THE SEARCH FOR THE QCD CRITICAL POINT

USING LATTICE QCD CALCULATIONS AND HEAVY ION COLLISION EXPERIMENTS

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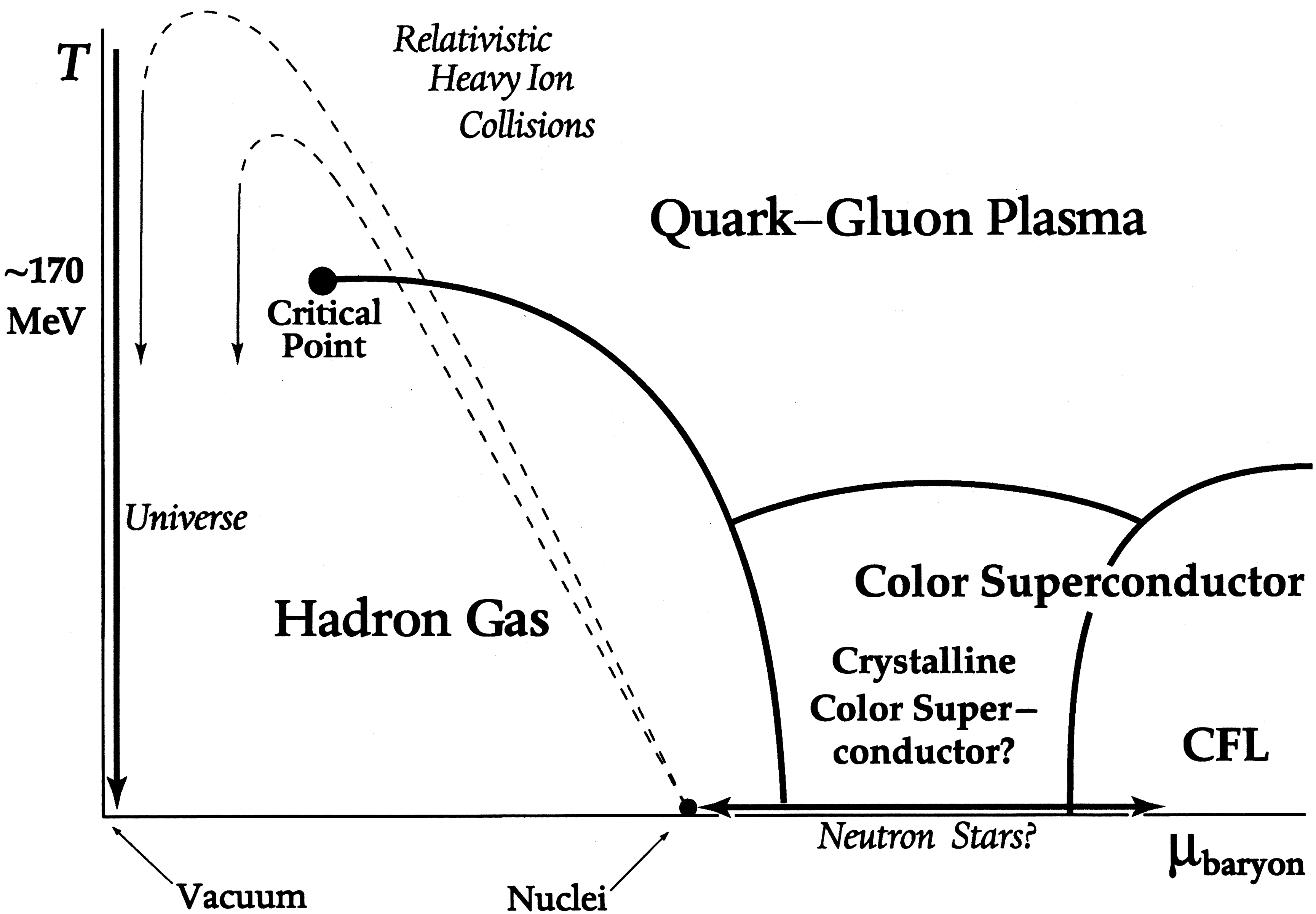
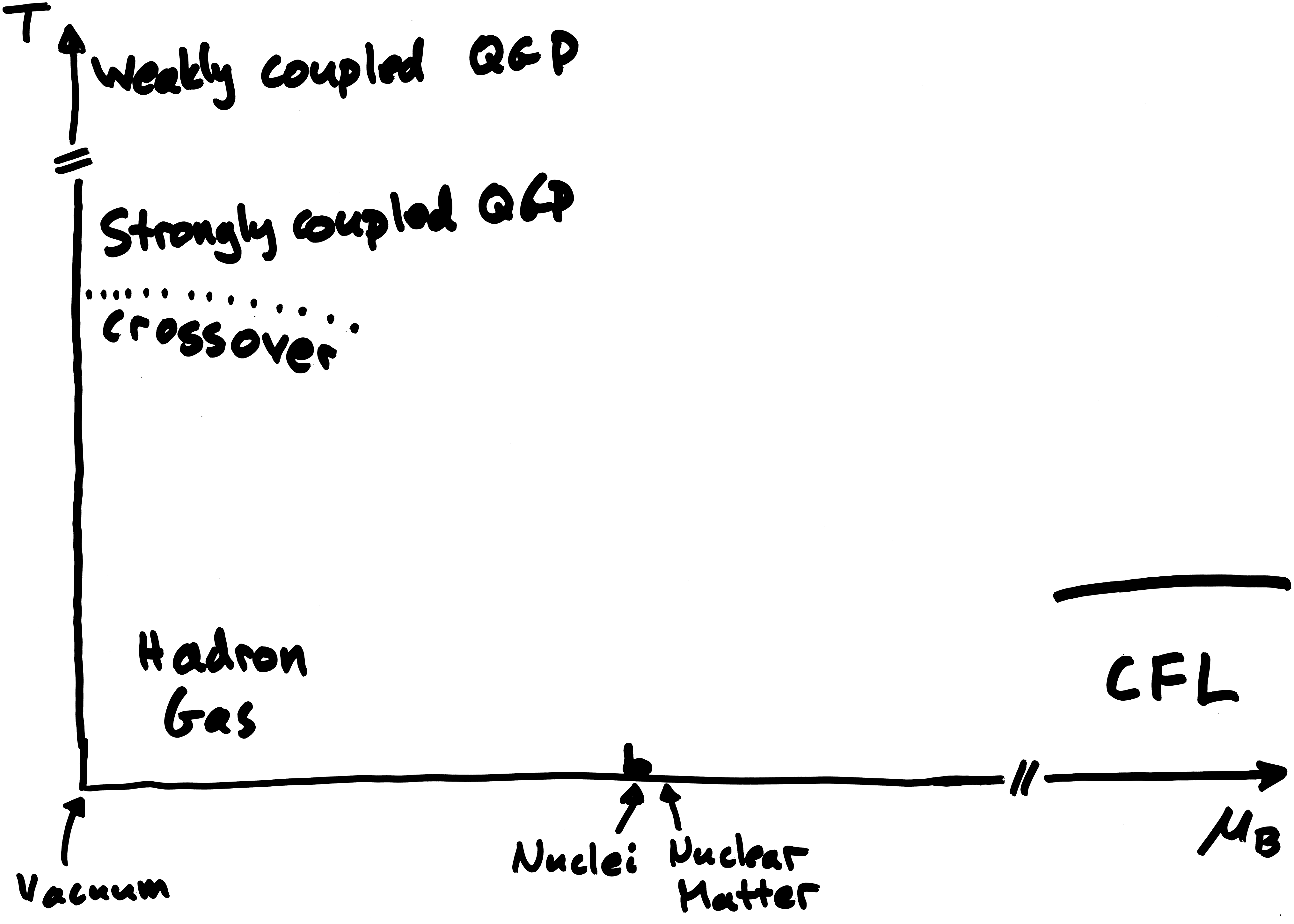
INT, Seattle. 8/11/08

