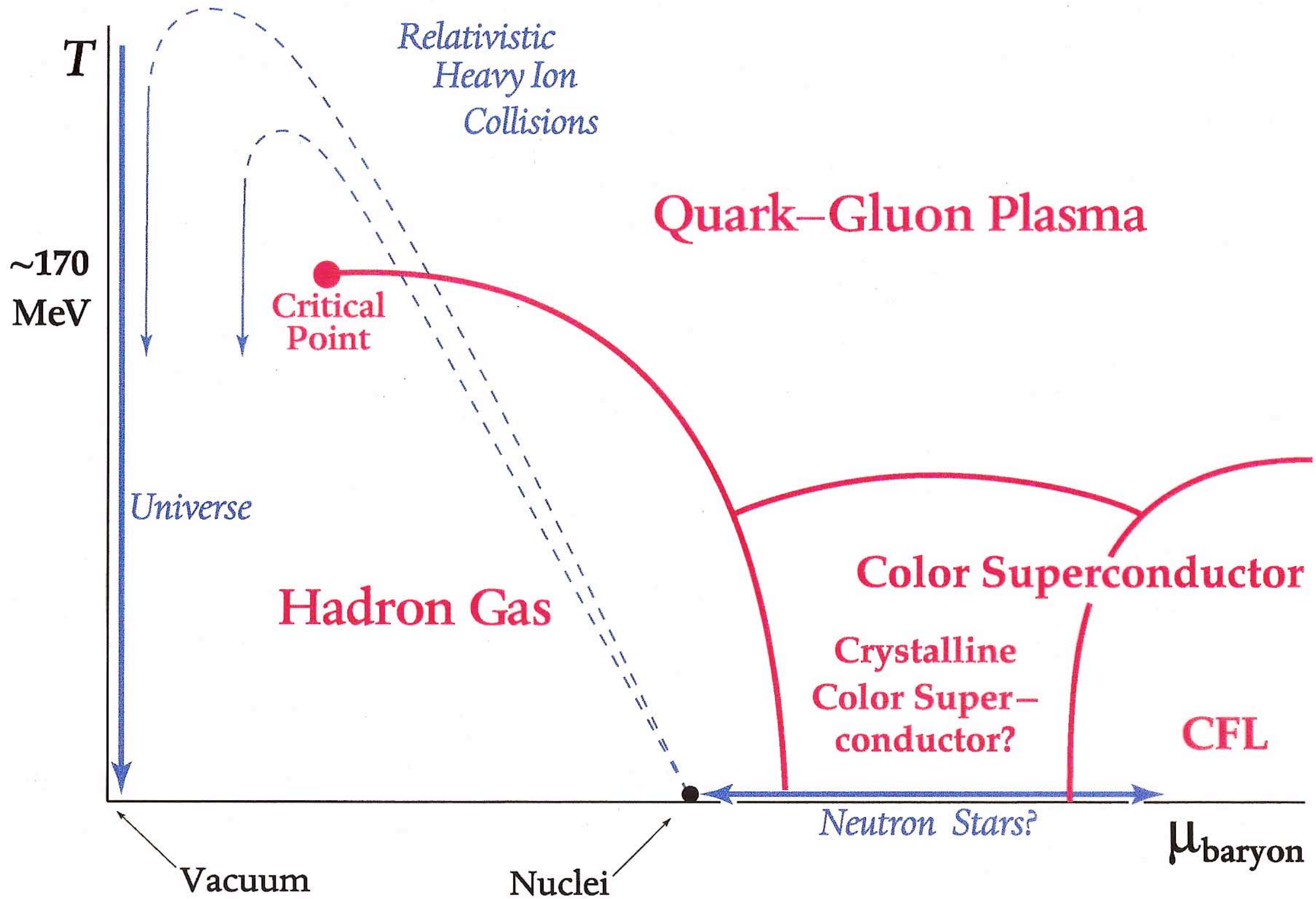
THE SEARCH FOR THE QCD CRITICAL POINT

USING LATTICE QCD CALCULATIONS