Hydrodynamic Expansion with the QCD Critical Point in Heavy Ion Collisions

Nagoya University

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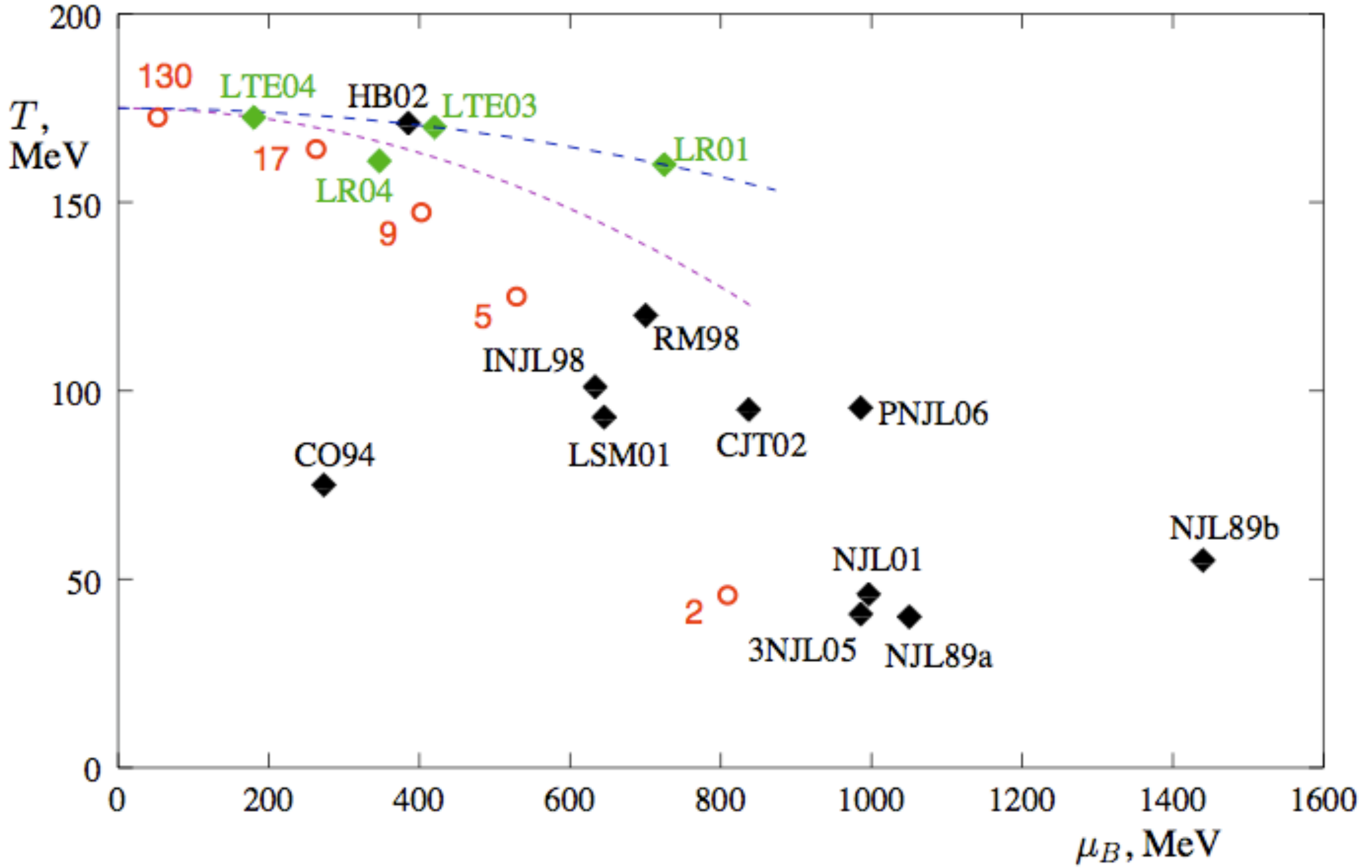
In collaboration with

M.Asakawa(Osaka), S.A.Bass(Duke) and B.Mueller(Duke)

August 12, 2008 @The QCD Critical Point, INT

Where is the QCP?

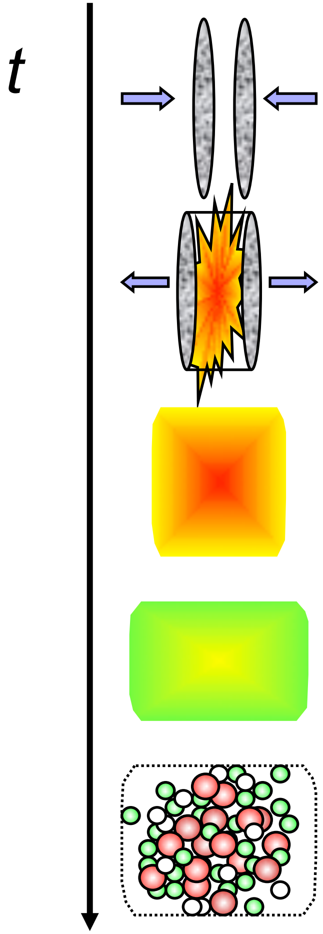
- **Lattice QCD, Effective models…**

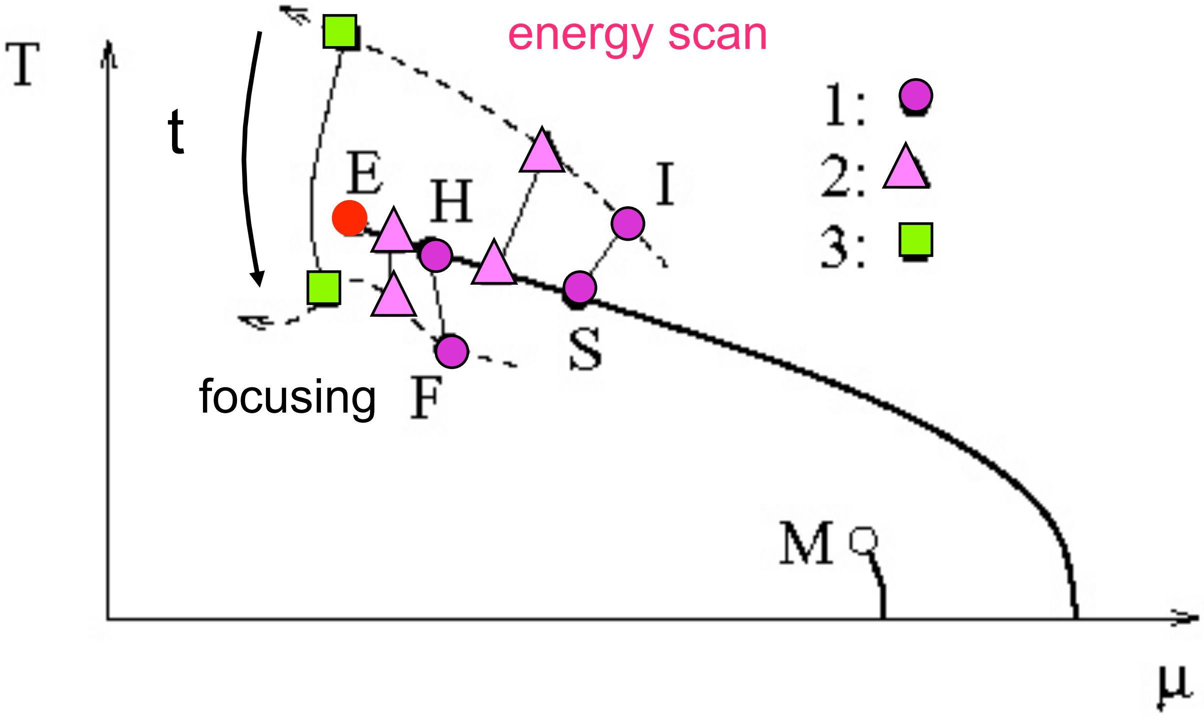


Stephanov,hep-lat/0701002

QCP Search in HIC

- **The QCD critical point search from phenomenology and experiments**





M. Stephanov, K. Rajagopal, and E.Shuryak, PRL81 (1998) 4816

Toward Quantitative Analyses

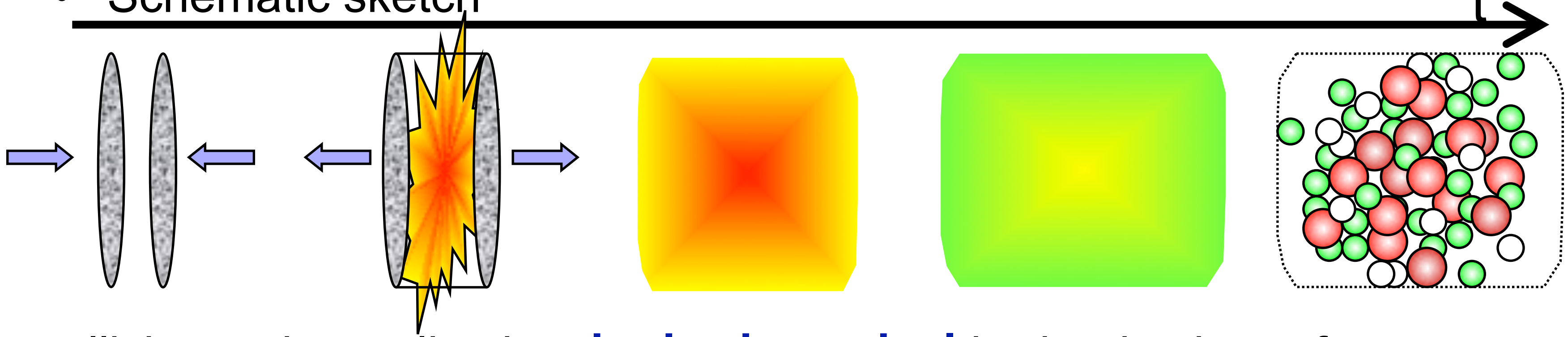
- **Realistic Dynamical Model**
  - 3D Hydro + UrQMD Model
- **The QCD Critical Point**
  - Focusing effect near the QCD critical point in isentropic trajectories on the T-μ_B plane
- **Emission Time Dependence**
  - High P_T particles emit at earlier time

3D Hydro+UrQMD Model

Nonaka and Bass PRC75:014902(2007)

Relativistic Heavy Ion Collision

- Schematic sketch



⦿3D Hydro + UrQMD

Full 3-d Hydrodynamics

Hadronization

EoS :1st order phase transition
QGP + excluded volume model ➡ Cooper-Frye formula

