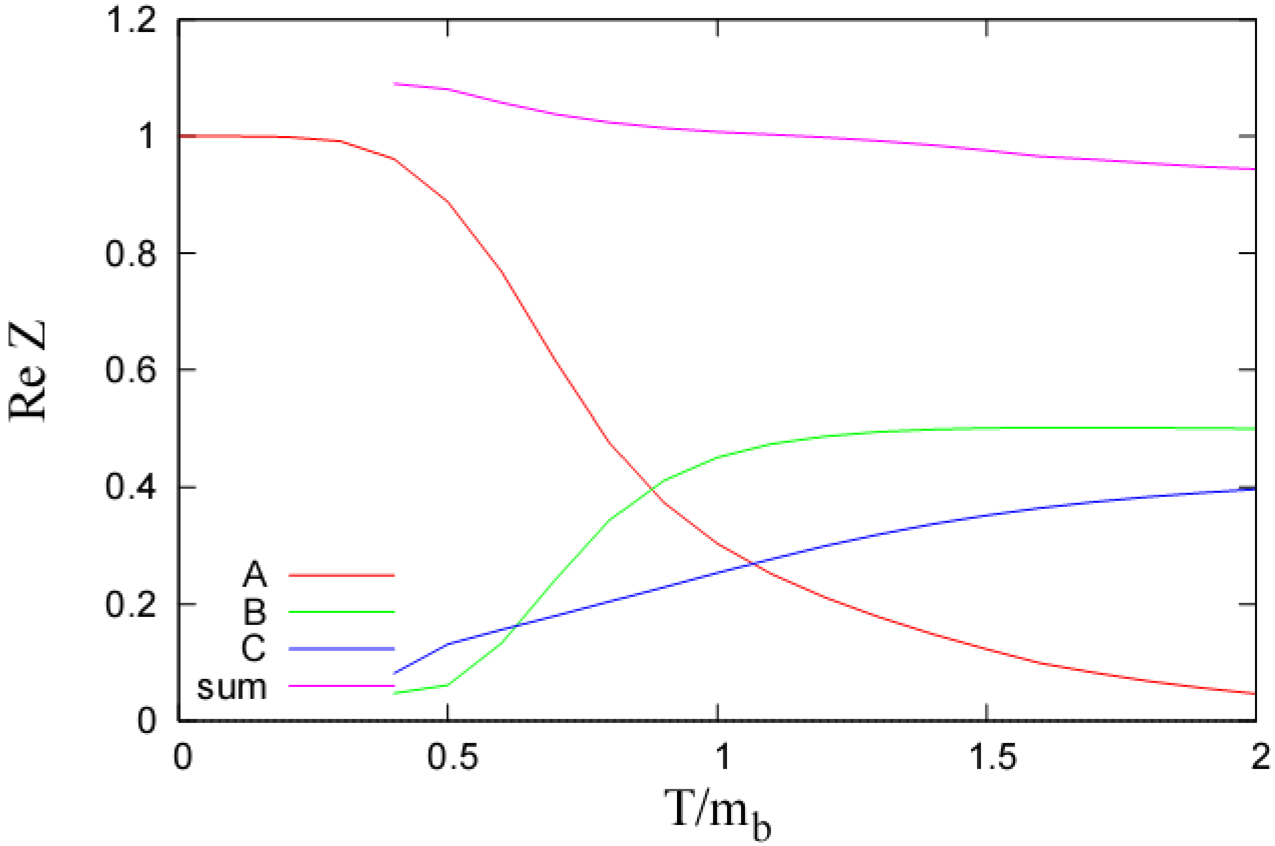






- The spectral function is well described by the three poles we picked up !