AND HEAVY ION COLLISION EXPERIMENTS

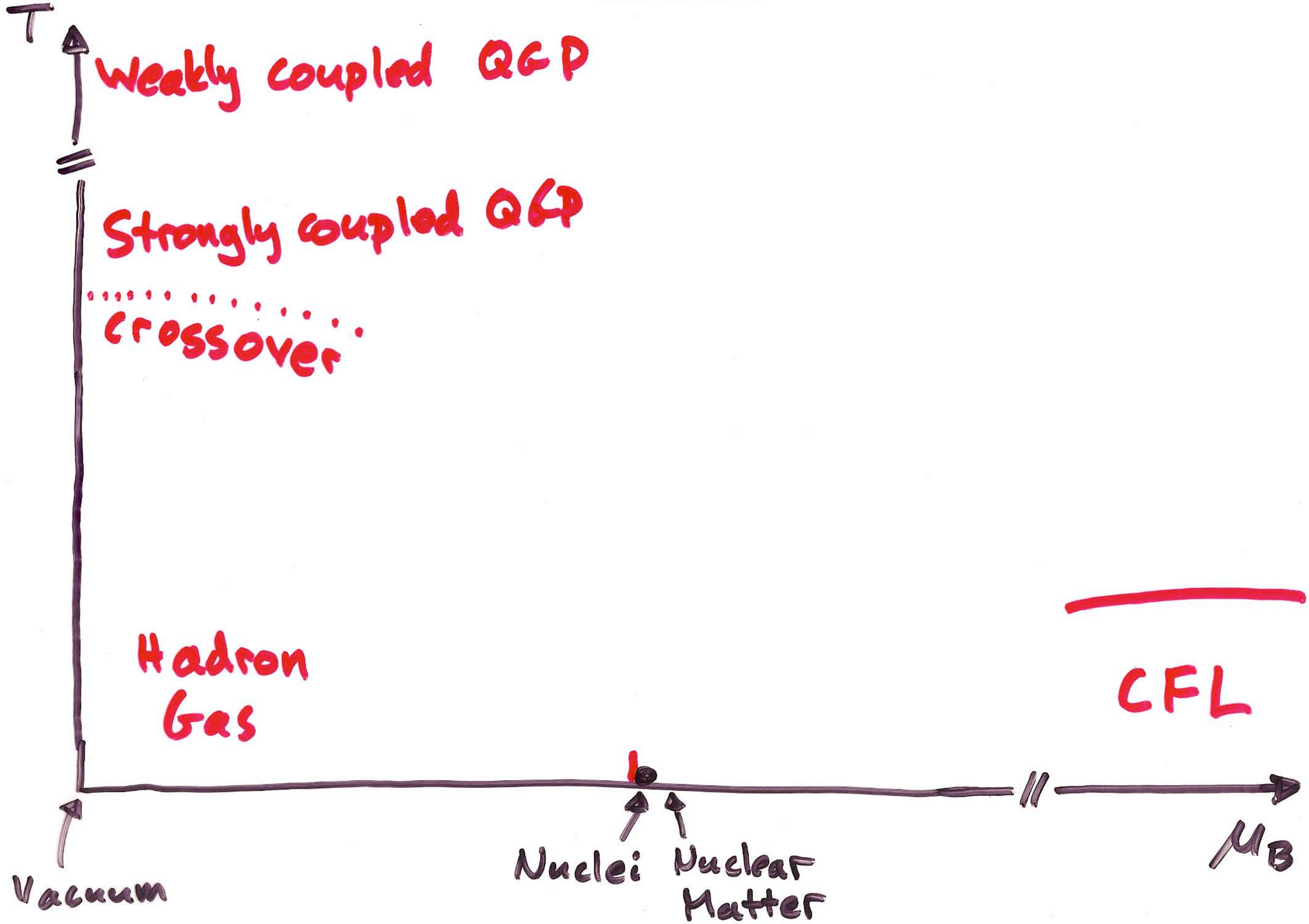
KRISHNA RAJAGOPAL
(MIT)