Monte Carlo ➡

UrQMD

final state interactions

T_C T_{SW} t fm/c

T_C:critical temperature > T_{SW}: Hydro ➔ UrQMD

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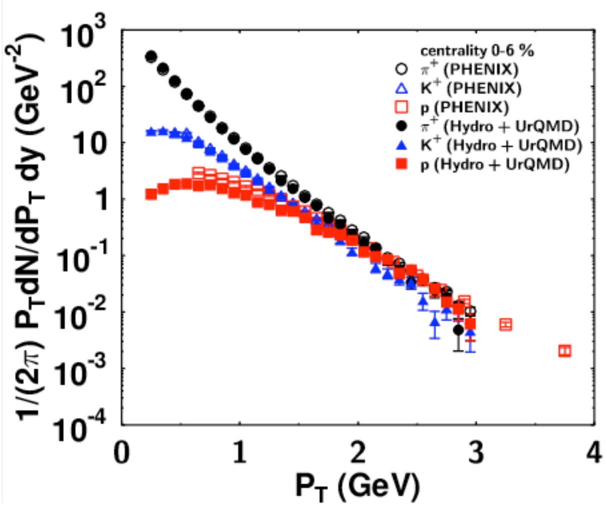
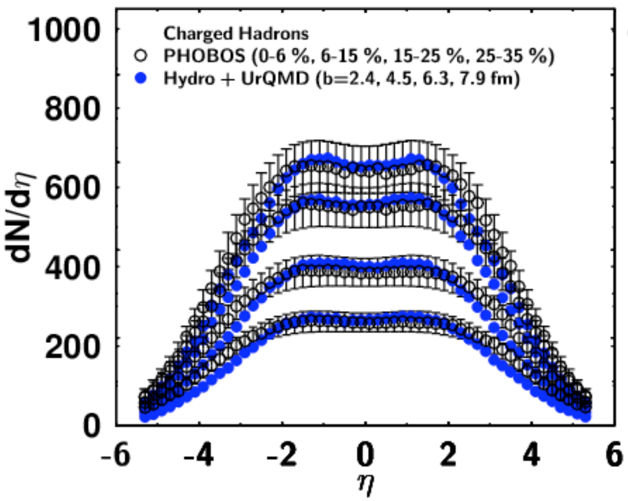
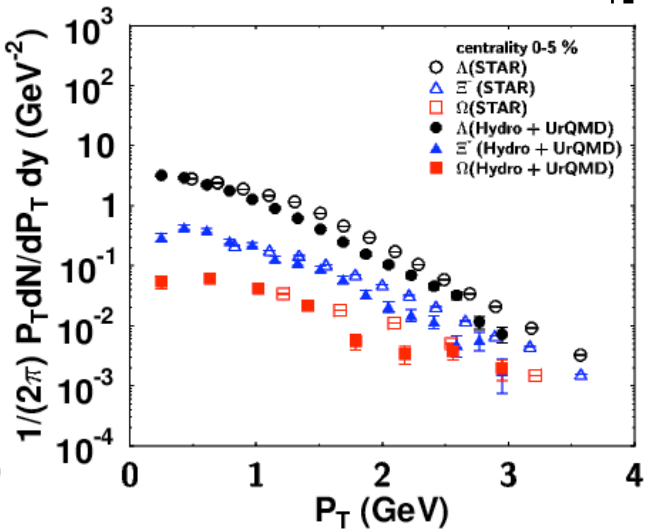
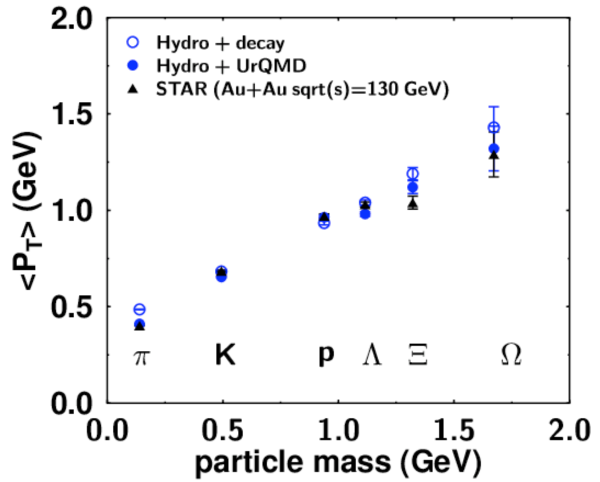
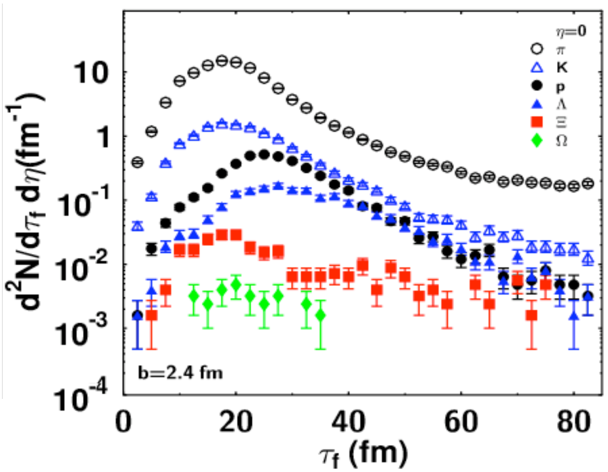
Highlights of 3D Hydro+UrQMD

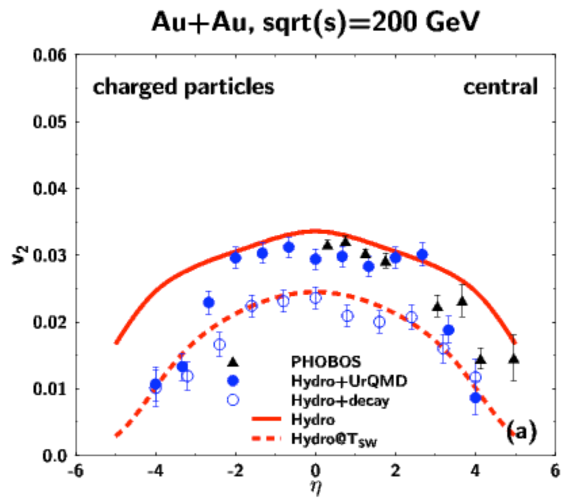












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Realistic Equation of States

- **3D Hydro + UrQMD**

Full 3-d Hydrodynamics

Hadronization

EoS :1st order phase transition
QGP + excluded volume model

➡ Cooper-Frye formula

Monte Carlo ➡

UrQMD

final state interactions

T_C T_{SW} t fm/c

T_C:critical temperature > T_{SW}: Hydro ➔ UrQMD

initial conditions
- parametrization

equation of states
- bag model

freezeout process
- Viscosity effect of hadron phase
- Final state interactions

Realistic EOS with QCD critical point

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Toward Quantitative Analyses

- **Realistic Dynamical Model**
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- **The QCD Critical Point**
  - Focusing effect near the QCD critical point in isentropic trajectories on the T-μ_B plane
- **Emission Time Dependence**
  - High P_T particles emit at earlier time

EOS with QCD Critical Point

Nonaka and Asakawa, PRC71,044904(2005)

- **Singular part near QCD critical point + Non-singular part**
  - Non-singular part

    QGP phase and hadron phase
  - Singular part

3d Ising Model ⟷ **QCD**

Same Universality Class

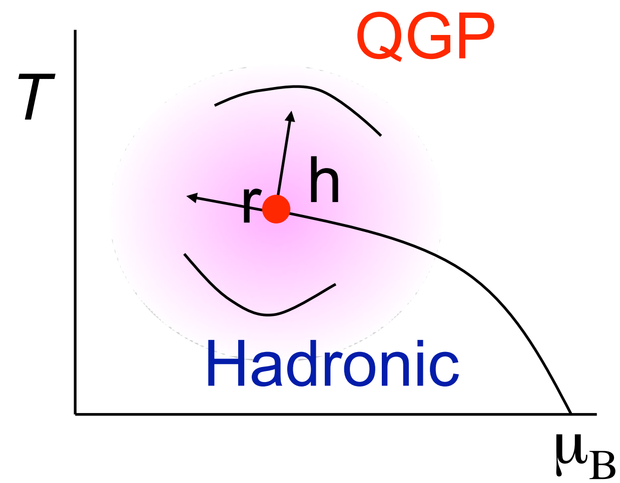
$r = \frac{T - T_C}{T_C}$

h : extermal magnetic field

$(r, h) \longleftrightarrow (T, \mu_B)$

T , μ_B

- Mapping (r, h) ➔ (T, μ_B)
- Matching with known QGP and hadronic entropy density
- Thermodynamical quantities



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EOS of 3-d Ising Model

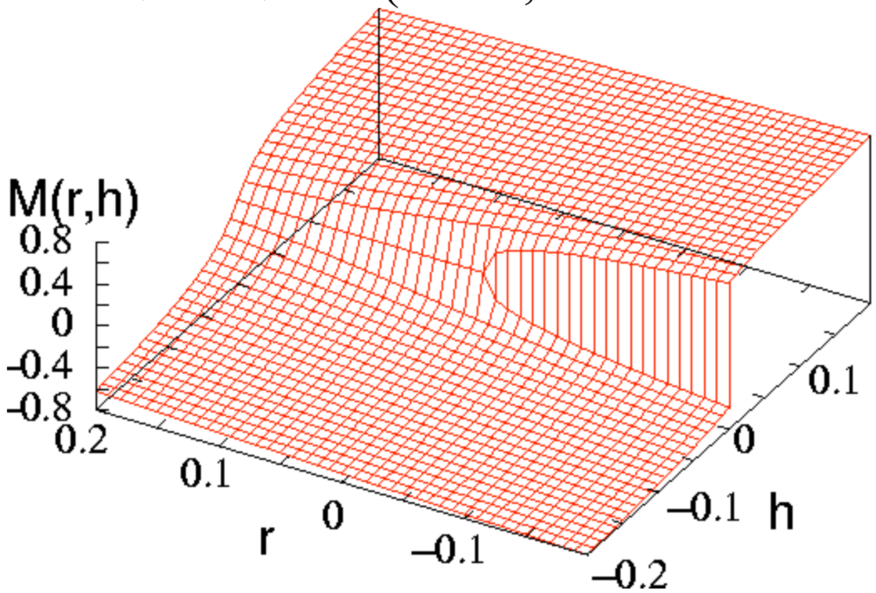
- **Parametric Representation of EOS**

$$\begin{cases} M = M_0 R^{\beta}\theta \\ h = h_0 R^{\beta}\tilde{h}(\theta) = hR_0^{\beta\delta}(\theta - 0.76201\theta^3 + 0.00804\theta^5) \\ r = R(1-\theta^2) \end{cases} \quad (R \geq 0, \ -1.154 \leq \theta \leq 1.154)$$

$$r = \frac{T - T_C}{T_C}$$

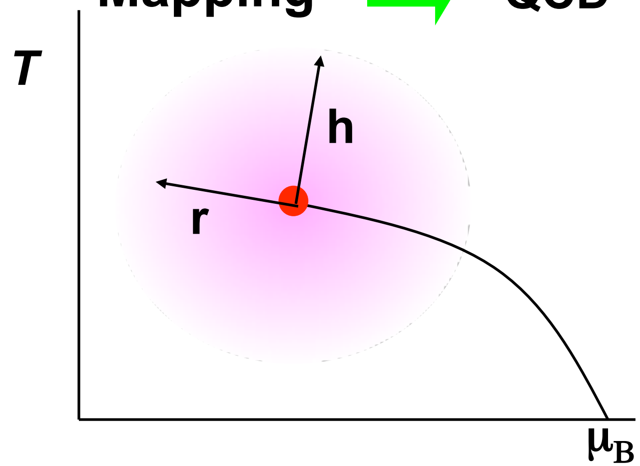
h **: external magnetic field**

$$\beta = 0.326$$
$$\delta = 4.8$$



Guida and Zinn-Justin NPB486(97)626

Mapping ➡ **QCD**



Order parameter

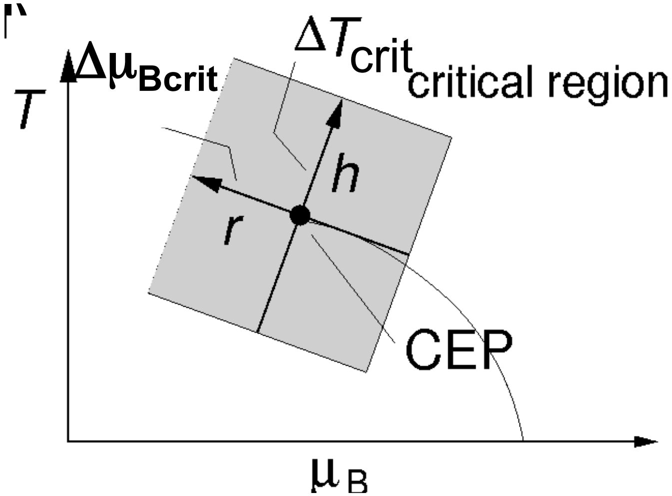
$$M \leftrightarrow \langle \bar{q}q \rangle$$

Singular Part + Non-singular Part

❋ Entropy Density

$$S_{\rm real}(T,\mu_B) = \frac{1}{2}\{1-\tanh[S_c(T,\mu_B)]\}S_H(T,\mu_B) + \frac{1}{2}\{1+\tanh[S_c(T,\mu_B)]\}S_Q(T,\mu_B)$$

- $S_H(T,\mu_B)$ **Hadron Phase (excluded volume model)**
  $S_Q(T,\mu_B)$ **QGP phase**