The residues and a sum rule

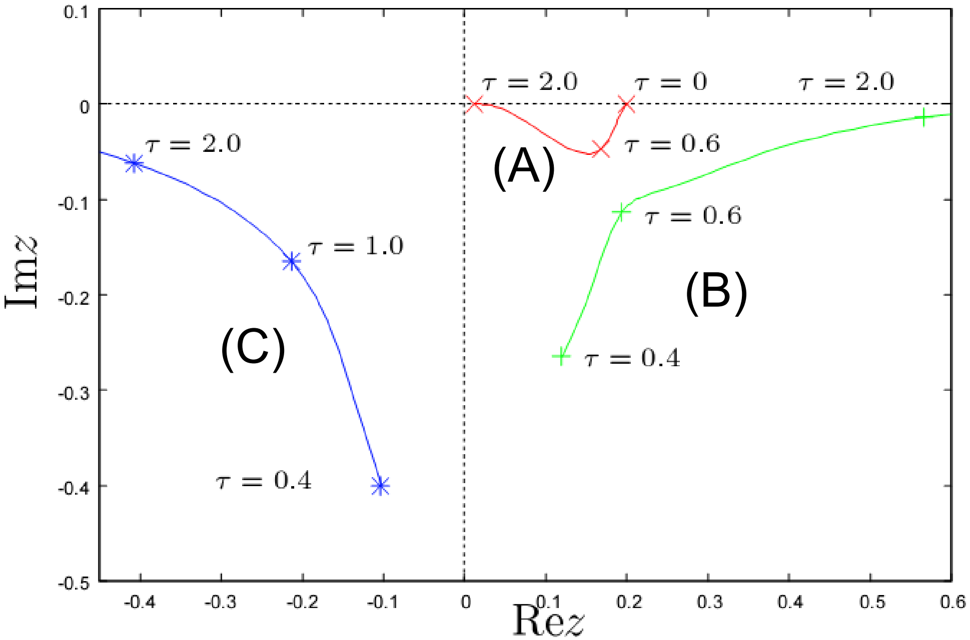


Sum rule is approximately satisfied

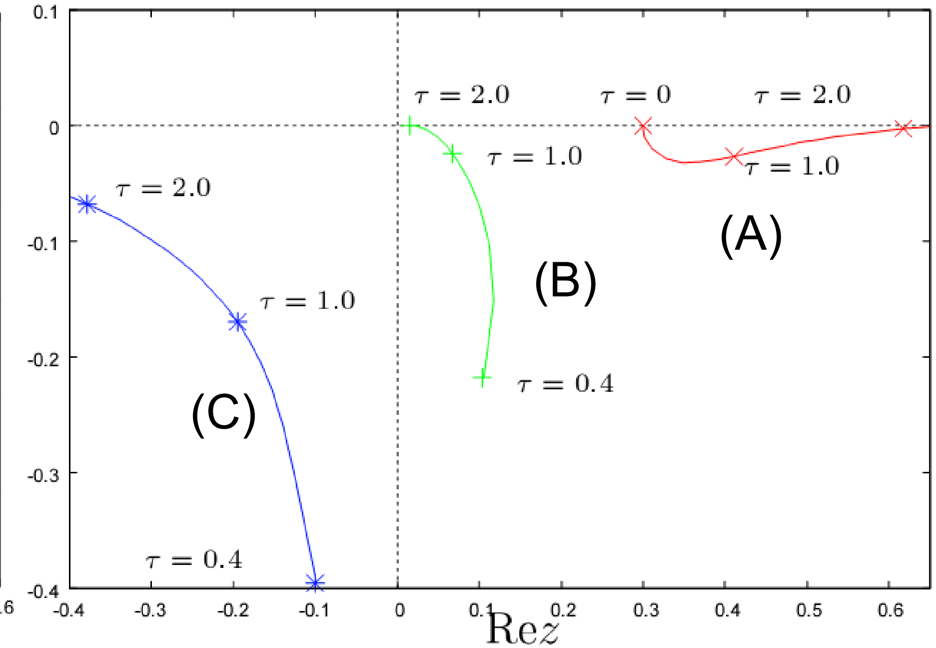
- Residue at pole (A) decrease at high T
- Residue at pole (B) is larger than that for pole (C) at $T \sim m_b$