INT, Seattle. 8/11/08

EXPLORING *the* PHASES *of* QCD



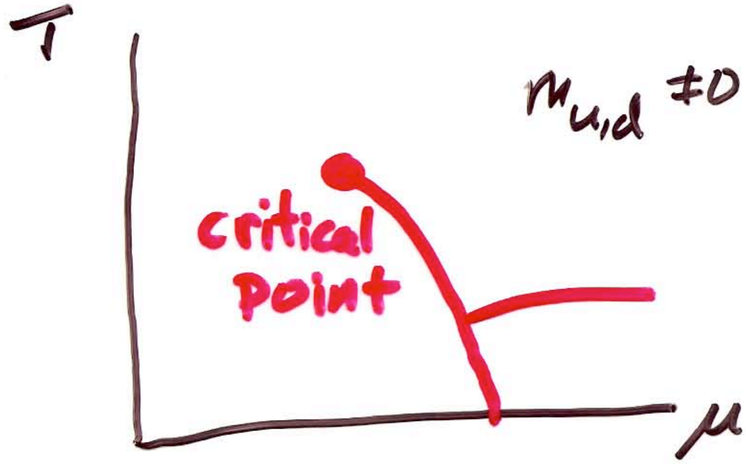
WHAT WE KNOW, SO FAR



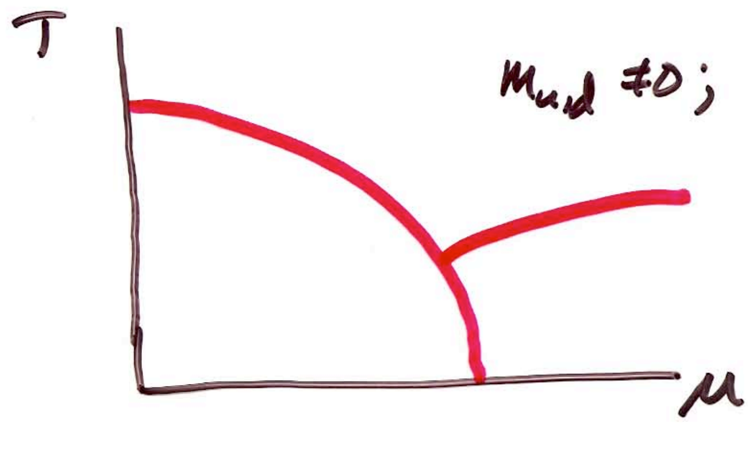
WHY EXPECT A CRITICAL POINT?

- Models; lattice QCD calculations at $\mu = 0$ with varying quark masses; suggest:

$m_{u,d} = 0$; m_s = physical



$m_{u,d} \neq 0$; m_s = physical



$m_{u,d} \neq 0$; m_s < physical

- Need lattice calculations with $T \neq 0$, $\mu \neq 0$ to locate it
- Universality class known (Ising)

LOCATING THE CRITICAL POINT...

- either via lattice calculations
- or via experimental detection of its signatures

would add a point and a line to the known QCD phase diagram.

OUTLINE OF TALK (AND WEEK)

- Lattice calculations
- Experimental signatures and searches

$T \neq 0$; $\mu \neq 0$; μ/T NOT LARGE

- a regime explored by heavy ion collisions
- a regime explored by lattice calculations that rely on smallness of μ/T to keep fermion sign problem under control. [$\mu \neq 0 \rightarrow$ complex Euclidean action $\rightarrow$ sign problem that makes difficulty of standard Monte Carlo $\sim \exp V$.]
- Either method may be used to locate the CRITICAL POINT, a 2^{nd} order point where a line of 1^{st} order transitions ends, if it is located at a μ/T that is not too large....

SEVERAL LATTICE METHODS

① Reweighting Fodor + Katz

Want physics at ⓐ $\equiv (\mu, T_a)$

Simulate using an ensemble of configurations at ⓑ $\equiv (0, T_b)$, and "reweight": lump difference between physics at ⓑ and ⓐ into observables.

Difficulty ~ $\exp\left[\frac{|F_b - F_a| V}{T}\right]$

F+K: choose T_b to minimize

BUT: still cannot use method at large volumes....

The endpoint is at $T_E = 162 \pm 2$ MeV, $\mu_E = 360 \pm 40$ MeV. As expected, μ_E decreased as we decreased the light quark masses down to their physical values (at approximately three-times larger $m_{u,d}$ the critical point was at μ_E=720 MeV; see [8]).