❋ Dimensionless parameter: S_c

$$S_c(T,\mu_B) = s_c\sqrt{(\Delta T_{\rm crit})^2 + (\Delta\mu_{\rm crit})^2} \times D$$

Critical domain

❋ Choice of parameters:

$$\Delta T_{\rm crit}, \Delta\mu_{\rm crit}, D$$

Thermodynamical inequalities

$$\left(\frac{\partial S}{\partial T}\right)\Bigg|_{V,N} \geq 0, \quad \left(\frac{\partial P}{\partial V}\right)\Bigg|_{T,N} \geq 0, \quad \left(\frac{\partial \mu}{\partial N}\right)\Bigg|_{T,V} \geq 0$$

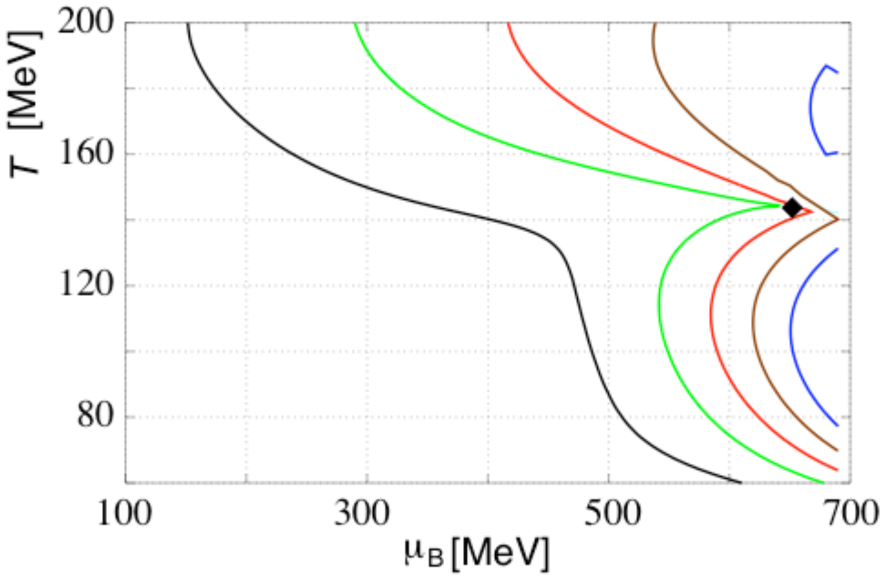
Critical exponent near CEP keeps correctly.

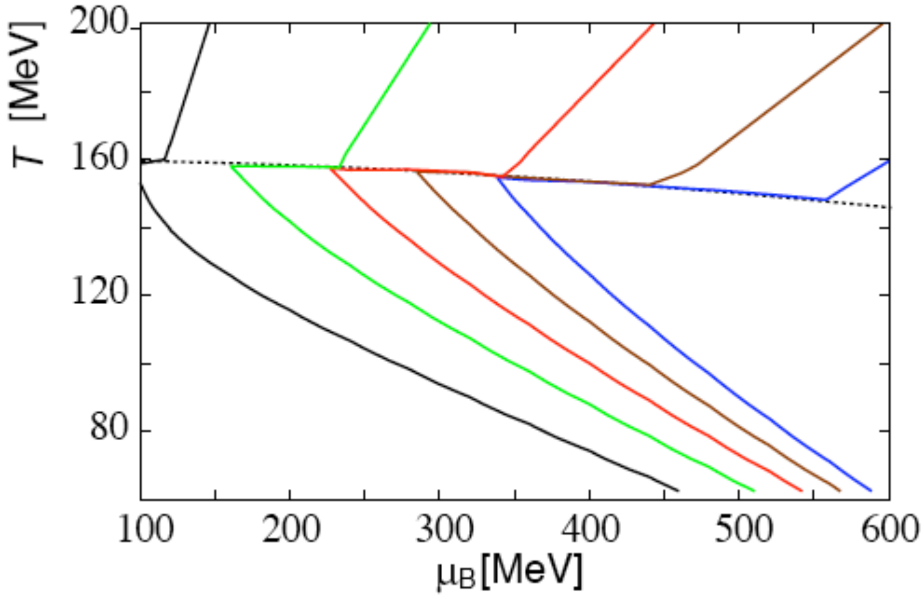
Focusing Effect

- **Isentropic trajectories on T- μ_B plane**

With QCD critical point

Bag Model +
Excluded Volume Approximation
(No Critical Point)
= Usual Hydro Calculation





Focused

Not Focused

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Toward Quantitative Analyses

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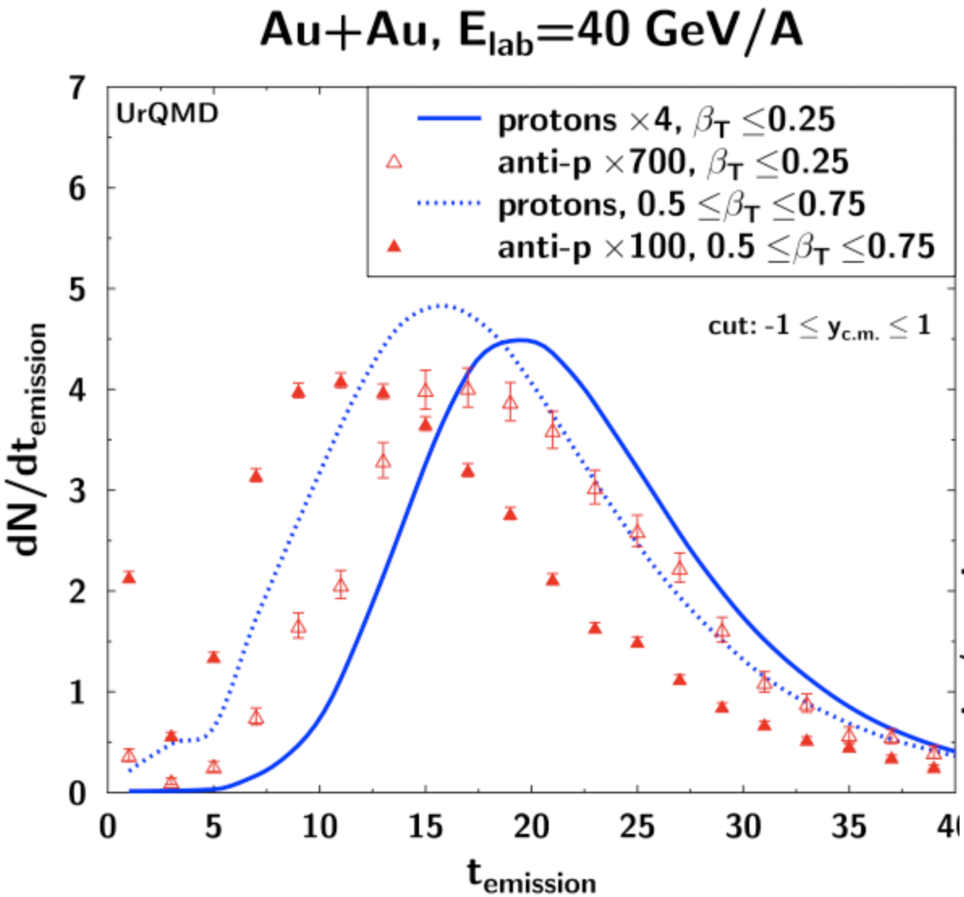
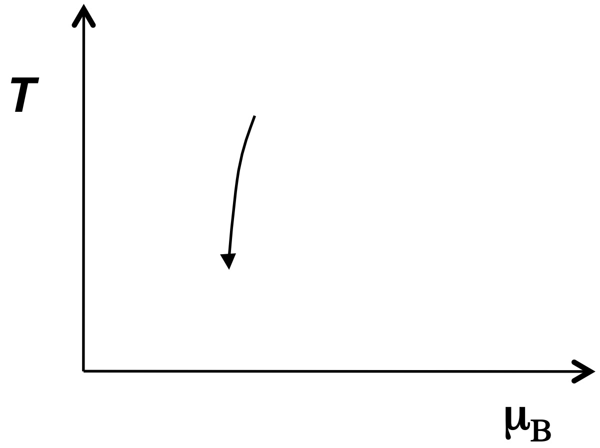
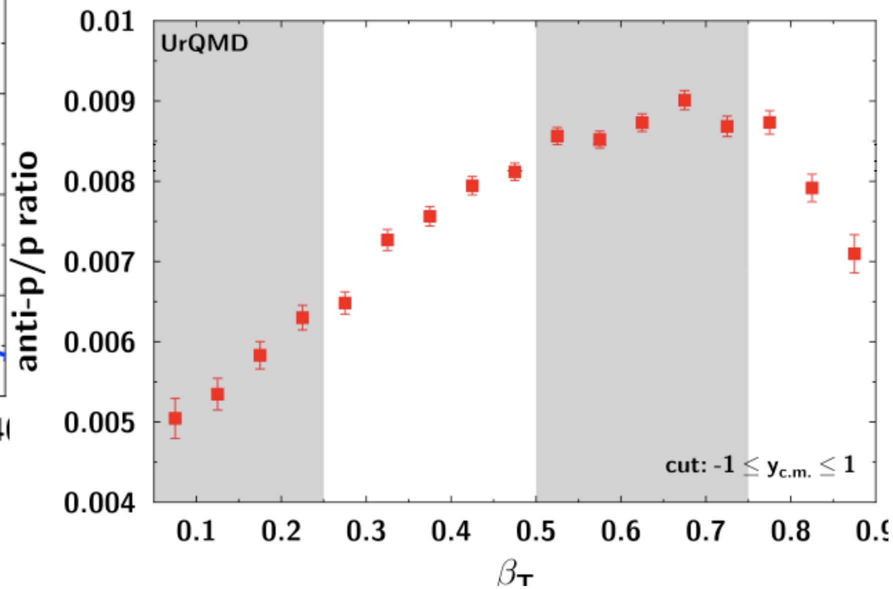
  - High P_T particles emit at earlier time

Emission Time Distribution

UrQMD : no QCD critical point







Toward Quantitative Analyses

- **Realistic Dynamical Model**
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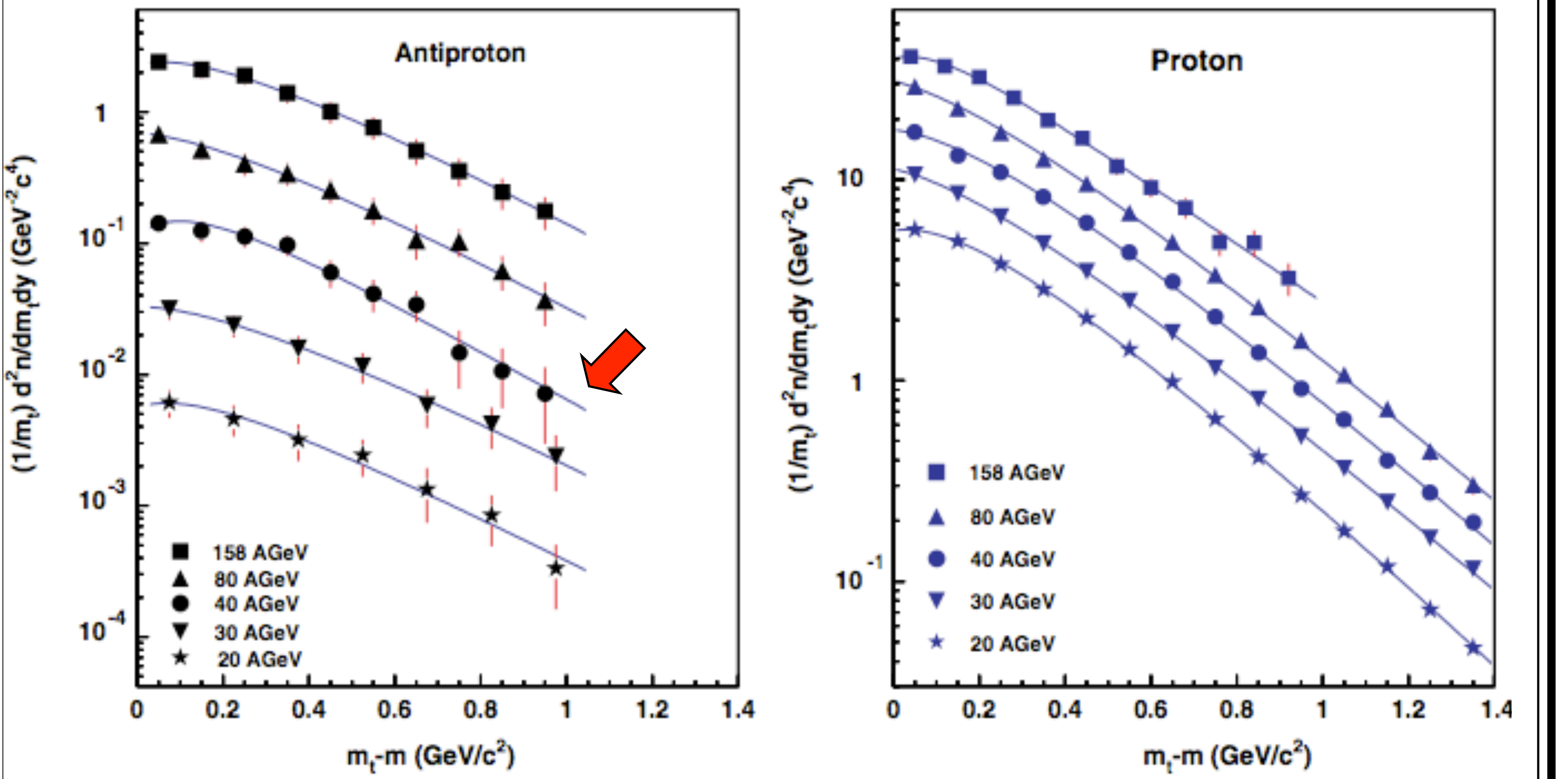
- **The QCD Critical Point**
  - Focusing effect near the QCD critical point in isentropic trajectories on the T-μ_B plane