Structural change in the pole behavior

m_f / m_b = 0.2



m_f / m_b = 0.3

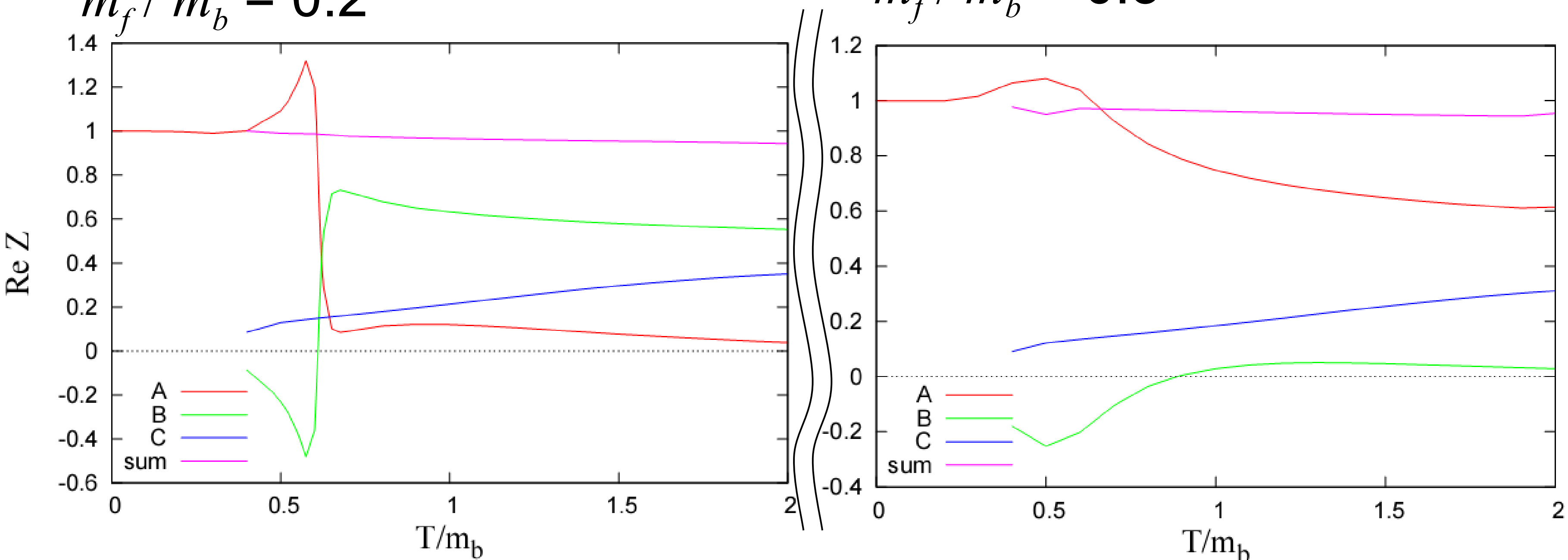


- The T-dependence of the poles qualitatively change

Residues

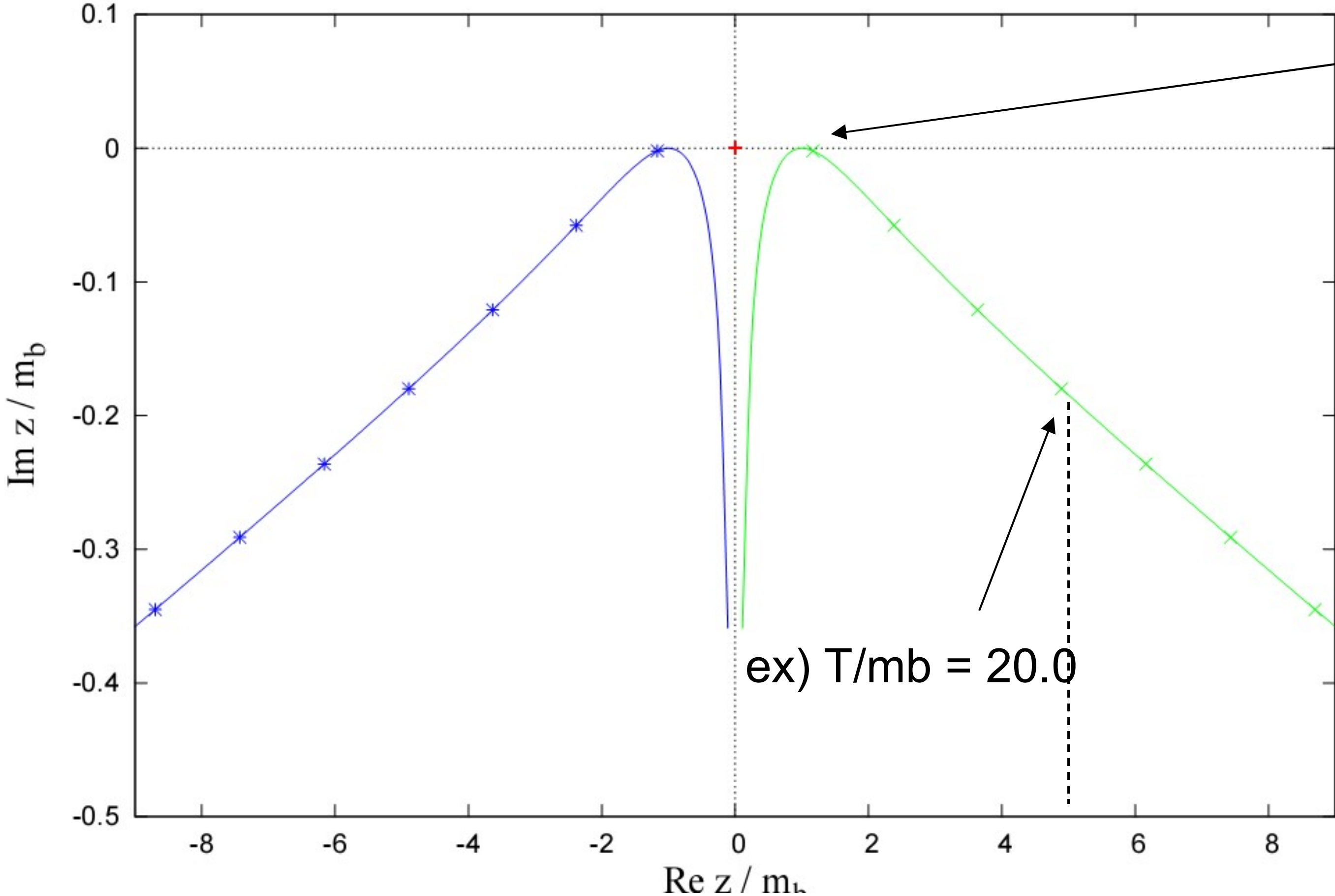
- It seems that behavior of poles A and B is exchanged.

m_f / m_b = 0.2 $\qquad$ m_f / m_b = 0.3



critical mass is $m_f / m_b \sim 0.21$

The behavior at high temperature



$\omega_Q = m_b$

phase space for decays vanish

- $\mathrm{Im}\, z$ / $\mathrm{Re}\, z$ is small at high T
- m_T of HTL well approximate the real part at high T $\quad m_T = gT/4$

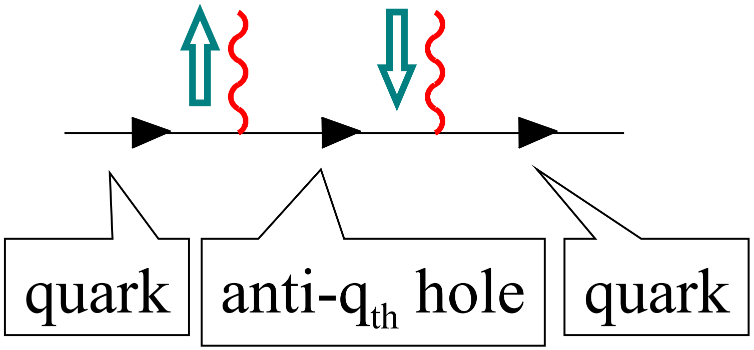
The Level Mixing

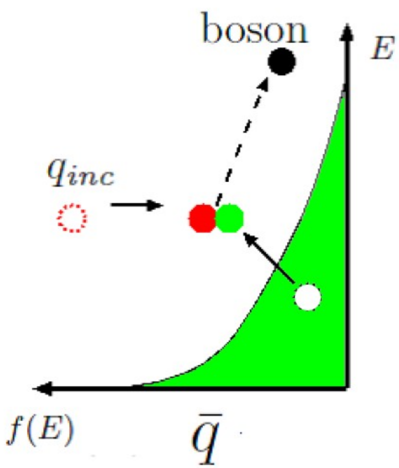
The Physical Origin
Of Multi Peak Structures