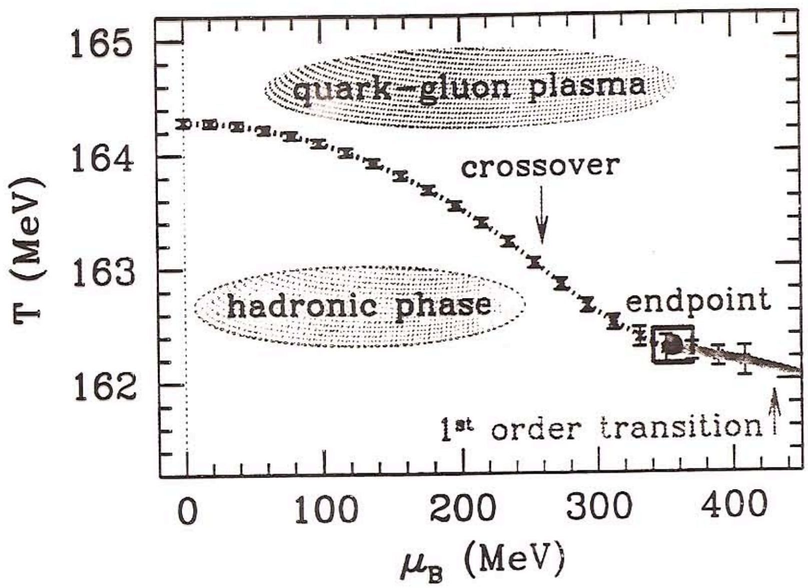


Figure 2: The phase diagram in physical units. Dotted line illustrates the crossover, solid line the first order phase transition. The small square shows the endpoint. The depicted errors originate from the reweighting procedure. Note, that an overall additional error of 1.3% comes from the error of the scale determination at T=0. Combining the two sources of uncertainties one obtains $T_E = 162 \pm 2$ MeV and $\mu_E = 360 \pm 40$ MeV.

The above result is a significant improvement on our previous analysis [8] by two means. We increased the physical volume by a factor of three and decreased the light quark masses by a factor of three. Increasing the volumes did not influence the results, which indicates the reliability of the finite volume analysis. Clearly, more work is needed to get the final values. Most importantly one has to extrapolate to the continuum limit.

Fodor, Katz
2004

$$\left.\begin{aligned} \mu_E &= 360 \pm 40 \text{ MeV} \\ \frac{\mu_E}{T_E} &= 2.22 \pm .25 \end{aligned}\right\} \text{statistical errors only}$$

CONCERNS, aka "SYSTEMATIC ISSUES"

- $N_\tau = 4$ (no continuum limit)
- $V = 12^3$, and method must break down for $V \to \infty$
- $\frac{\mu_E}{3} \simeq \frac{m_\pi}{2}$. This was also the case in older F+K calculation at larger m_π. If this is not a coincidence, it is a problem. Splittorf

  [$\mu_q = m_\pi/2$ is where phase quenched QCD has onset of pion condensation.]
- $\frac{m}{T}$ held fixed during reweighting, not m.

ALL these, except for $V \to \infty$, are IMPROVABLE.

② Continue from imaginary μ.

deForcrand + Philipsen

D'Elia + Lombardo et al

Simulate at $\mu = i\mu_I$; calculate

$T_c(\mu_I)$; Taylor expand:

$$= c_0 + c_2 \mu_I^2 + c_4 \mu_I^4 + \cdots$$

- valid for $\frac{\mu_I}{T} < \frac{\pi}{3}$
- Good luck ... $c_4, c_6, \ldots$ terms all small over this range.
- So, boldly continue:

$$T_c(\mu) = c_0 - c_2 \mu^2 + c_4 \mu^4 \cdots$$

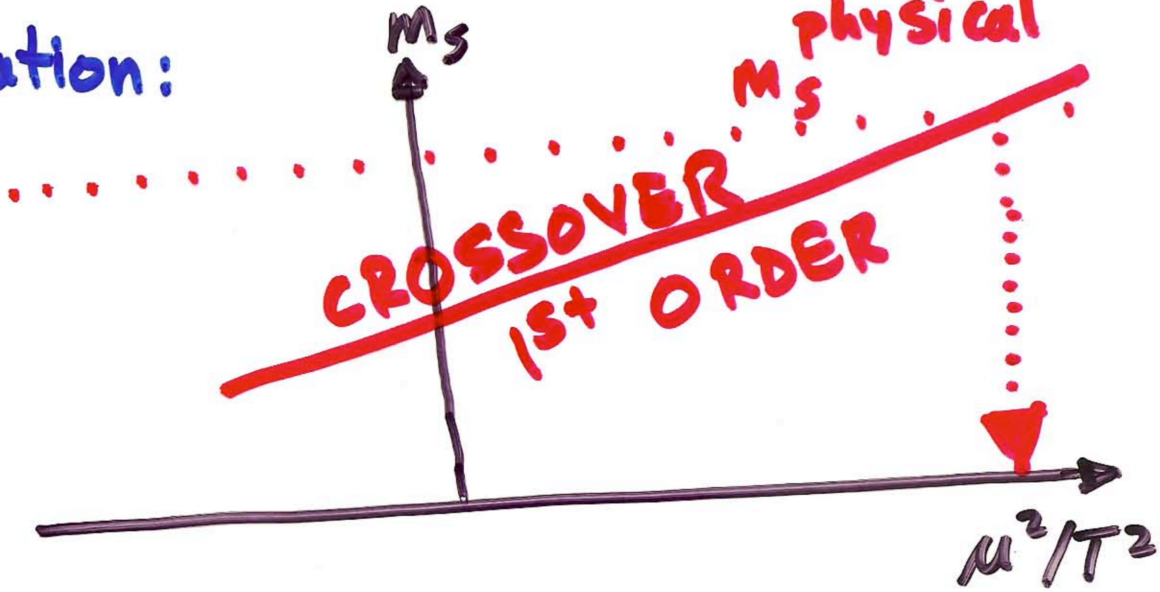
Curvature of crossover line on phase diagram