EXPLORING *the* PHASES *of* QCD



WHAT WE KNOW, SO FAR



$T \neq 0$; $\mu \neq 0$; μ/T NOT LARGE

- a regime explored by heavy ion collisions
- a regime explored by lattice calculations that rely on smallness of μ/T to keep fermion sign problem under control. [$\mu \neq 0 \rightarrow$ complex Euclidean action $\rightarrow$ sign problem that makes difficulty of standard Monte Carlo $\sim \exp V$.]
- Either method may be used to locate the CRITICAL POINT, a 2^{nd} order point where a line of 1^{st} order transitions ends, if it is located at a μ/T that is not too large....

LOCATING THE CRITICAL POINT...

- either via lattice calculations
- or via experimental detection of its signatures

would add a point and a line to the known QCD phase diagram.

OUTLINE OF TALK (AND WEEK)

- Lattice calculations
- Experimental signatures and searches

SEVERAL LATTICE METHODS

① Reweighting   Fodor + Katz

Want physics at ⓐ $\equiv (\mu, T_a)$

Simulate using an ensemble of configurations at ⓑ $\equiv (0, T_b)$, and "reweight": lump difference between physics at ⓑ and ⓐ into observables.

$$\text{Difficulty} \sim \exp\left[\frac{|F_b - F_a| V}{T}\right]$$

F+K: choose T_b to minimize ↺

BUT: still cannot use method at large volumes....

The endpoint is at $T_E = 162 \pm 2$ MeV, $\mu_E = 360 \pm 40$ MeV. As expected, μ_E decreased as we decreased the light quark masses down to their physical values (at approximately three-times larger $m_{u,d}$ the critical point was at μ_E=720 MeV; see [8]).

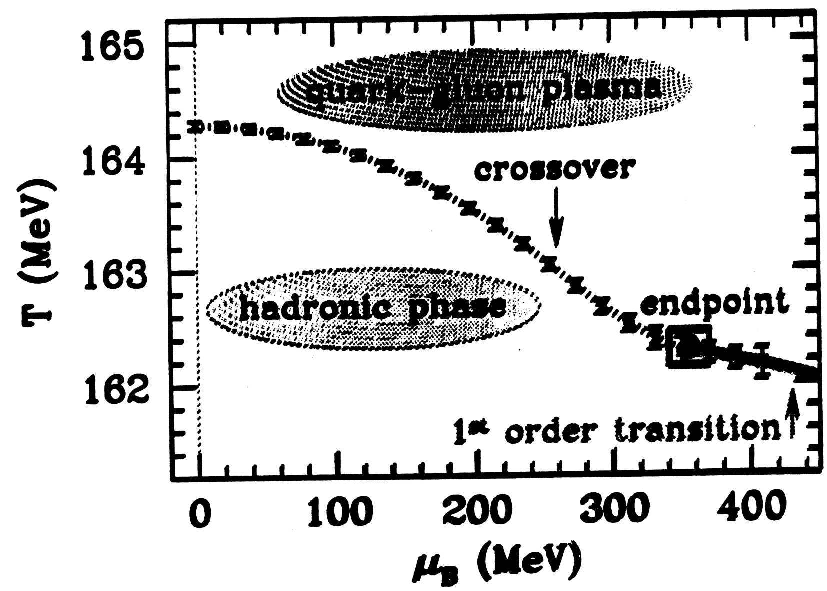


Figure 2: The phase diagram in physical units. Dotted line illustrates the crossover, solid line the first order phase transition. The small square shows the endpoint. The depicted errors originate from the reweighting procedure. Note, that an overall additional error of 1.3% comes from the error of the scale determination at T=0. Combining the two sources of uncertainties one obtains $T_E = 162 \pm 2$ MeV and $\mu_E = 360 \pm 40$ MeV.

The above result is a significant improvement on our previous analysis [8] by two means. We increased the physical volume by a factor of three and decreased the light quark masses by a factor of three. Increasing the volumes did not influence the results, which indicates the reliability of the finite volume analysis. Clearly, more work is needed to get the final values. Most importantly one has to extrapolate to the continuum limit.

Fodor, Katz
2004

$\mu_E = 360 \pm 40$ MeV

$\frac{\mu_E}{T_E} = 2.22 \pm .25$

statistical errors only

CONCERNS, aka "SYSTEMATIC ISSUES"

- $N_\tau = 4$ (no continuum limit)
- $V = 12^3$, and method must break down for $V \to \infty$

- $\frac{\mu_E}{3} \simeq \frac{m_\pi}{2}$. This was also the case in older F+K calculation at larger m_π. If this is not a coincidence, it is a problem. Splittorf
  [$\mu_q = m_\pi/2$ is where phase quenched QCD has onset of pion condensation.]