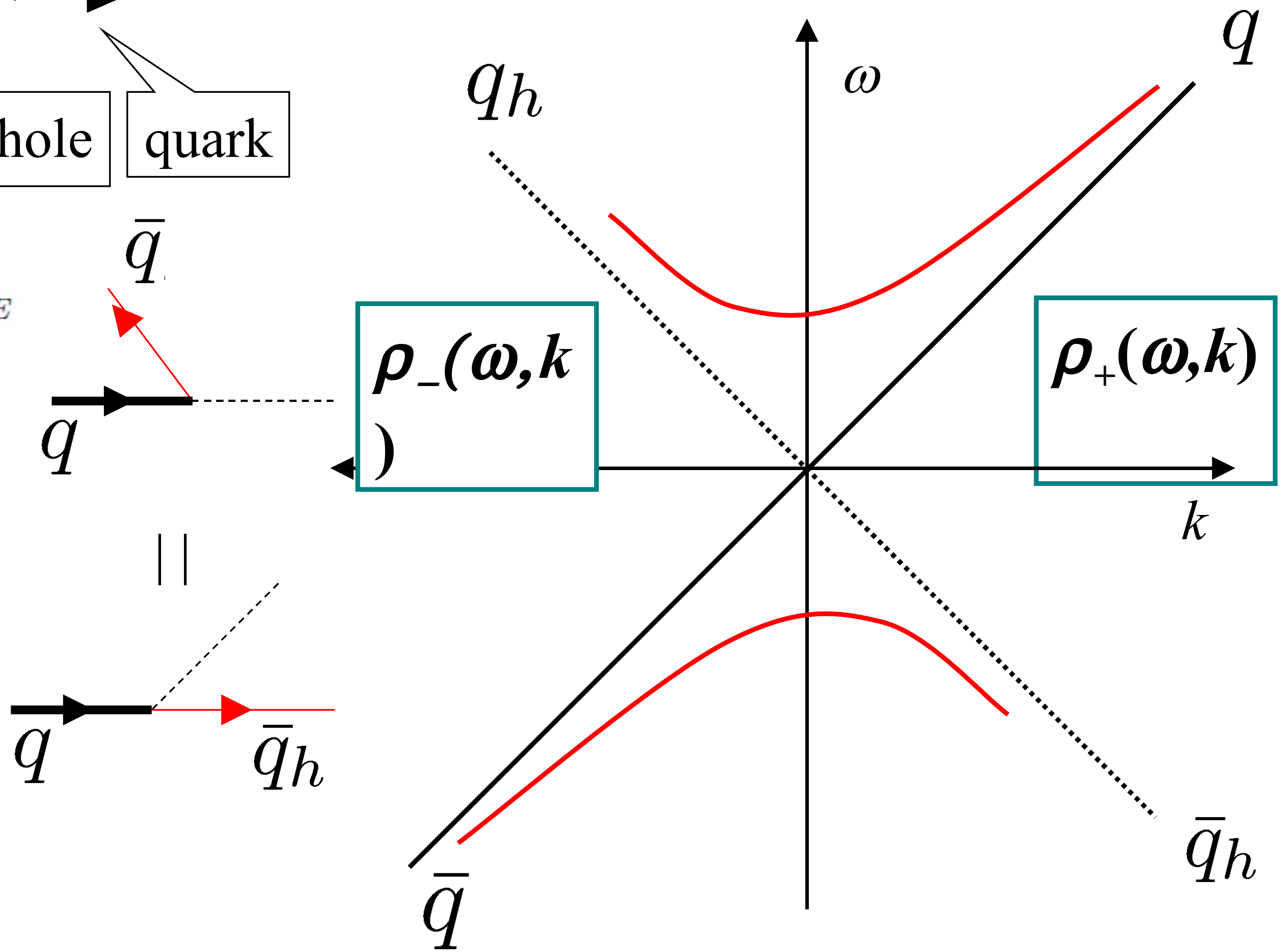
Level Mixing

~ massless fermions coupled with a massless boson ~

【 H.A.Weldon, PRD**40**(1989)2410 】





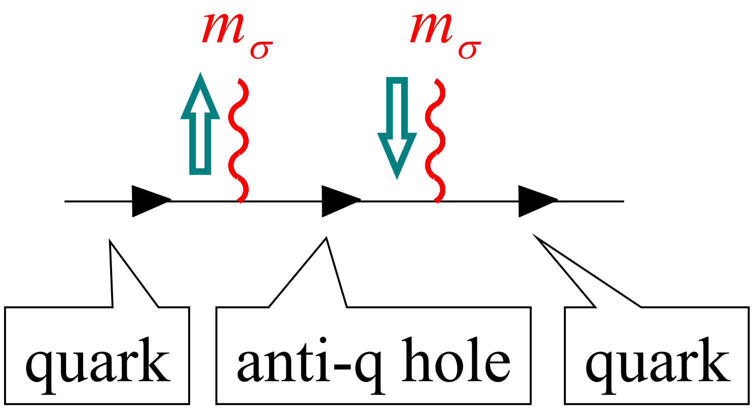