CRITICAL POINT ??

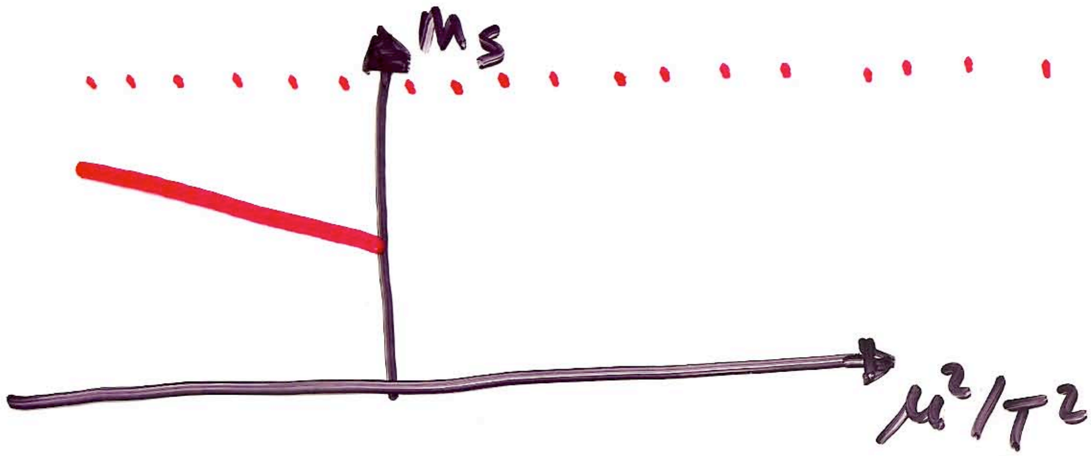
- Calculate

$$\frac{\partial}{\partial \mu^2}\left[\begin{array}{l} m_q \text{ at which transition goes} \\ \text{from } 1^{st} \text{ order to crossover} \end{array}\right]$$

- Expectation:



- deForcrand + Philipsen find:



- $\Rightarrow$ NO CRITICAL POINT with $\frac{\mu}{T} < \mathcal{O}(1)$.

CONCERNS, aka "SYSTEMATIC ISSUES"

Lets defer their discussion to after Philippe's talk, but here are two:

- $N_\tau = 4$
- Staggered fermions with
  $N_f = 3$ or $2+1$ .....
  - $Det^{3/4}$ or $Det^{1/2}\, Det^{1/4}$
  - First order phase transition at small m_s originates from 't Hooft $uds\bar{u}\bar{d}\bar{s}$ interaction, in low energy effective theory. Pisarski Wilczek
  - do staggered fermions describe this adequately ??
    Fukushima, Stephanov