- $\frac{m}{T}$ held fixed during reweighting, not m.

ALL these, except for $V \to \infty$, are IMPROVABLE.

② Continue from imaginary μ.

deForcrand + Philipsen

D'Elia + Lombardo et al

Simulate at $\mu = i\mu_I$; calculate $T_c(\mu_I)$; Taylor expand :

$$= C_0 + C_2 \mu_I^2 + C_4 \mu_I^4 + \dots$$

- valid for $\frac{\mu_I}{T} < \frac{\pi}{3}$
- Good luck ... $C_4, C_6, \dots$ terms all small over this range.
- So, boldly continue:

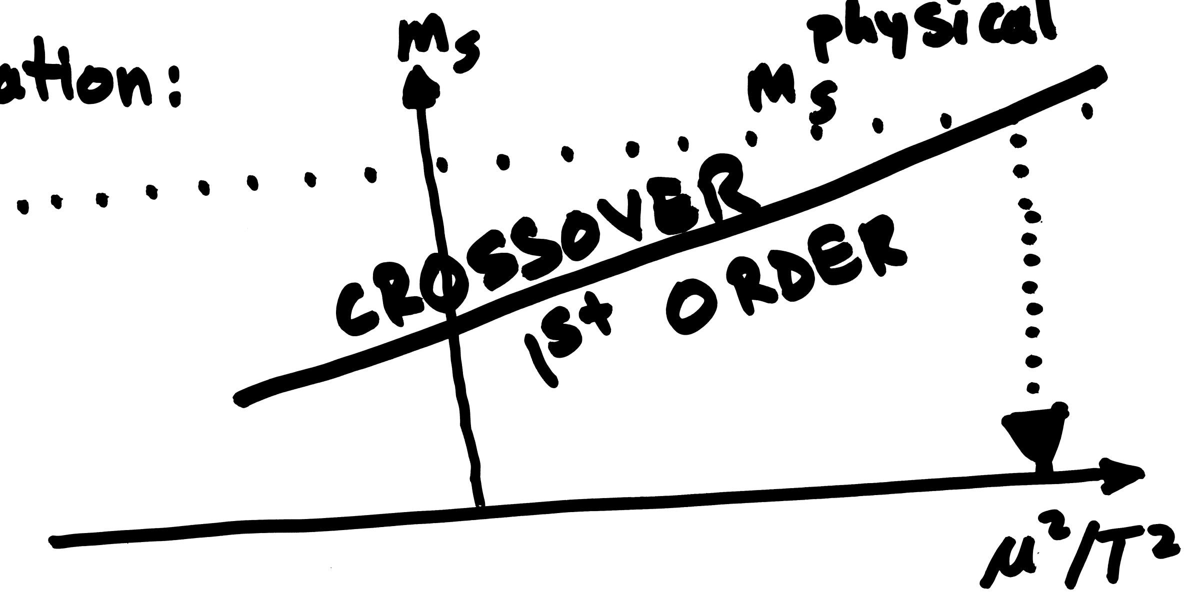
$$T_c(\mu) = C_0 - C_2 \mu^2 + C_4 \mu^4 \dots$$

Curvature of crossover line on phase diagram

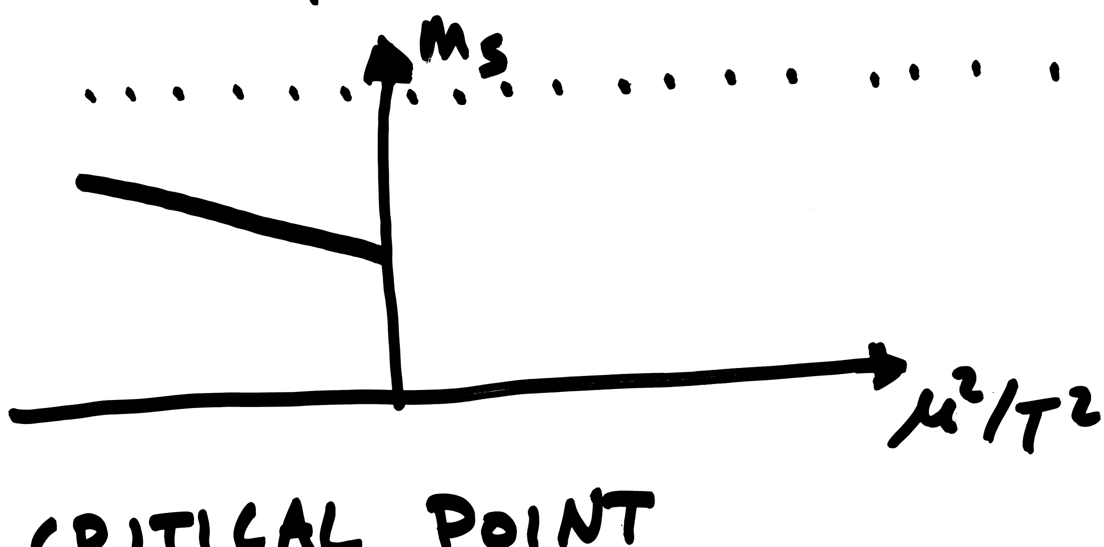
CRITICAL POINT ??

- Calculate
$$\frac{\partial}{\partial \mu^2}\left[\begin{array}{l} m_q \text{ at which transition goes} \\ \text{from } 1^{st} \text{ order to crossover}\end{array}\right]$$

- Expectation:



- deForcrand + Philipsen find:



- $\Rightarrow$ NO CRITICAL POINT
  with $\frac{\mu}{T} < \mathcal{O}(1)$.

CONCERNS, aka "SYSTEMATIC ISSUES"

Lets defer their discussion to after Philippe's talk, but here are two:

- $N_\tau = 4$
- Staggered fermions with

  $N_f = 3$ or $2+1$ .....

  also an issue for F+K

  - $\mathrm{Det}^{3/4}$ or $\mathrm{Det}^{1/2}\,\mathrm{Det}^{1/4}$
  - First order phase transition at small m_s originates from 't Hooft $uds\bar{u}\bar{d}\bar{s}$ interaction, in low energy effective theory. Pisarski Wilczek
  - do staggered fermions describe this adequately ??