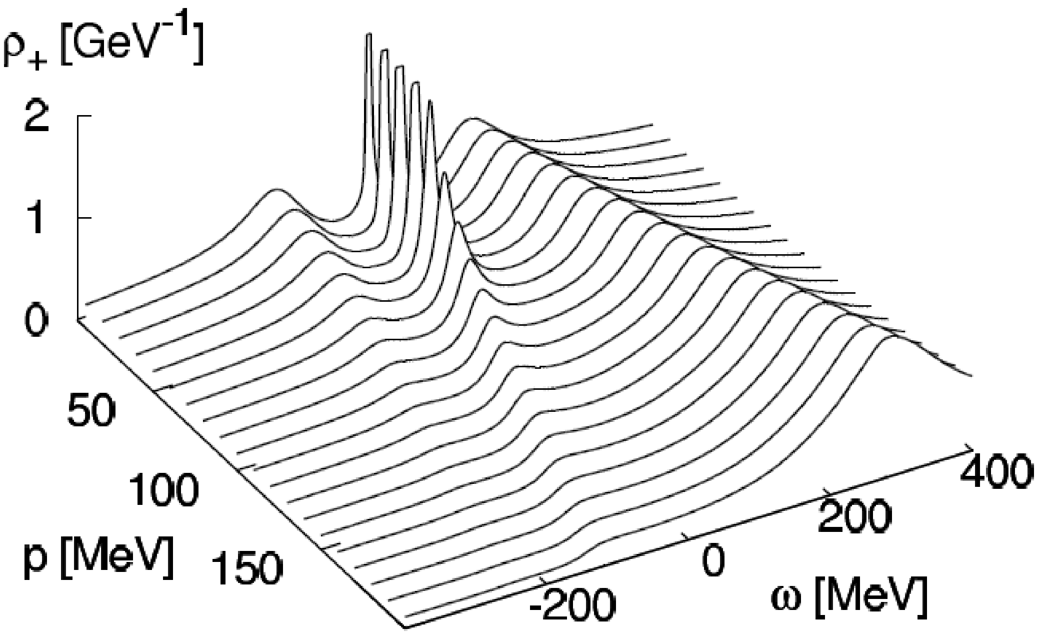
M.Kitazawa, et.al. Phys.Lett. **B633,** 269 (2006)

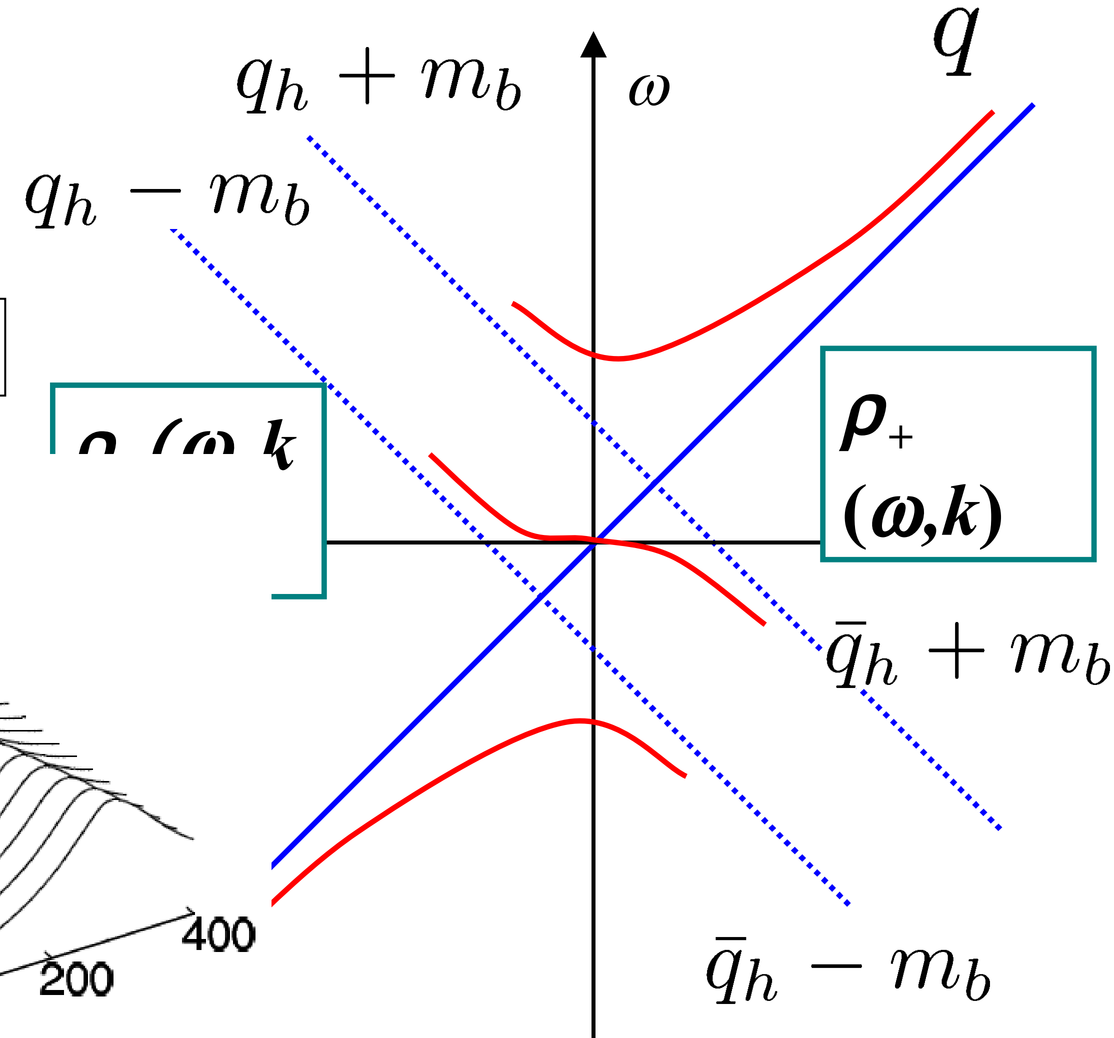
Kitazawa et al, Prog.Theor.Phys.**117,** 103 (2007)