Location of the QCD critical point ⇦ **lattice QCD**
Critical region **experiments**

- **Emission Time Dependence**
  - High P_T particles emit at earlier time

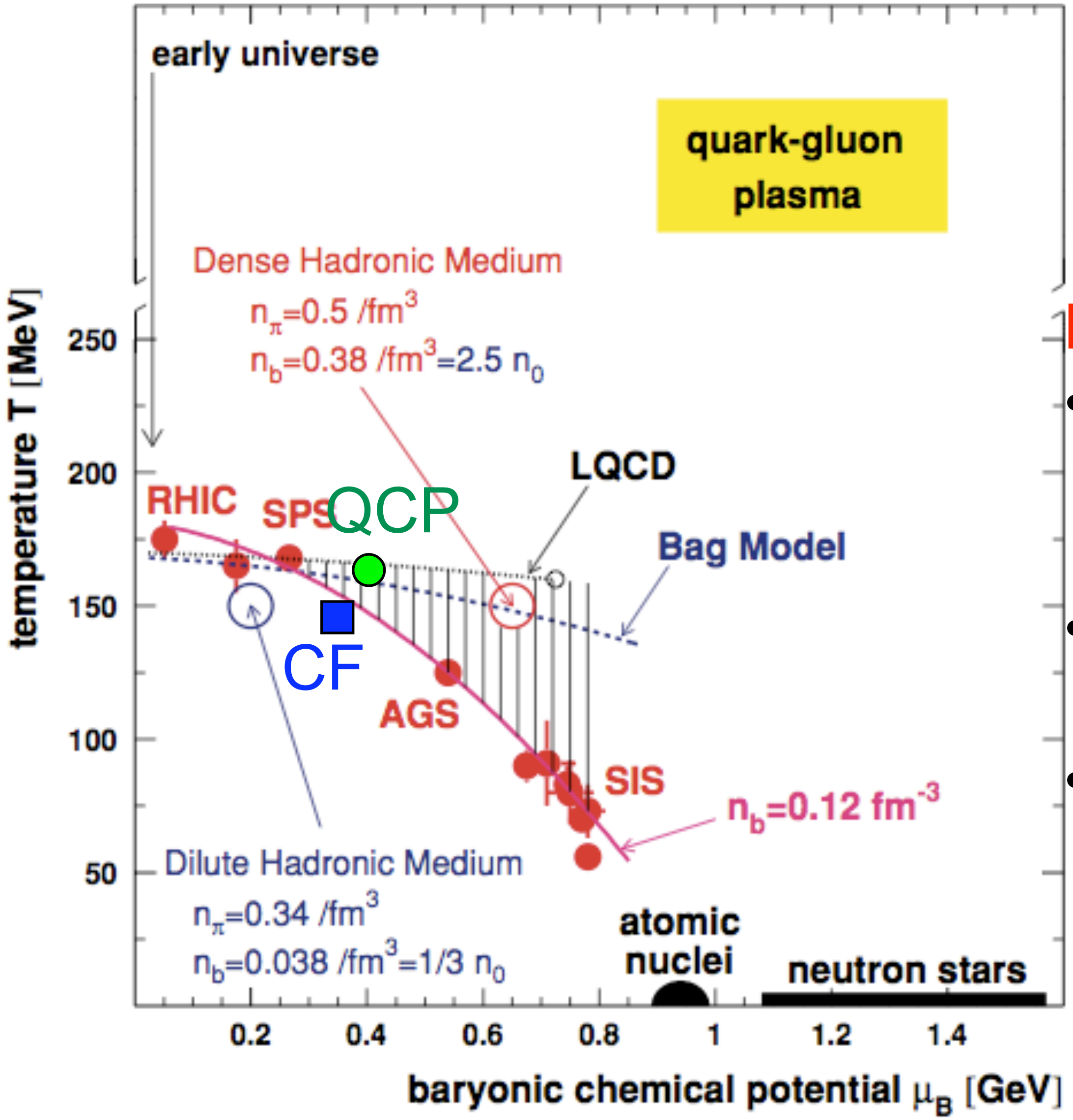
QCD Critical Point?



steeper $\bar{p}$ spectra at high P_T

NA49, PRC73,044910(2006)

Demonstration



Search of the QCD critical point from experiments

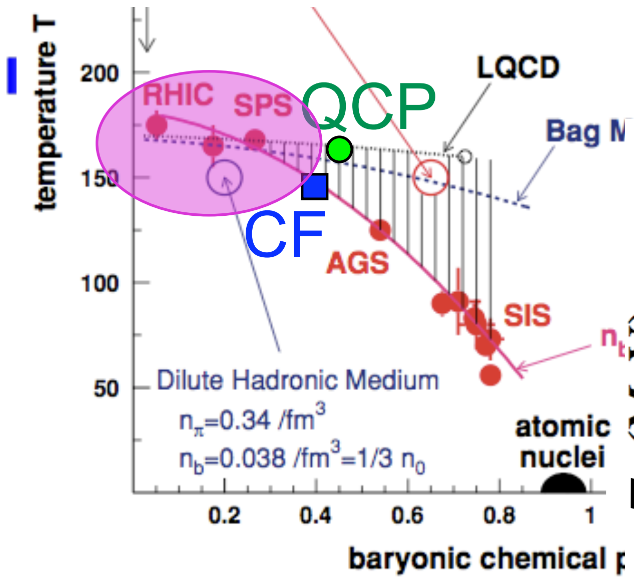
NA49:

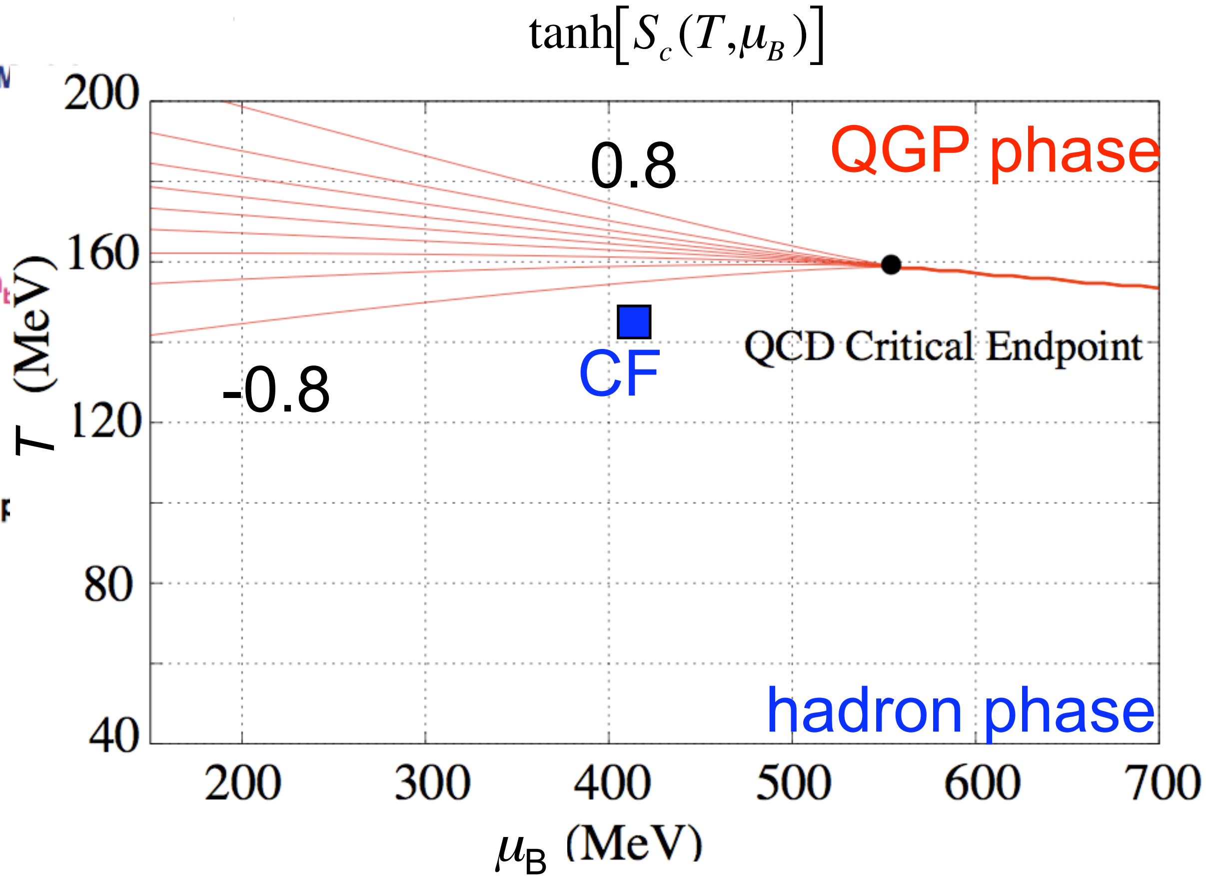
- Location of QCP
  (μ_B, T)=(550,159)

- Critical Region

- Chemical freezeout point
  (μ_B, T)=(406,145)
  from statistical model

Critical Region





QCD critical point

(μ_B, T)=(550,159)