    Fukushima, Stephanov

③ Taylor Expansion of the Pressure.

Bielefeld-Swansea; Gavai Gupta

Calculate the coefficients in:

$$\frac{P}{T^4} = b_0(T) + b_2(T)\mu^2 + b_4(T)\mu^4 + b_6(T)\mu^6 + \cdots$$

and hence in:

$$\chi_B \equiv \frac{\partial^2 P}{\partial \mu^2} = C_0(T) + C_2(T)\mu^2 + C_4(T)\mu^4 + C_6(T)\mu^6 + \cdots$$

which should diverge at critical point.

Several ways to look for critical point:

- Look for μ at which χ_B peaks
- Do Taylor expansion at varying m_q and evaluate

$$\frac{\partial}{\partial \mu^2}\left[\begin{array}{l} m_q \text{ at which crossover at} \\ \mu = 0 \text{ becomes } 1^{st} \text{ order} \end{array}\right]$$

[Defer discussion of these to Karsch.]

- And ...

RADIUS OF CONVERGENCE METHOD

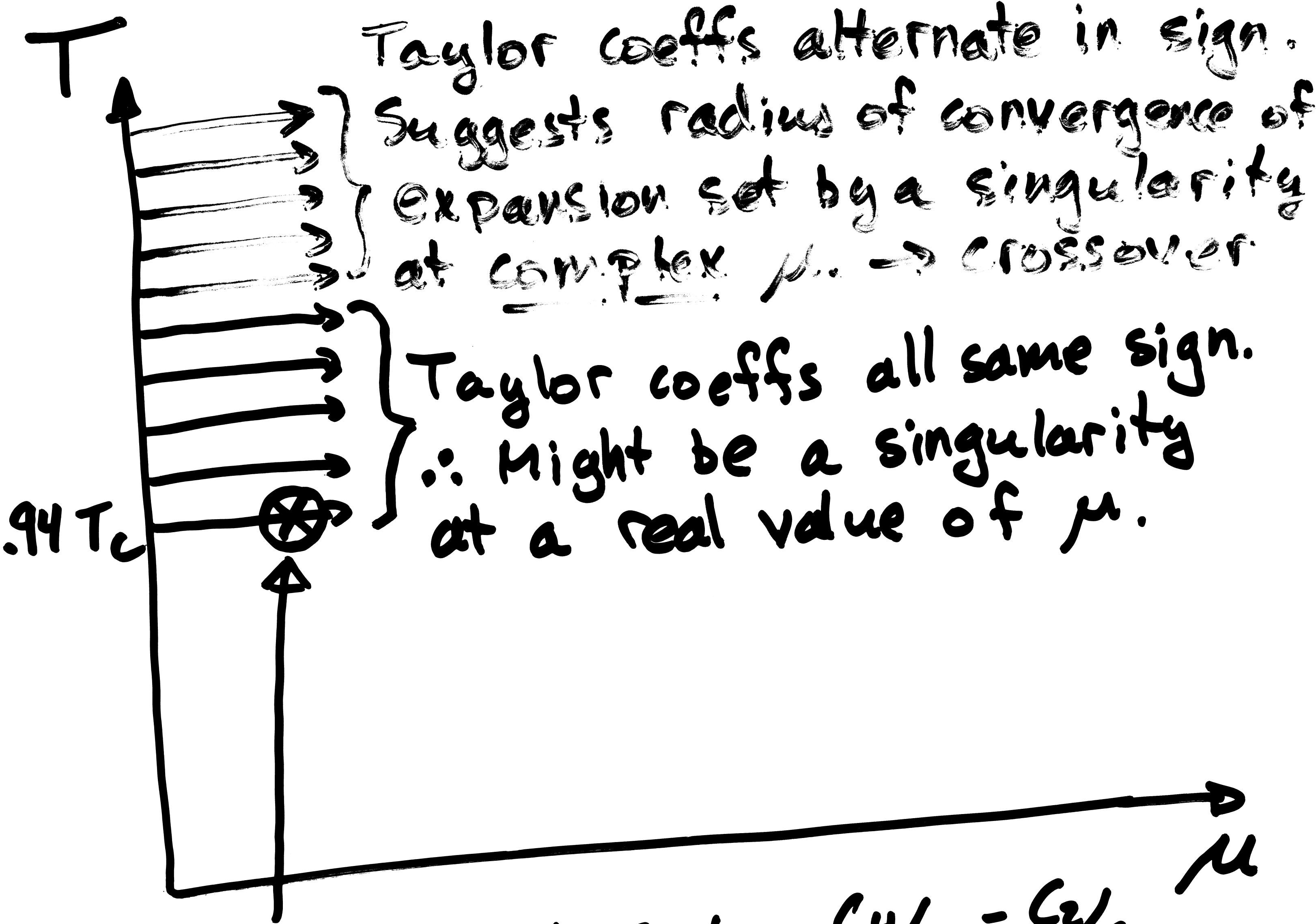
Use fact that Taylor expansion must break down at critical point.

Bielefeld Swansea ; Gavai Gupta

New results from Gavai + Gupta, June 2008 & earlier this workshop:

- $N_\tau = \underline{\underline{6}}$; $V = 24^3$
- $N_f = 2$; $m_\pi = 230$ MeV
- staggered fermions, so maybe not a bad thing that $N_f = 2$.
- Taylor coefficients $c_0(T)$, $c_2(T)$, $c_4(T)$, $c_6(T)$.

  [ie up to μ^8 term in P]



At *this* T, find $c_6/c_4 = c_4/c_2 = c_2/c_0$, as would be the case for a pole at real μ. And, as yields a consistent estimate of radius of conv.
Also, at the same T, coeffs have expected finite size scaling (upon comparing $LT = 2$ and 4).
I identify this T as T_E, and this radius of convergence as μ_E.