Level Mixing

~ massless fermions coupled with a **massive** boson ~



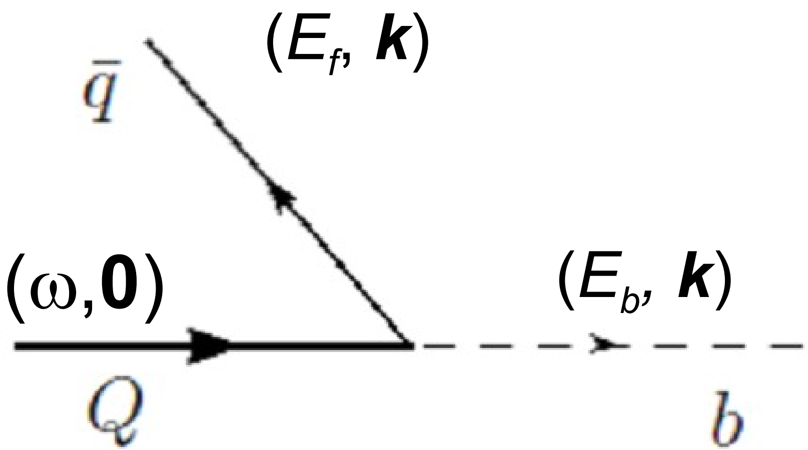




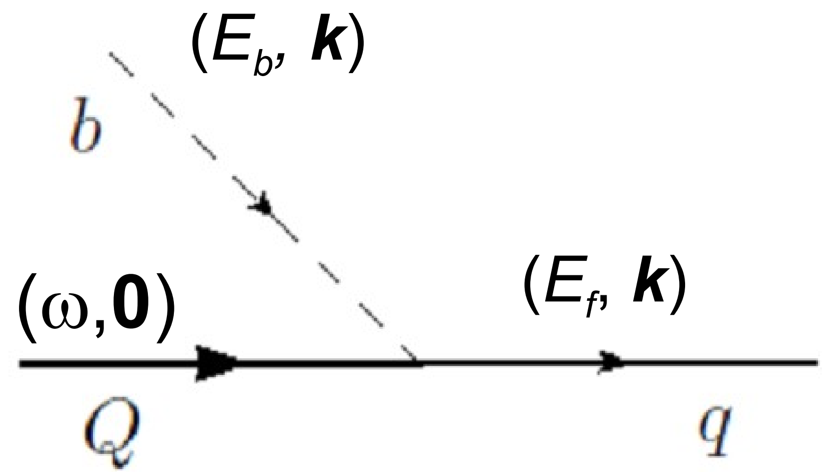
Energies of the mixed levels

In the discussions above, the momentum of the boson is set to zero.
Here I consider general values of the absorbed/emitted boson.

$0 < \omega < |m_b - m_f|$



$0 > \omega > -|m_b - m_f|$

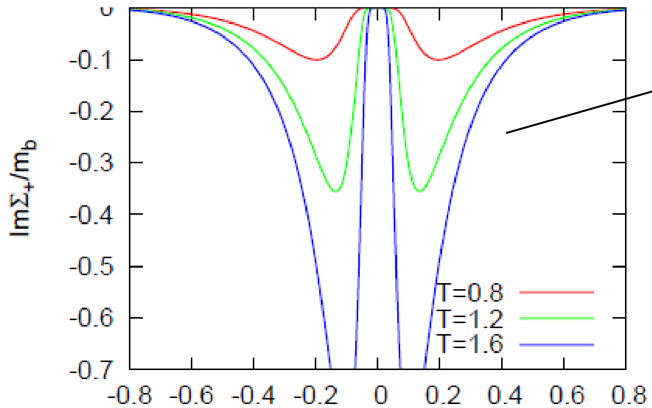


The energy of the state which mixed with the original (free) state

$$\boxed{\begin{aligned} \omega^{>} &= E_b - E_f \ (>0) \\ \omega^{<} &= E_f - E_b \ (<0) \end{aligned}}$$

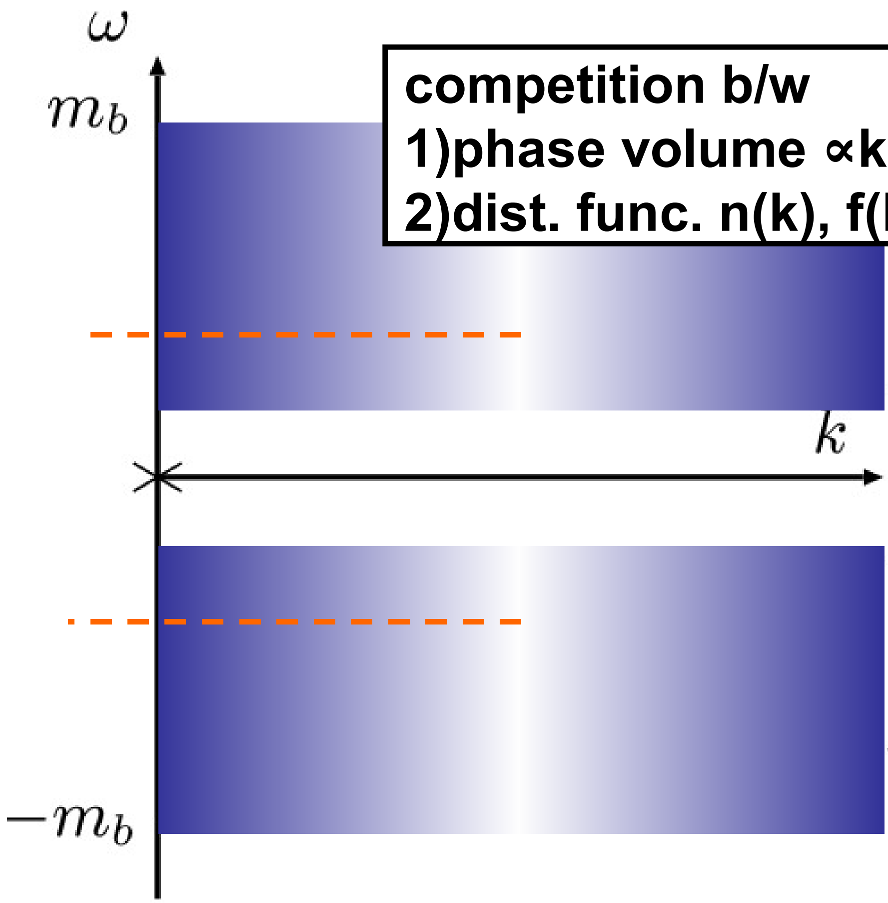
$$E_b = \mathrm{sqrt}(m_b^2 + k^2)$$
$$E_f = \mathrm{sqrt}(m_f^2 + k^2)$$