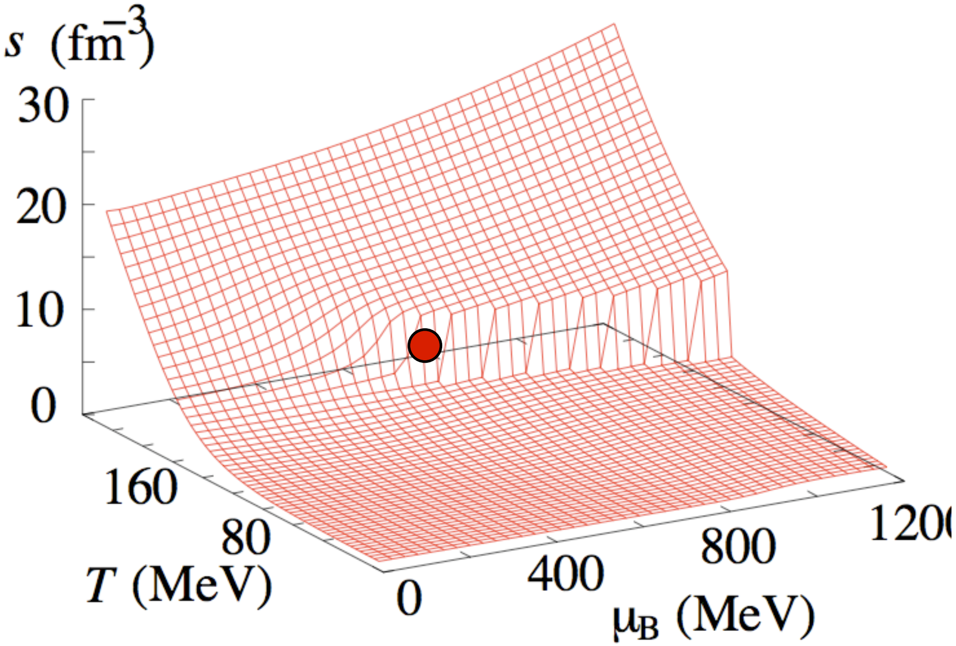
- Entropy density

$$S_{\mathrm{real}}(T,\mu_B) = \frac{1}{2}\{1-\tanh[S_c(T,\mu_B)]\}S_H(T,\mu_B) + \frac{1}{2}\{1+\tanh[S_c(T,\mu_B)]\}S_Q(T,\mu_B)$$

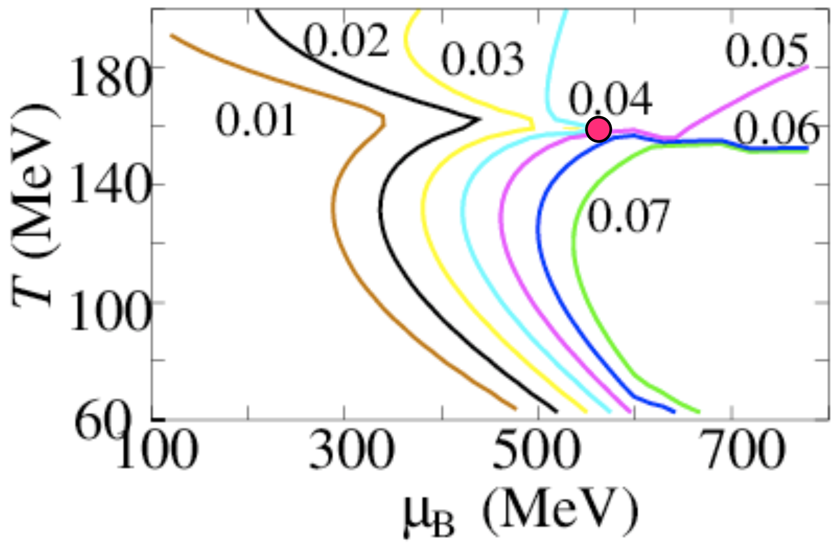
Equation of State

- **QCD critical point** $(\mu_B, T)=(550,159)$

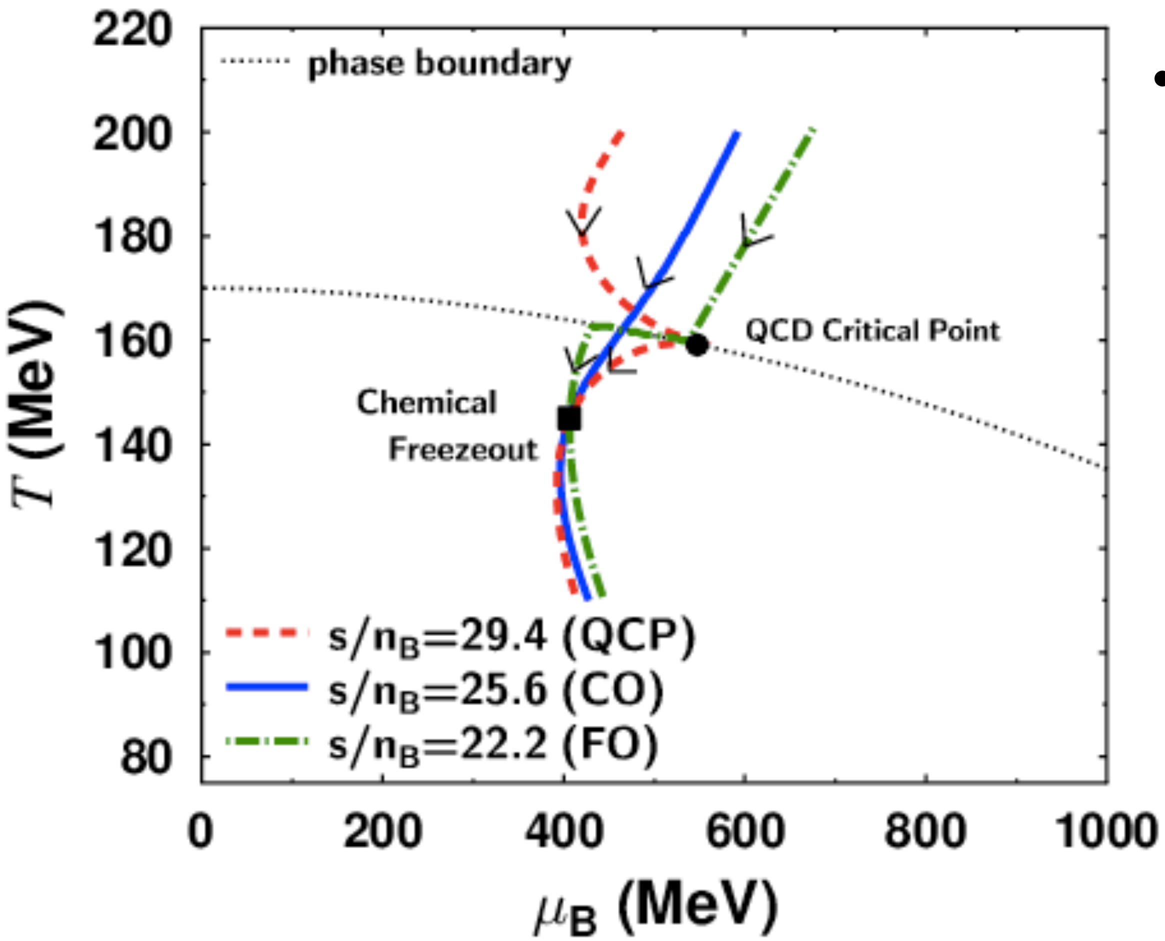
entropy density



trajectories



Isentropic Trajectories



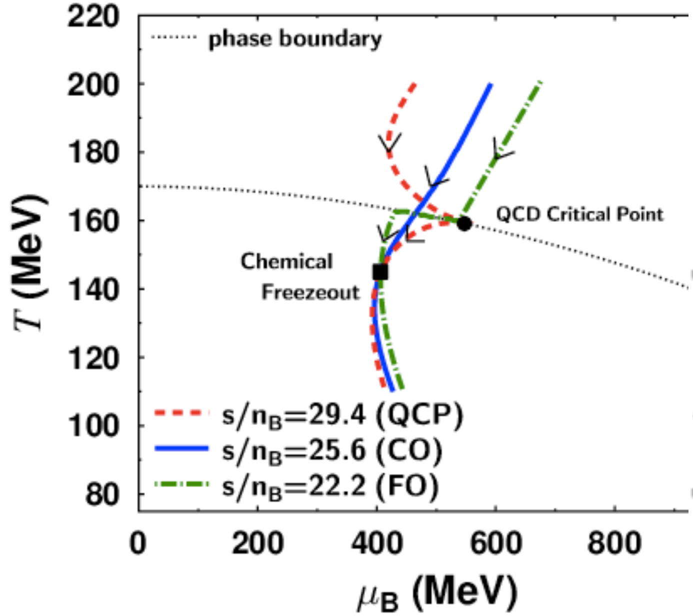
- Hadronization occurs from the phase boundary and chemical freezeout point

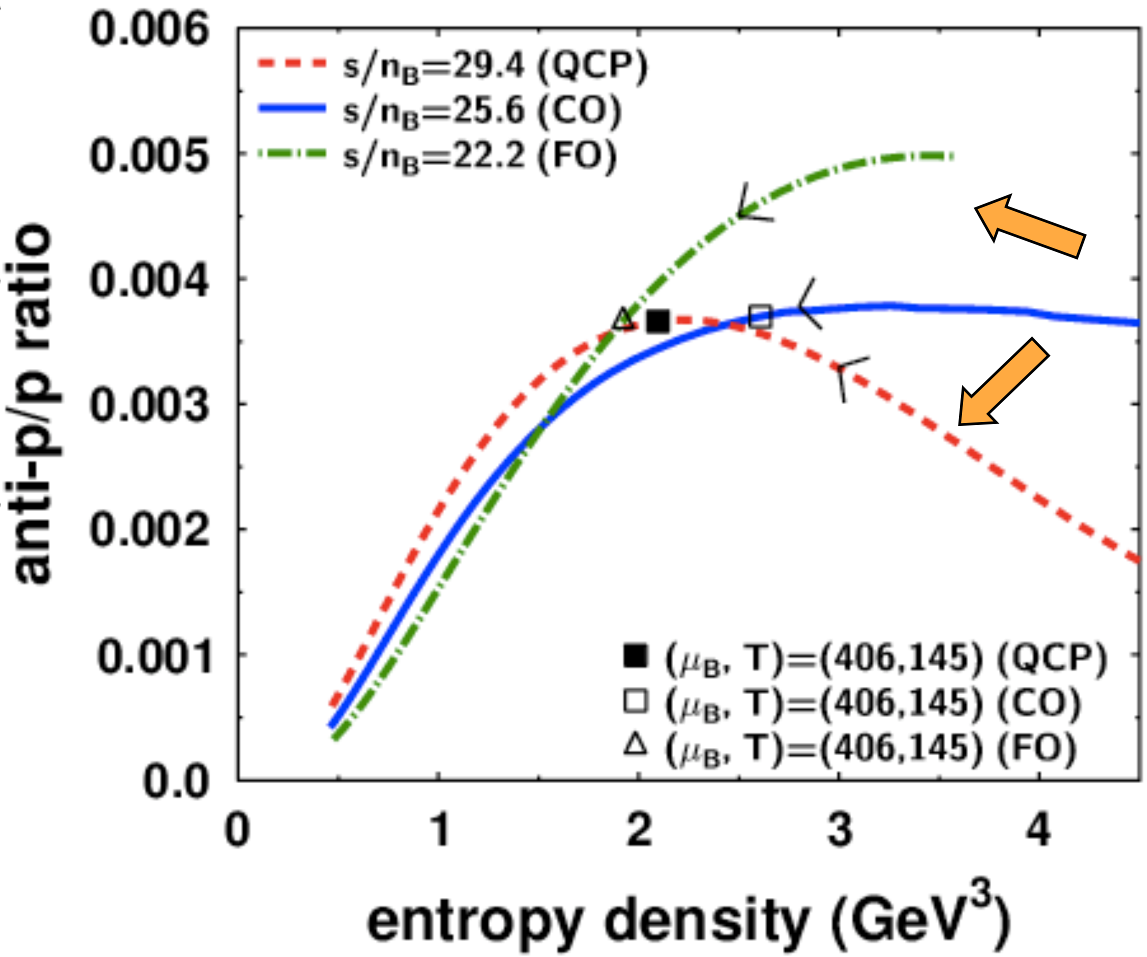
- FO, CO ↗ $\frac{\mu_B}{T}$
- QCP →

➡ $\bar{p}/p$ **ratio**

$$\bar{p}/p \sim \exp\left(-\frac{2\mu_B}{T}\right)$$

Signature of QCP





$$\bar{p}/p \sim \exp\left(-\frac{2\mu_B}{T}\right)$$

- decreases (FO,CO)
- increases (QCP)

with QCP
steeper $\bar{p}$ spectra at high P_T

Toward Detailed Analyses

- **Realistic Dynamical Model**
  - 3D Hydro + UrQMD Model
- **The QCD Critical Point**
  - Focusing effect near the QCD critical point in isentropic trajectories on the T-μ_B plane