Gavai and Gupta find:

$$\frac{T^E}{T_c} = 0.94 \pm 0.01$$

$$\frac{\mu_E}{T_E} = 1.8 \pm 0.1$$

Issues:

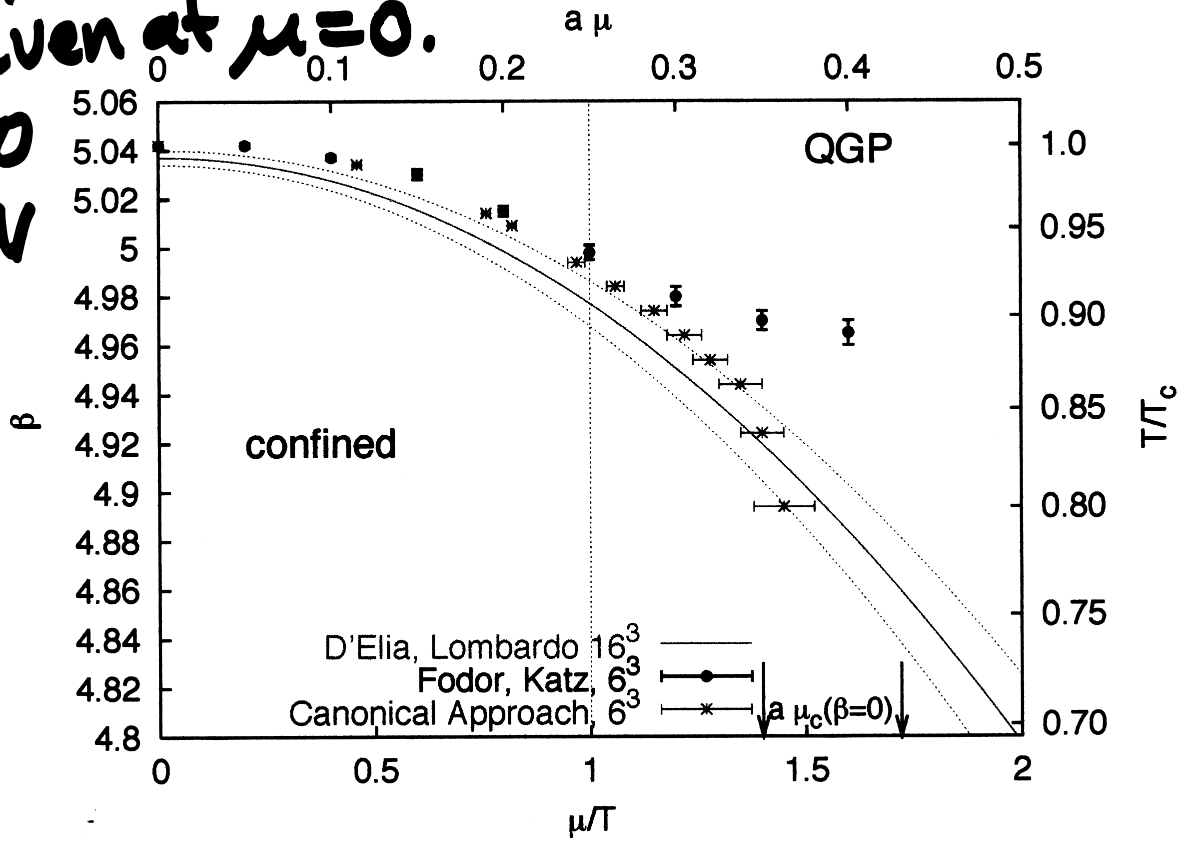
- $N_\tau = 6$. "Crawling towards the continuum limit." Gupta
- $N_f = 2 \rightarrow N_f = 2+1$
- What is the best estimator of T^E, ie what combination of criteria, given $C_0(T)$, $C_2(T)$, $C_4(T)$, $C_6(T)$?

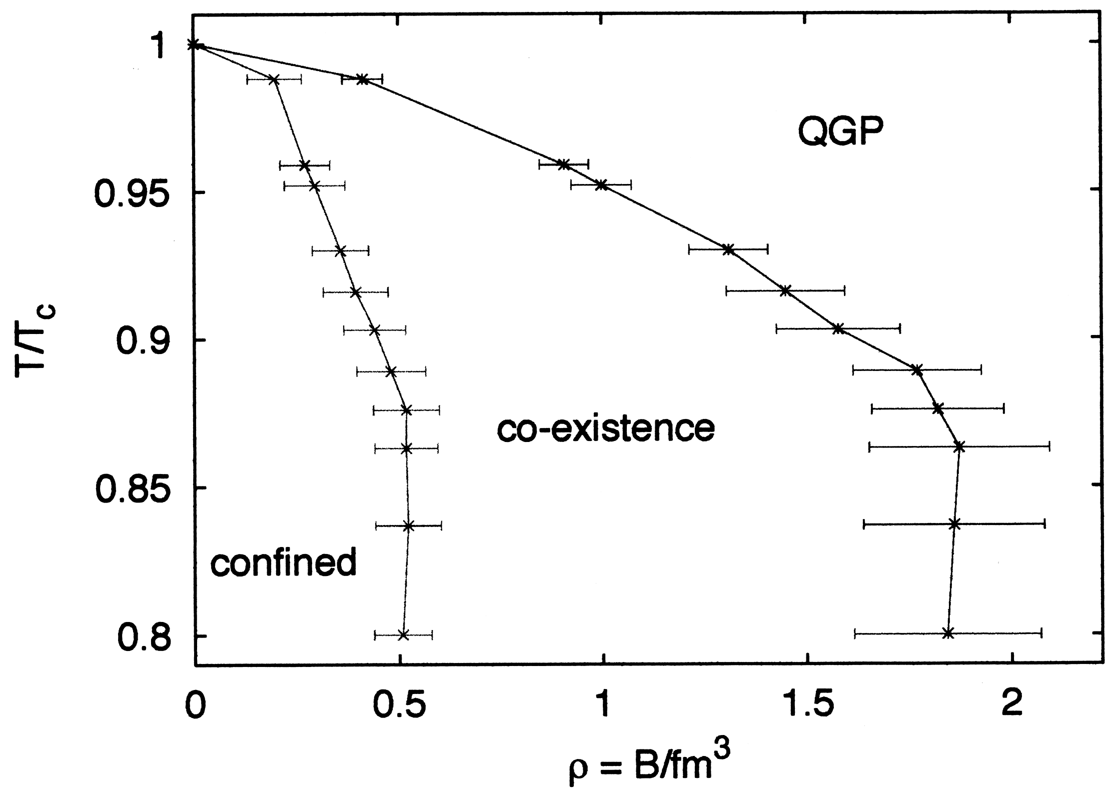
LATTICE CALCULATIONS AT FIXED n_B

$N_f = 4 \rightarrow 1^{st}$ order even at $\mu = 0$.

de Forcrand Kratochvila

$m_\pi = 350$ MeV





Calculation on 6^3 lattice, with $0 < B < 30$. $(V \sim (2\,fm)^3)$

- Will be very interesting to see what they find with $N_f = 2+1$.
- Determining the location of the critical point this way will have very different "systematic error" relative to calculations relying on $\mu/T < 1$. (ie reweighting Fodor Katz or Taylor expansion Ejiri et al, Gavai Gupta)
- In principle can be pushed to larger μ/T, but remains to be seen how large a V can be reached at a given μ or n_B.