Im Σ prop. to the decay rate

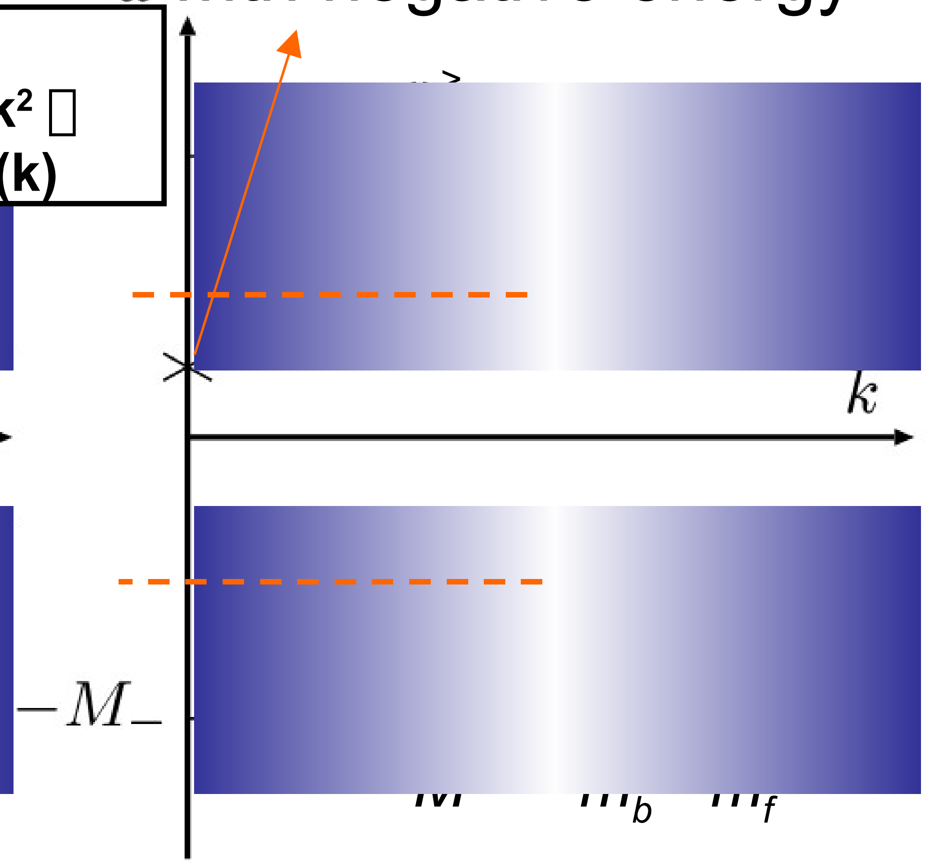


This suggest that the mixing processes occur most often at two energy value
→effectively the mixing b/w three states
→three peak structure

suppression of the peak with negative energy



competition b/w
1)phase volume ∝k²
2)dist. func. n(k), f(k)



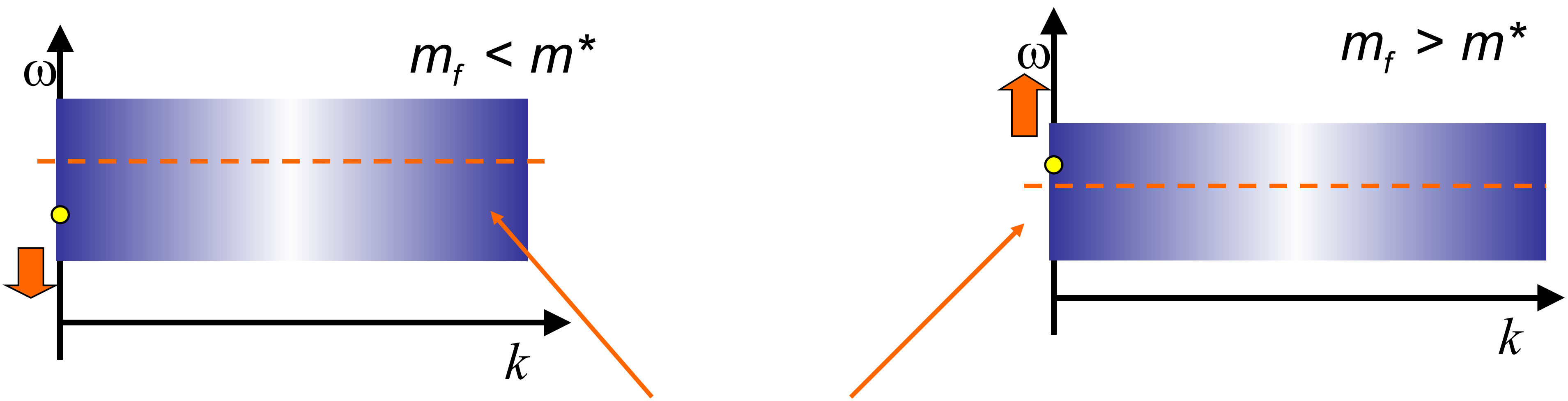
the structural change from the aspect of level mixing