  Location of the QCD critical point ⇐ **lattice QCD**
  Critical region **experiments**
- **Emission Time Dependence**
  - High P_T particles emit at earlier time

3D Hydro+UrQMD with QCP

QCP: $T_E = 159$ MeV, $\mu_E = 550$ MeV

T_c=170 MeV at μ_B=0 MeV

Initial Conditions

- Energy density

  $\varepsilon(x,y,\eta) = \varepsilon_{\max} W(x,y;b)H(\eta)$

- Baryon number density

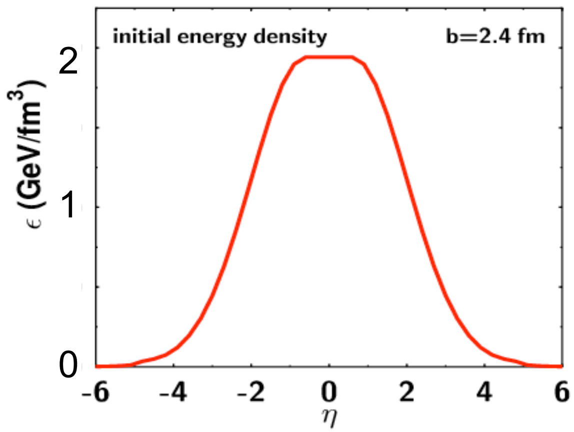
  $n_B(x,y,\eta) = n_{B\max} W(x,y;b)H(\eta)$

- Parameters $\begin{cases} \tau_0 = 0.6 \text{ fm/c} \\ \eta_0 = 0.5 \quad \sigma_\eta = 1.5 \end{cases}$

- Flow

  $v_L = \eta$ (Bjorken's solution); $v_T = 0$

•longitudinal direction: $H(\eta)$



$\varepsilon_{\max}$ GeV/fm^3	$n_{B\max}$ fm^{-3}
2.0	0.15

EOS: QCP, Bag Model

Switching temperature

T_{SW}=150 [MeV]

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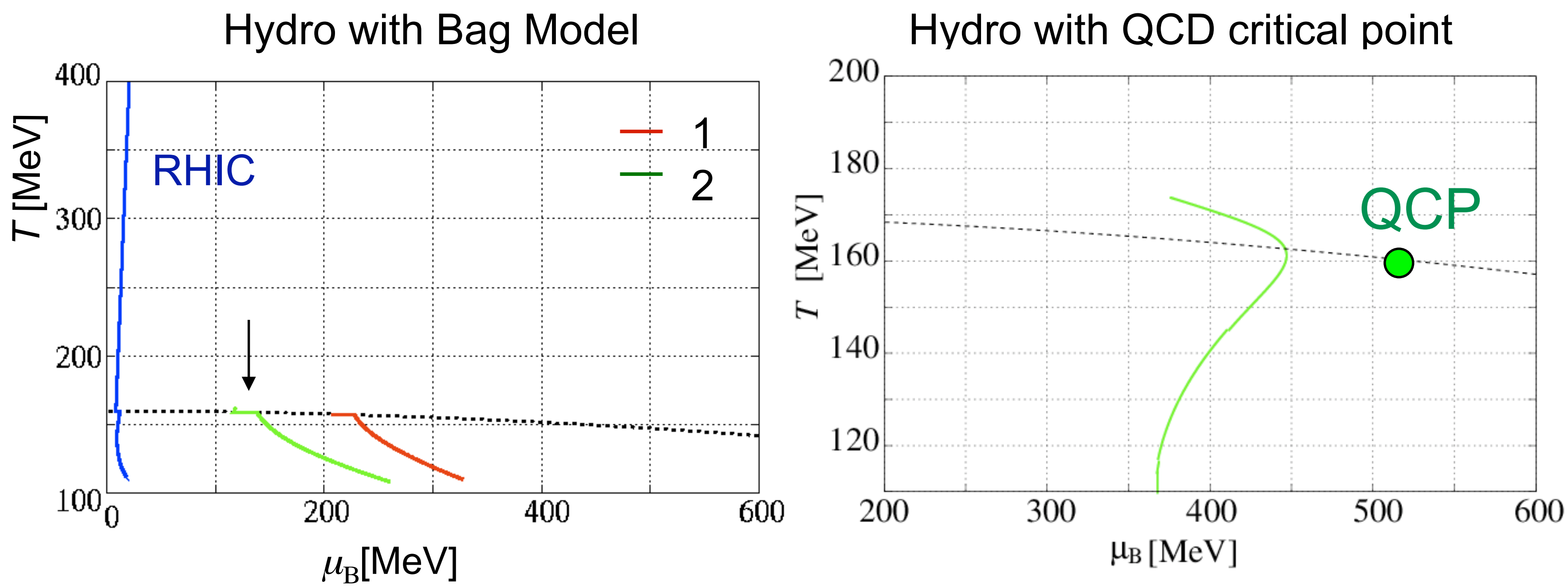
Isentropic Trajectories in Hydro

- *T* and μ_B in one volume element close to center



- T_f=110 MeV
- Behavior of isentropic trajectories in hydro with QCP is different from one in hydro with bag model.
- Focusing effect appears in hydro with QCD critical point.

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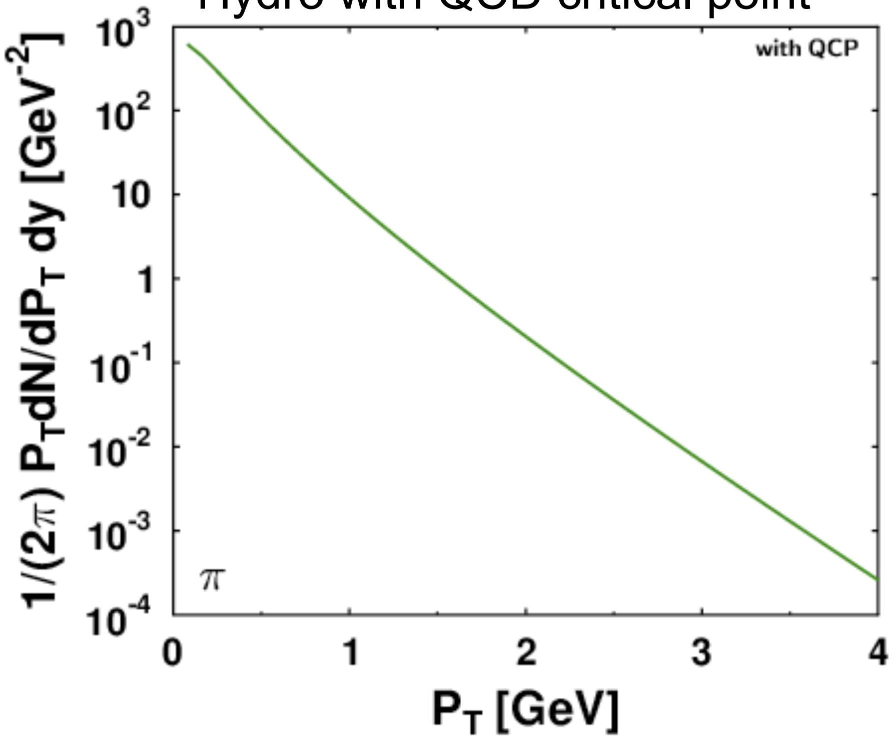
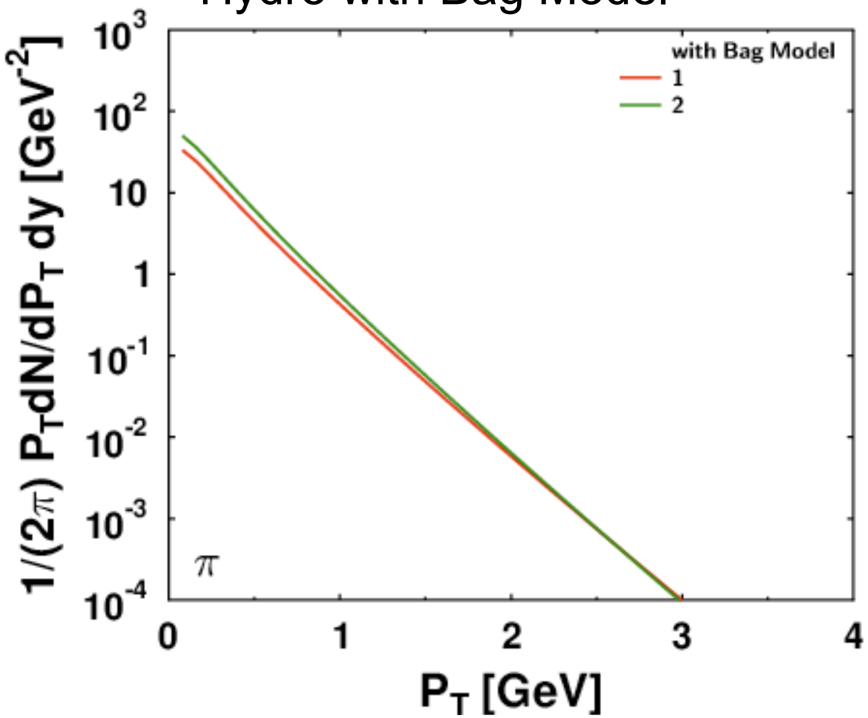
P_T Spectra in 3D Hydro

Hydro with QCD critical point



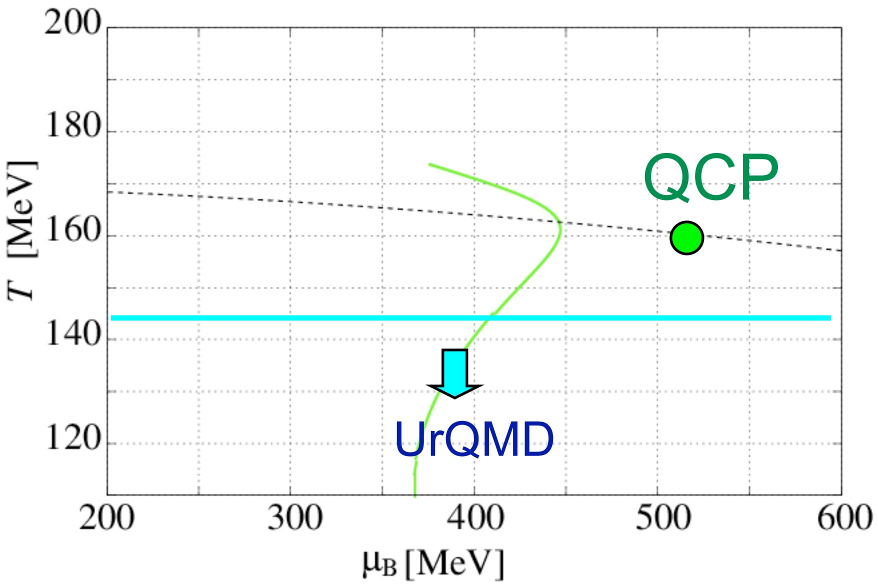
Hydro with Bag Model



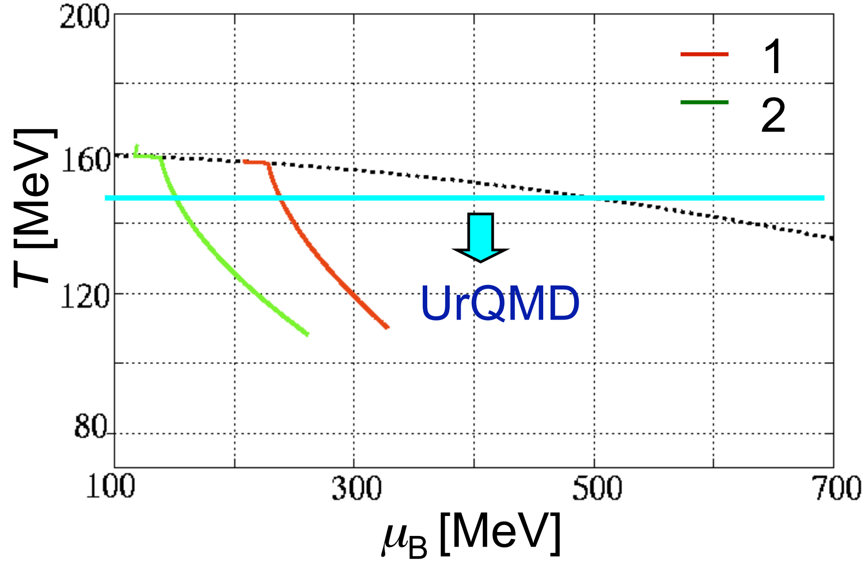
- T_f=110 MeV
- P_T slope is almost the same.