LOCATING THE CRITICAL POINT

Location still uncertain:

$$\frac{\mu_B^{\text{critical point}}}{T_c(\mu=0)} \sim 2\,,\ > \mathcal{O}(3)\,,\ \sim 1.7$$

Fodor Katz, Philipsen deForcrand, Gavai Gupta

- gaining confidence will require "crawling towards the continuum limit", and several methods agreeing.
- If $\mu_B^{c.p.} < 3T$, this ↑ will happen
- If $\mu_B^{c.p.} > 3T$, all methods should come to agree on this. But, barring an unforeseen algorithmic breakthrough, unlikely that lattice calculations will locate it with confidence.

In the race between lattice calculations and experimental searches to locate the critical point, the lattice team is running strongly but not yet threatening to end the race.

So, lets turn to experimental searches ....

HOW CAN EXPERIMENTS LOCATE THE CRITICAL POINT?

① Need evidence that at large $\sqrt{s}$, ie small μ, collisions equilibrate well above the crossover. V_2@RHIC.

② Decrease $\sqrt{s}$, moving freezeout point to larger and larger μ_B.

③ Look for signatures:

a) Of the critical point itself. Those relying on the long wavelength gaussian fluctuations occurring only near ●. Rise and then fall as μ_B increases.

b) Onset of signatures of non-equilibri[um] "lumpy" final state expected after cooling through a first order transitio[n]

Mishustin; Dumitru Pasch Stöcker; Randrup; Koch Majumder Randrup; ...

→ NON Gaussian fluctuations

SIGNATURES OF THE CRITICAL POINT

In those collisions that pass near the critical point as they cool, find long wavelength oscillations of a mode that is a linear combination of σ (means fluctuations couple to $\pi\pi$) and baryon number. Fujii Ohtani; Son Stephanov

The more effectively equilibrium is maintained, the longer the correlation length ξ gets, the bigger the signatures:

- Gaussian event-by-event fluctuations of specific observables, calculable in magnitude in terms of ξ. Stephanov KR Shuryak
- Vary μ by varying $\sqrt{s}$, search for enhancement of these fluctuations in a window in $\sqrt{s}$, ie μ.
- Examples .... But first:

HOW LARGE CAN ξ GET?

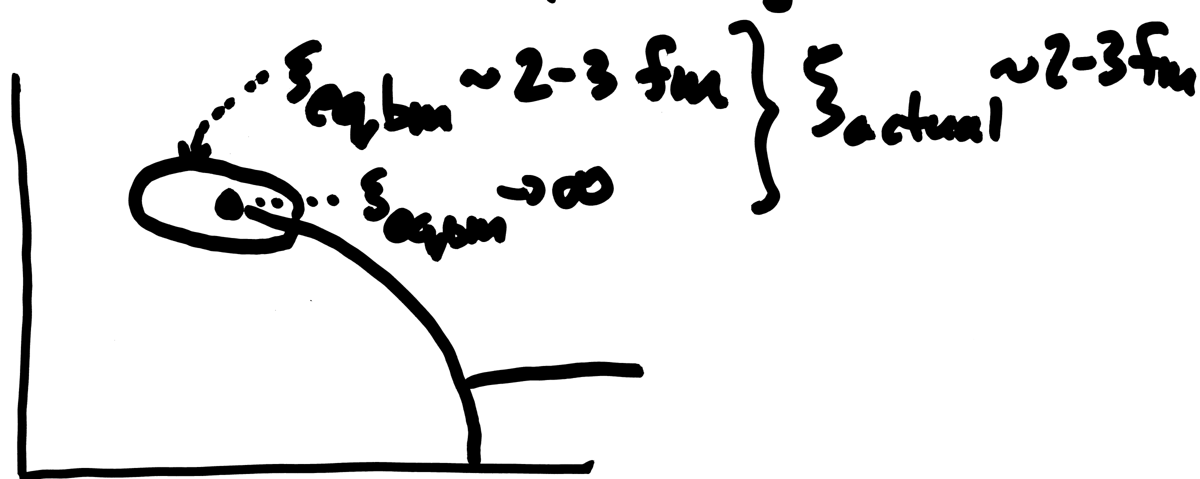
HOW CLOSE TO ● NEED WE BE?

- Obviously ξ limited by finite size of system. But, turns out that finite time is a more severe limitation.

  Berdnikov KR; Asakawa Nonaka

- Finite time spent in critical region means that even if equilibrium value of ξ is much larger, actual ξ won't grow bigger than 2-3 fm.

- Means no need to hit ● precisely.