Intermediated states are thermally excited

- at low *T* effect of level mixing is weak
- graphs are schematic picture at enough high *T* so that level mixing become effective and pole (A) start to move

original level is pushed down in energy at high *T*

original level is pushed up in energy at high *T*



m_f= 0,0.1,0.2

Effective mixed level

m_f= 0.3

Subtracted Dispersion Relation

Kramers-Kroenig Relation for f(x)

$$\boxed{\mathrm{Re} f(z) = \frac{1}{\pi}\mathcal{P}\int \frac{dz'}{z-z'}\mathrm{Im} f(z)}$$

Finiteness of ReΣ require | f(z)| to converge as z → ∴ . Else one should use

$$\mathrm{Re} f(z) = f(a) + \frac{(z-a)}{\pi}\mathcal{P}\int \frac{dz'}{(z'-z)(z-a)}\mathrm{Im} f(z')$$

$$\mathrm{Re} f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{\pi}\mathcal{P}\int \frac{dx'}{(x'-a)^2(x'-x)}\mathrm{Im} f(x')$$

Those are called once “subtracted” and twice “subtracted” dispersion relation respectively.

Explicit expression of T = 0 part

$$\Sigma = p_0\Sigma_0 - \hat{p}\cdot\vec{\gamma}\Sigma_V + \Sigma_2$$

$$\mathrm{Im}\Sigma_0(p_0,\vec{p}) = -\frac{g^2}{32\pi}\frac{(p_\mu^2 - \tilde{m}_I\cdot M)}{p_\mu^2}\frac{\sqrt{(p_\mu^2-\tilde{m}_I{}^2)(p_\mu^2-M^2)}}{p_\mu^2}p_0\epsilon(p_0)\theta(p_\mu^2-M^2)$$

$$\mathrm{Im}\Sigma_V(p_0,\vec{p}) = -\frac{(p_\mu^2 - \tilde{m}_I\cdot M)}{p_\mu^2}\frac{\sqrt{(p_\mu^2-M^2)(p_\mu^2-\tilde{m}_I{}^2)}}{p_\mu^2}p\epsilon(p_0)\theta(p_\mu^2-M^2)$$

$$\mathrm{Im}\Sigma_0 \to p_0 \qquad \text{as } p_0 \to \infty$$

$$\mathrm{Re}\Sigma(p_0,p) = \frac{1}{\pi}\mathcal{P}\int\frac{dz}{z-p_0}\mathrm{Im}\Sigma(z,p)$$ Diverge !

Renormalization Of T = 0 Part

- We use twice subtracted disp. rel. for regularization of integral

General complex function f(x) obey to the following relation :

$$\mathrm{Re}f(z) = \frac{1}{\pi}\mathcal{P}\int \frac{dz'}{z-z'}\mathrm{Im}f(z)$$ □ Dispersion Relation □

Even if above expression diverge, following expression sometimes converge.

$$\mathrm{Re}f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{\pi}\mathcal{P}\int \frac{dx'}{(x'-a)^2(x'-x)}\mathrm{Im}f(x')$$

This expression is called “twice-subtracted” dispersion relation

$$\mathrm{Re}\Sigma(p_0,p) = \Sigma(\pm\tilde{E}_p,p) + (p_0 \mp \tilde{E}_p, p)\Sigma'(\pm\tilde{E}_p) + \frac{(p_0 \mp \tilde{E}_p)^2}{\pi}\mathcal{P}\int \frac{dx'}{(x' \mp \tilde{E}_p)^2(x'-p_0)}\mathrm{Im}\Sigma(x')$$

Renormalization Of T = 0 Part

$$\mathrm{Re}\Sigma(p_0,p)_{T=0} = \Sigma_{T=0}(\pm\tilde{E}_p,p)+(p_0\mp\tilde{E}_p)\Sigma'(\pm\tilde{E}_p,p)_{T=0}+\frac{(p_0\mp\tilde{E}_p)^2}{\pi}\mathcal{P}\int\frac{dx'}{(x'\mp\tilde{E}_p)^2(x'-p_0)}\mathrm{Im}\Sigma(x')_{T=0}$$

□ dbl sign : for $p_0 > 0$ upper , for $p_0 < 0$

Mass Shell Renormalization

Mass Renorm.

$$\delta m(p^2 = m_R^2) = 0$$

Wv. Fnc. Renorm.

$$\delta Z_2(p^2 = m_R^2) = 0$$

In terms of Σ

$$\Sigma(p^2 = m_R^2) = 0$$

$$\Sigma'(p^2 = m_R^2) = 0$$

$$\mathrm{Re}\Sigma(p_0,p) = \mathcal{P}\int\frac{dx'}{(x'\mp\tilde{E}_p)^2(x'-p_0)}\mathrm{Im}\Sigma(x')$$

□ dbl sign : for $p_0 > 0$ upper , for $p_0 < 0$ lower □

- Finite temperature part converge → □ disp. rel. without subtraction.

Decomposition of the integral

$$\mathrm{Im}\Pi^{\mu}_{\mu}(q_0+i\eta)$$

$$\propto \quad \bullet \quad \int\frac{d^3p}{(2\pi)^3}\int d\omega\int d\omega' f(\omega)f(\omega')\rho_\pm(\omega,p)\rho_\pm(\omega',p)\delta(q_0-\omega-\omega')$$

±

decomposition of spectral function

$$\rho_\pm(\omega,p)=\sum_j Z_p^{\pm,j}\delta(\omega-E_p^j)+\rho_\pm^{\mathrm{reg}}(\omega,p)$$

The integral is decomposed in to three blocks

□□	: pole-pole term
$\rho^{\mathrm{reg}}_{\pm}$□	: pole-regular term
$\rho^{\mathrm{reg}}_{\pm}\rho^{\mathrm{reg}}_{\pm}$	: regular-regular term