3D Hydro + UrQMD

Hydro with QCD critical point



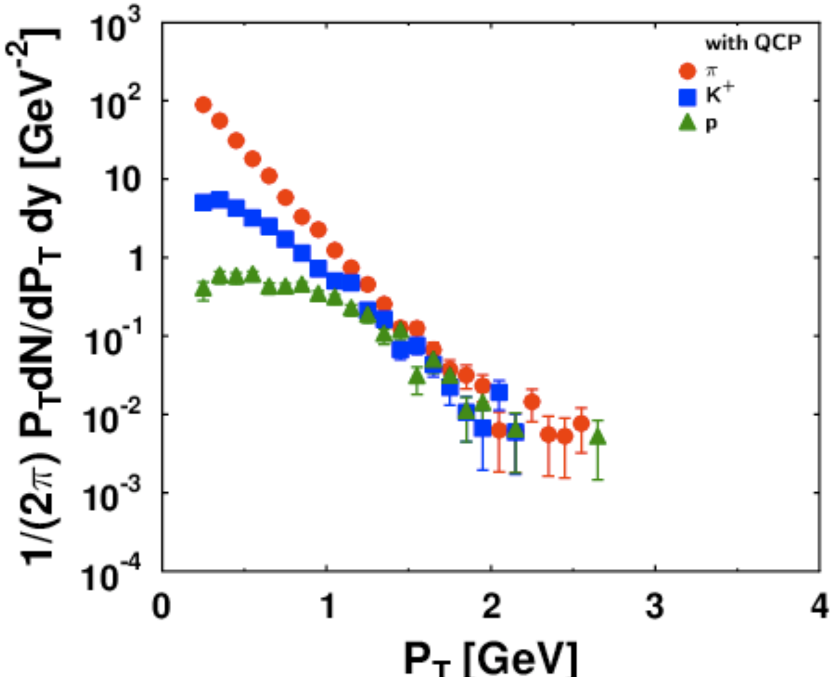
Hydro with Bag Model



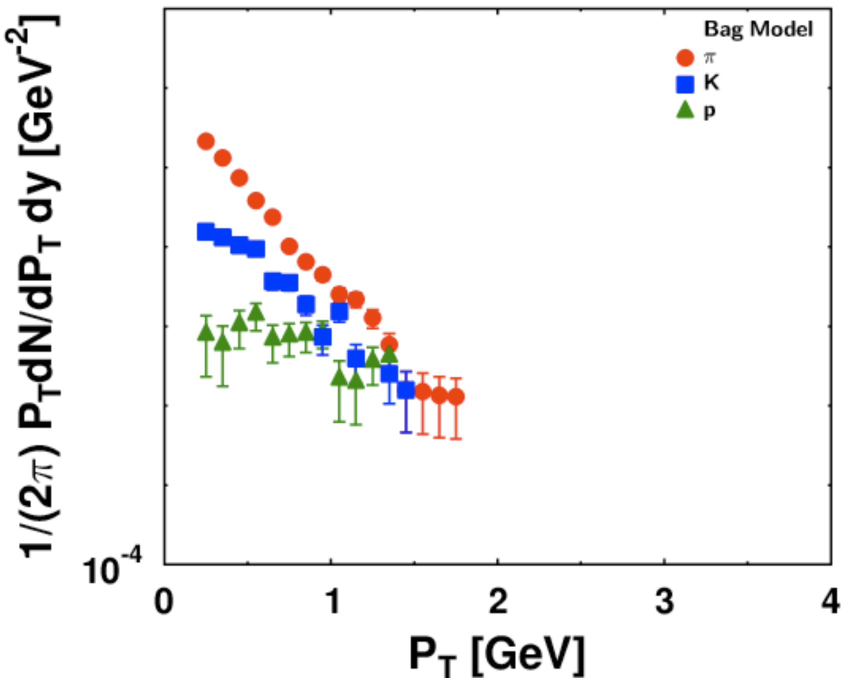
Switching temperature: 150 MeV

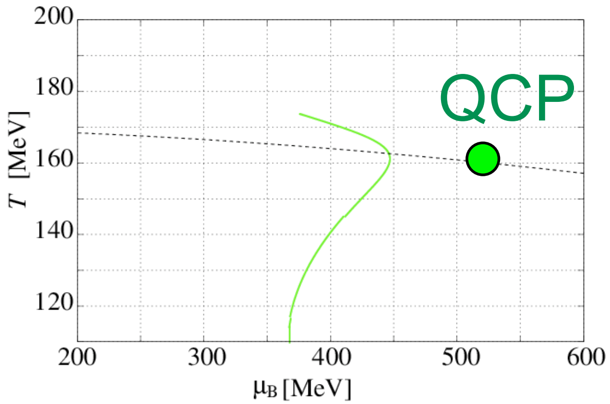
P_T Spectra

QCD critical point



Bag Model





Because of focusing effect

At T_{SW} $\langle \mu_B \rangle_{QCP} > \langle \mu_B \rangle_{BG} \Rightarrow \frac{p}{\pi}_{QCP} > \frac{p}{\pi}_{BG}$

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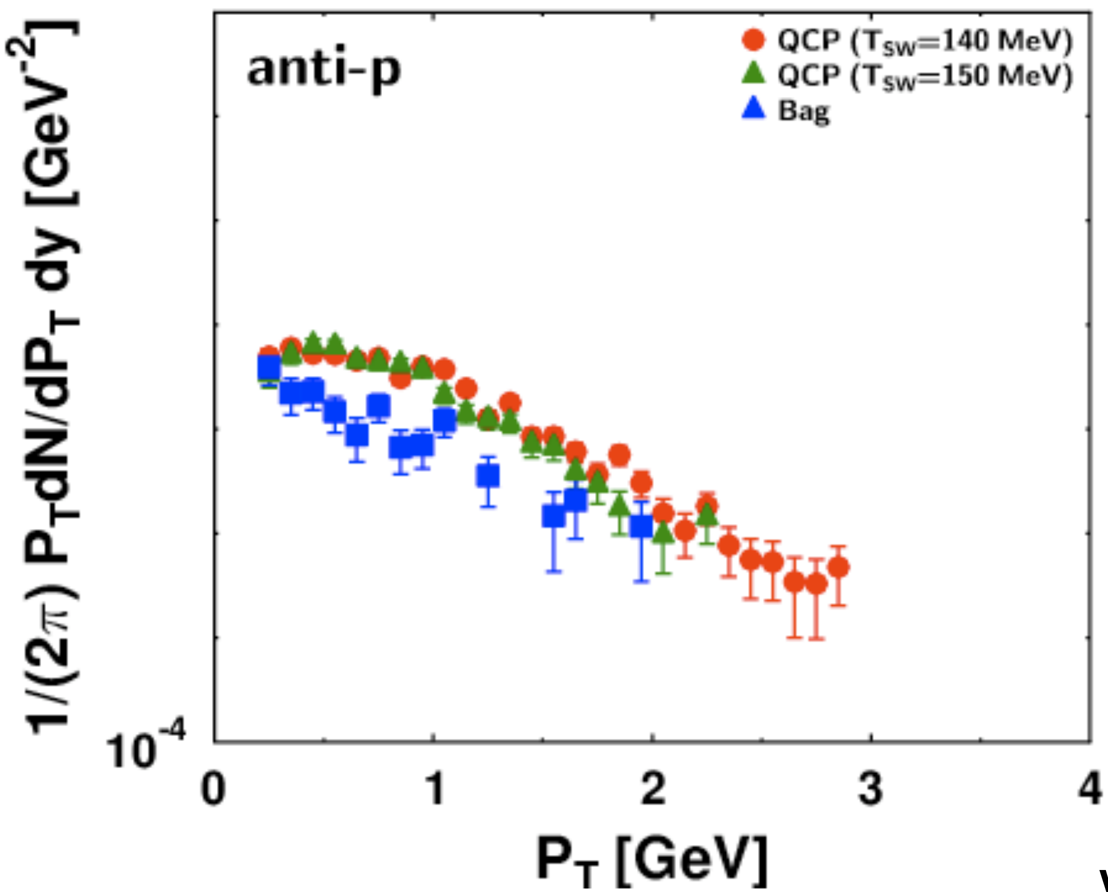
$\overline{p}$ Spectra



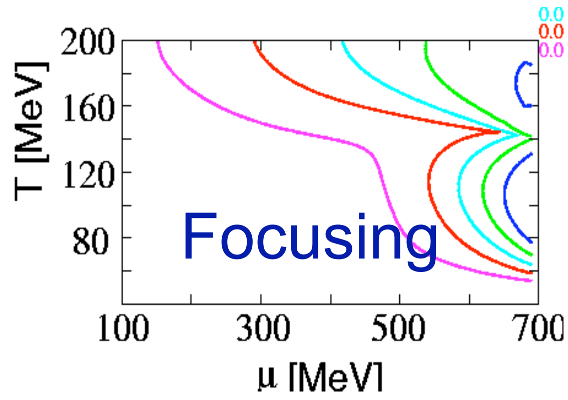
work in progress

▪Narrow collision energy window is needed. ← NA49: anti-p spectra

•Careful choice for initial condition of hydro calculation

Summary

- **3D Hydro + UrQMD Model with the QCD critical point**
  - Isentropic trajectories
  - P_T spectra, hadron ratio



- **From experiments**

  information of the QCD critical point

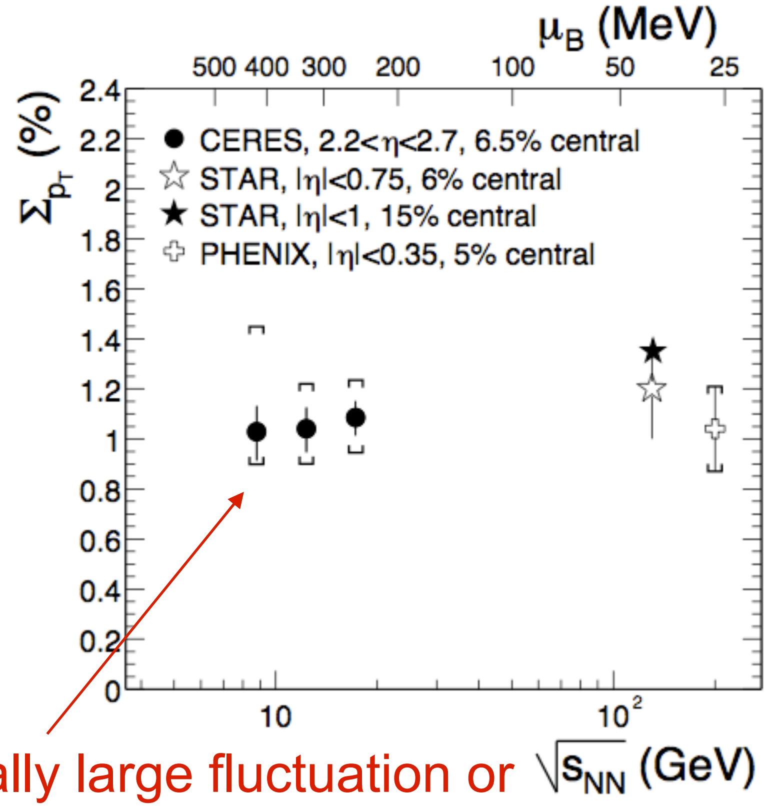
  location, critical region, existence…
- **Physical observables**
  - Anti-p/p ratio : promising and clear signature

Back Up

Consequence of QCP?