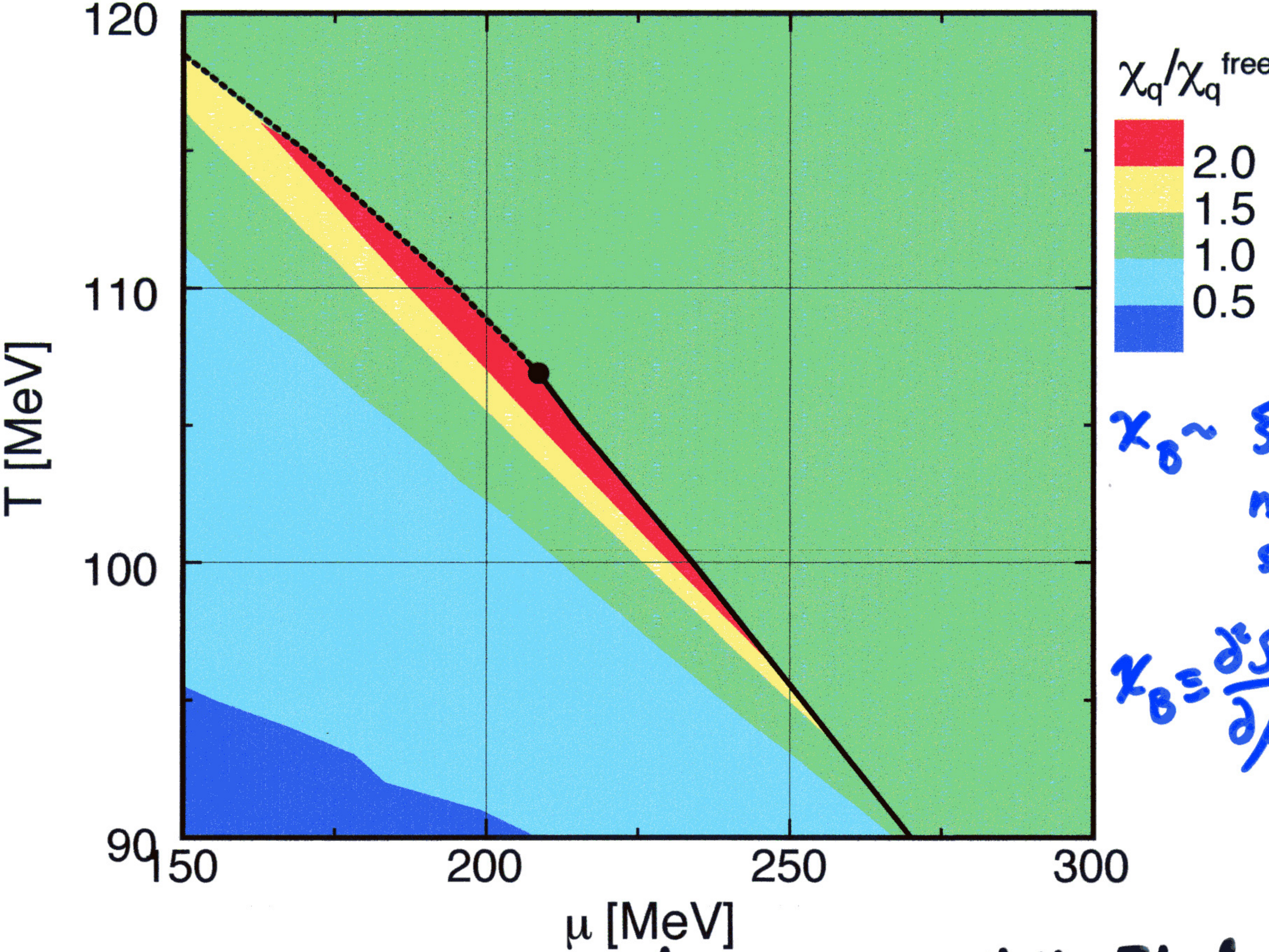
Signatures will be just as big if you pass anywhere in ⬭. No bigger, even if you hit ●.

- Hatta + Ikeda calculated "⬭'s" in a model, but did so with contours of χ_B rather than ξ. → Figs.

  The robust point is that the extent of these ⬭'s in μ_B is not small. Width in μ_B is ~ 100 MeV, an estimate that is both crude and uncertain. Can this be obtained on lattice??
- NB also: since ξ cannot be > 2-3 fm, heavy ion collision experiments can never be used to measure the critical exponents of the 2nd order critical point. That's OK: we know it is Ising. What we don't know, and need experiments for, is where it is located.