CERES:Nucl.Phys.A727(2003)97

Fluctuations
CERES
40,80,158 AGeV Pb+Au collisions

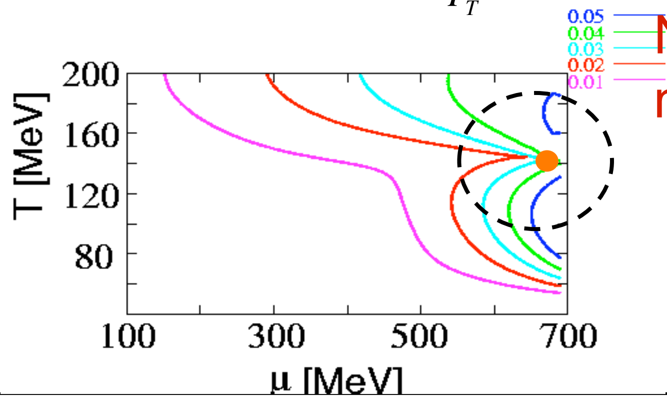


Mean P_T Fluctuation

$$\sigma^2_{PT,dyb} \equiv \langle \Delta M^2_{PT} \rangle - \frac{\Delta P_T^2}{\langle N \rangle}$$

$$\langle M_x^2 \rangle \equiv \frac{\sum_{j=1}^{n} N_j \left(M_x^j - \langle M_x \rangle\right)^2}{\sum_{j=1}^{n} N_j}$$

$$\Sigma_{PT} \equiv \mathrm{sgn}(\sigma^2_{PT,dyn}) \frac{\sqrt{|\sigma^2_{dyn}|}}{P_T}$$



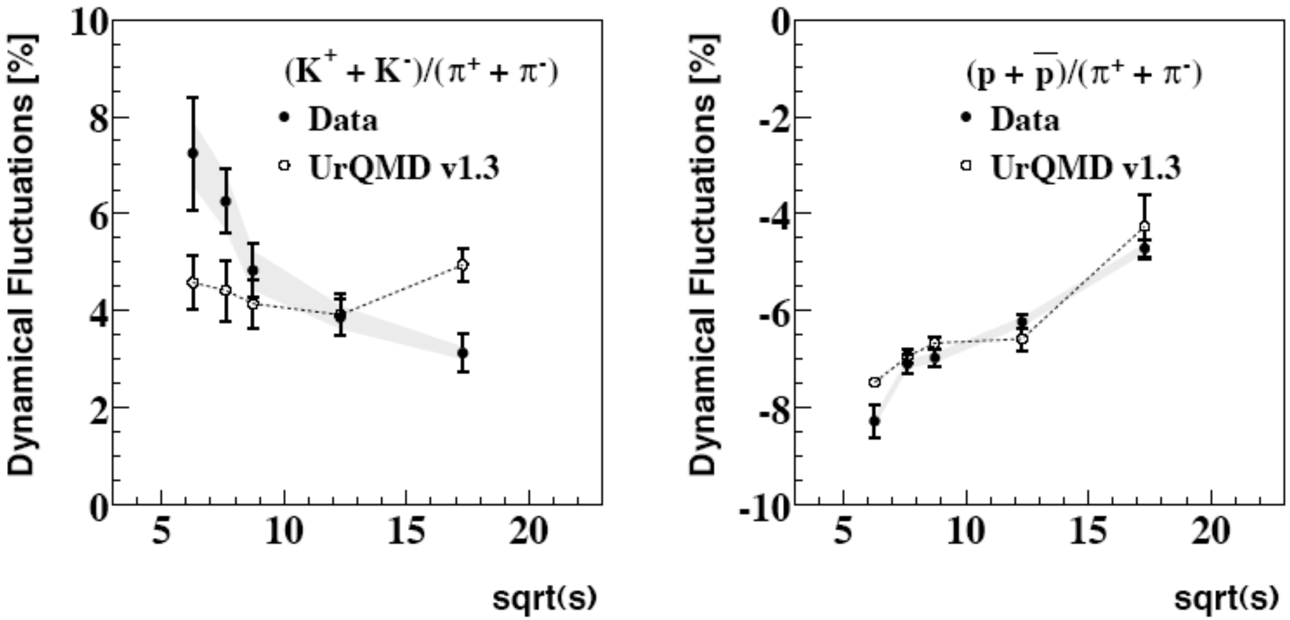
No unusually large fluctuation or non-monotonic behavior

CEP: attractor of isentropic trajectories

Similar correlation length and fluctuation is observed near QCP.

QCP?

NA49



- Suggestion of QCD critical end point?
- Fluctuation ← Conservation

Singular Part of EOS

- **Gibbs Free Energy**

Guida and Zinn-Justin NPB486(97)626

$$G(h,r) = F(M,r) - Mh$$

Free energy: $F(M,r) = h_0 M_0 R^{2-\alpha} g(\theta)$ ← $h = \left(\frac{\partial F}{\partial M}\right)\Big|_r$ $\quad \alpha = 0.11$

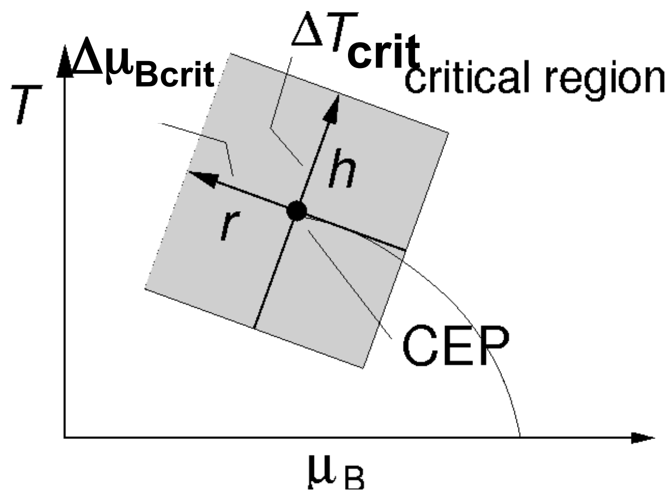
- **Entropy Density for Singular Part**

$$s_c = -\frac{\partial G}{\partial T}\Big|_\mu = -\frac{\partial G}{\partial h}\Big|_r \frac{\partial h}{\partial T} - \frac{\partial G}{\partial r}\Big|_h \frac{\partial r}{\partial T}$$

$$\begin{cases} \frac{\partial G}{\partial h}\Big|_r = -M \\ \frac{\partial G}{\partial r}\Big|_h = \frac{\partial F}{\partial r}\Big|_h - \frac{\partial M}{\partial r}\Big|_h h \end{cases}$$

mapping

$$(r,h) \leftrightarrow (T,\mu_B)$$



Thermodynamical Quantities

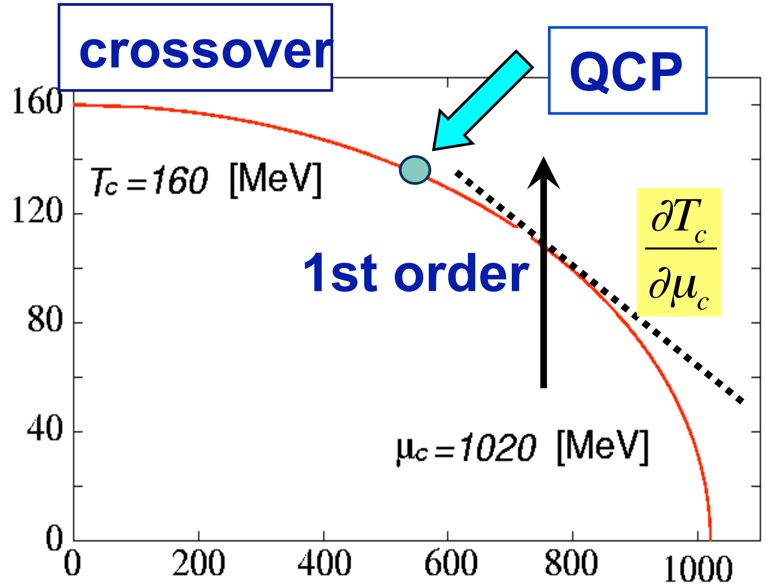
$$S_{\rm real}(T,\mu_B)=\frac{1}{2}\{1-\tanh[S_c(T,\mu_B)]\}S_H(T,\mu_B)+\frac{1}{2}\{1+\tanh[S_c(T,\mu_B)]\}S_Q(T,\mu_B)$$

Baryon number density

$$n_B(T,\mu_B)=\frac{\partial P}{\partial \mu_B}=\int_0^T \frac{\partial s}{\partial \mu_B}(T',\mu_B)dT'+n_B(0,\mu_B)$$

$$+\left|\frac{\partial T_C(\mu_B)}{\partial \mu_C}\right|\left(S_Q(T_C,\mu_B(T_C))-S_H(T_C,\mu_B(T_C))\right)$$

1st order



Pressure

$$P(T,\mu_B)=\int_0^T S_{\rm real}(T',\mu_B)dT'+P(0,\mu_B)$$

Energy Density

$$\varepsilon=TS_{\rm real}-P-\mu_B n_B$$

Chiho NONAKA

Focusing Effect

3d Ising Model ⟷ **QCD**

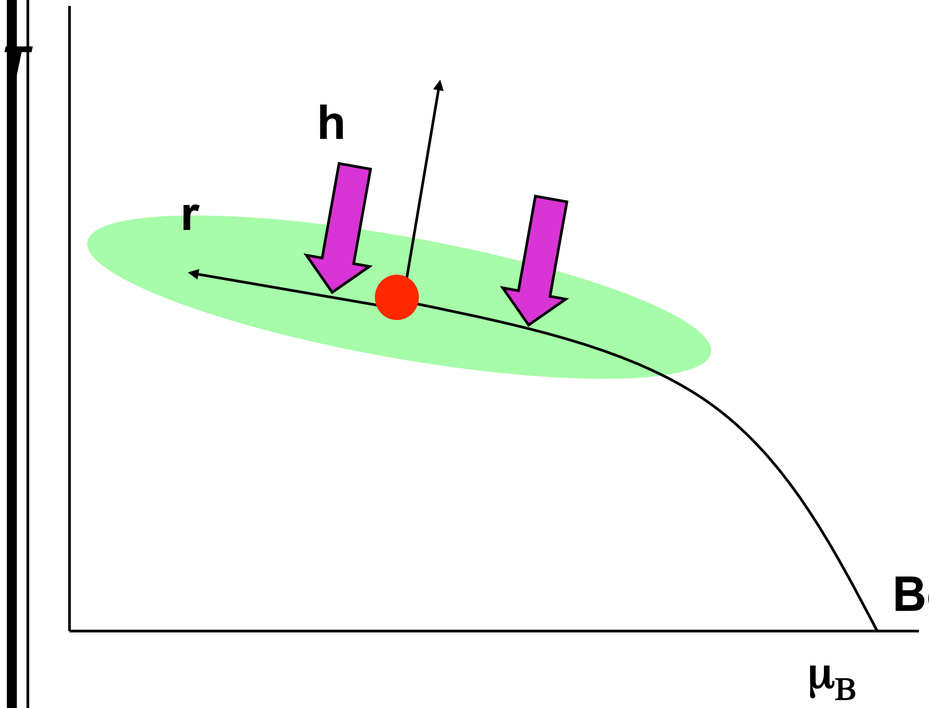
Same Universality Class

$r = \frac{T - T_C}{T_C}$

h: extermal magnetic field

$T\,,\ \mu_B$

$(r, h) \longleftrightarrow (T, \mu_B)$



•Critical exponent

$$M = \begin{cases} |r|^{\beta} & h = +0 \\ -|r|^{\beta} & h = -0 \end{cases} \qquad M = \mathrm{sgn}(h)|h|^{1/\delta} \quad r = 0$$

$$\beta = 0.36, \delta = 4.86$$

Guida and Zinn-Justin, NPB486(97)626

•Critical slowing down

dynamical critical exponent along *h* is dominant.

Berdnikov and Rajagopal,PRC61,105017(99)