$m_u = m_d = 0$

MODEL ANALYSIS OF EXTENT OF CRITICAL REGION



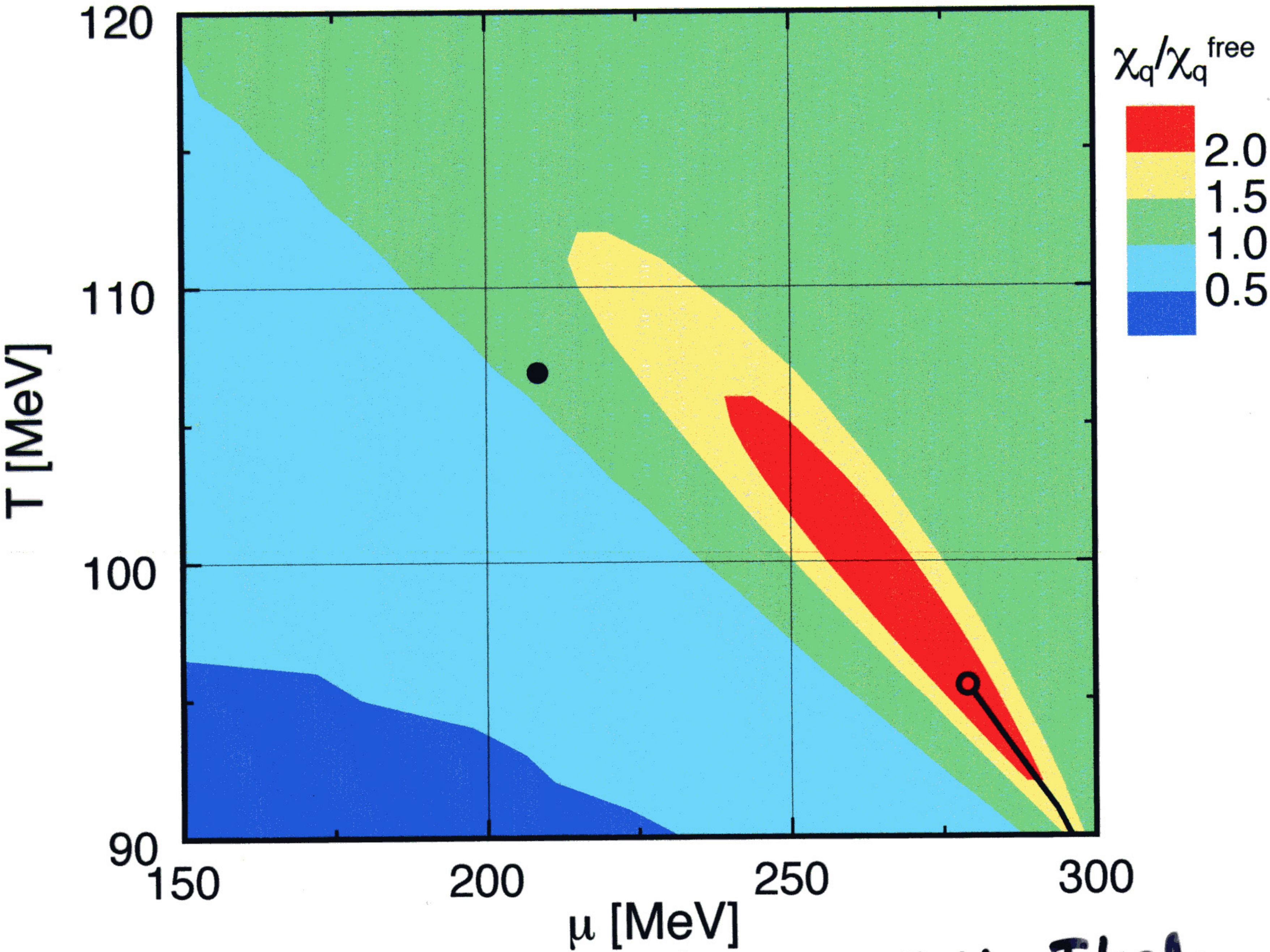
$= \mu_B/3$

$\chi_B \sim \xi^2 +$ non-singular

$\chi_B = \frac{\partial^2 \Omega}{\partial \mu_B^2}$

Hatta Ikeda

$m_u = m_d = 5\ \mathrm{MeV}$



$= \mu_B/3$

Hatta Ikeda

SIGNATURES OF CRITICAL POINT

Decreasing $\sqrt{s}$ → Increasing μ_B

$\sqrt{s}$:	200 GeV	12 GeV	5 GeV
$\mu_B^{freezeout}$:	25 MeV	300 MeV	550 MeV

Vary $\sqrt{s}$, and hence μ_B, and look for nonmonotonic enhancement (rise and then fall) of Gaussian event-by-event fluctuations of:

i) Mean p_T of low p_T pions

ii) proton number

iii) Particle ratios involving pions and/or protons.

And, also, signatures due to focussing of trajectories:

iv) elevation of $T_{freezeout}$

v) steepening of $\bar{p}$ spectrum

MEAN P_T OF LOW P_T PIONS

Stephanov KR Shuryak

Advantage: directly controlled by long wavelength fluctuations of the chiral order parameter.

Disadvantage: will they survive the late time hadron gas??

Result: NA49 has done a very nice analysis of Pb Pb collisions at $\sqrt{s} = 6.3, 7.6, 8.8, 12.3, 17.3$ and sees no $\sqrt{s}$ dependence $\rightarrow$ Fig

So.....

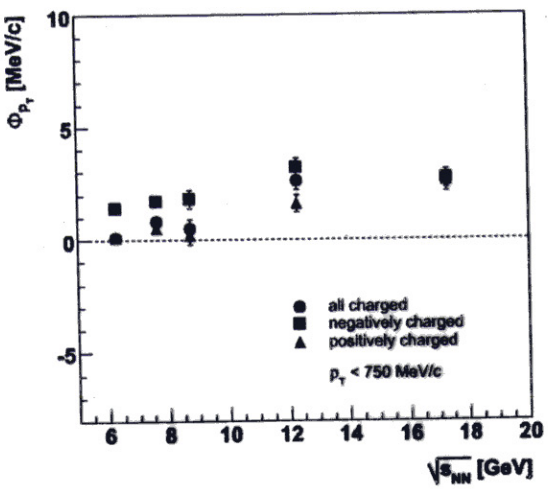
- try lighter ions, so $T_{freezeout}$ higher, shorter time in hadron gas phase. $\rightarrow$ NA61
- try other observables that are harder to wash out....

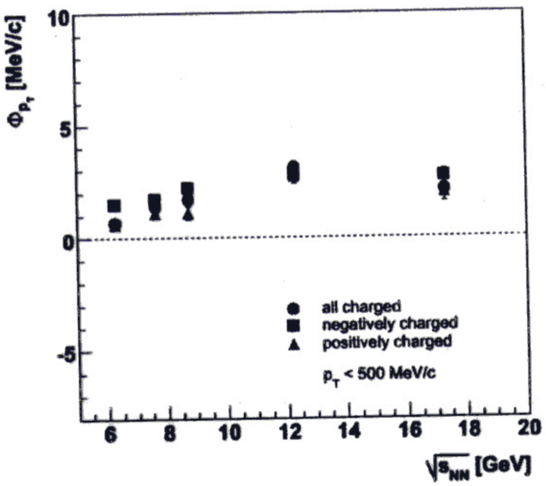
Fluctuations due to the critical point should be dominated by fluctuations of pions with $p_T \leq 500$ MeV/c

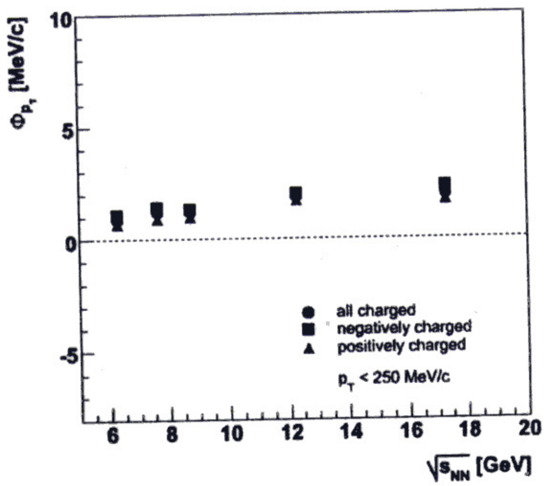
M. Stephanov, K. Rajagopal, E. V. Shuryak (Phys. Rev. **D60**, 114028, 1999):

suggestion to do analysis with several upper p_T cuts

$p_T < 750$ MeV/c | $p_T < 500$ MeV/c | $p_T < 250$ MeV/c







No significant energy dependence of Φ_{PT} measure

also when low transverse momenta are selected.

Remark: predicted fluctuations at the critical point should result in $\Phi_{PT} \cong 20$ MeV/c, the effect of limited acceptance of NA49 reduces them to $\Phi_{PT} \cong$ **10 MeV/c**

NA49 data; slide from K. Grebieszkow talk at CPOD 2007