also an issue for F+K

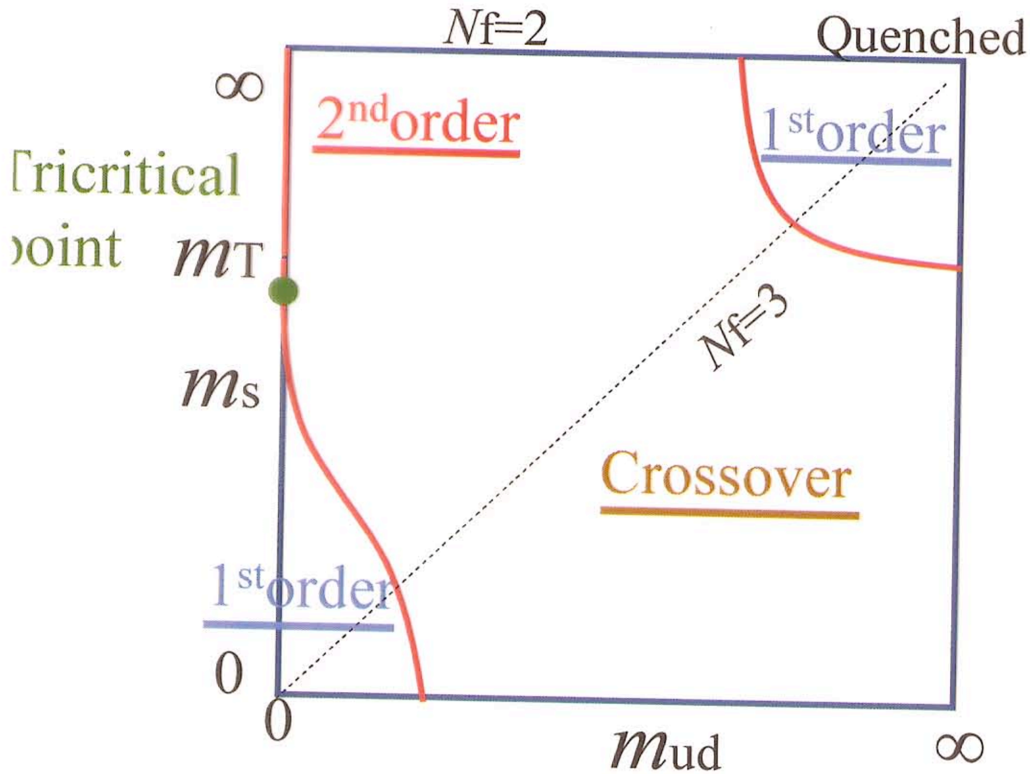
Mean field argument

Ejiri

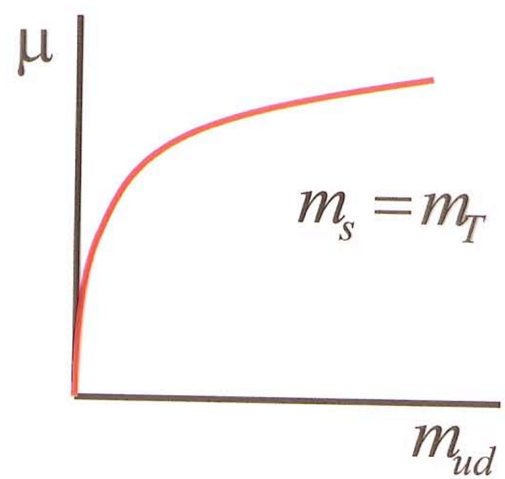
- Sigma model prediction near tri-critical point on the ms axis.

$$\boxed{V_{\rm eff}(\sigma)=\frac{1}{2}a\sigma^2+\frac{1}{4}b\sigma^4+\frac{1}{6}c\sigma^6-h\sigma}$$

Critical point: $\dfrac{\partial^n V_{\rm eff}}{\partial\sigma^n}=0 \quad (n=1,2,3)$



$$\boxed{b\sim(m_T-m_s)} \Rightarrow \boxed{b\sim\mu^2}$$



$$\boxed{m_{ud}^{\rm crit}\sim(m_T-m_s)^{5/2}} \Rightarrow \boxed{m_{ud}^{\rm crit}\sim\mu^5}$$

③ Taylor Expansion of the Pressure.

Bielefeld-Swansea; Gavai Gupta

Calculate the coefficients in:

$$\frac{P}{T^4} = b_0(T) + b_2(T)\mu^2 + b_4(T)\mu^4 + b_6(T)\mu^6 + \ldots$$

and hence in:

$$\chi_B \equiv \frac{\partial^2 P}{\partial \mu^2} = c_0(T) + c_2(T)\mu^2 + c_4(T)\mu^4 + c_6(T)\mu^6 + \ldots$$

which should diverge at critical point.

Several ways to look for critical point:

- Look for μ at which χ_B peaks
- Do Taylor expansion at varying m_q and evaluate

$$\frac{\partial}{\partial \mu^2}\left[\begin{array}{l} m_q \text{ at which crossover at} \\ \mu = 0 \text{ becomes } 1^{st} \text{ order} \end{array}\right]$$

[Defer discussion of these to Karsch.]

- And ...

RADIUS OF CONVERGENCE METHOD

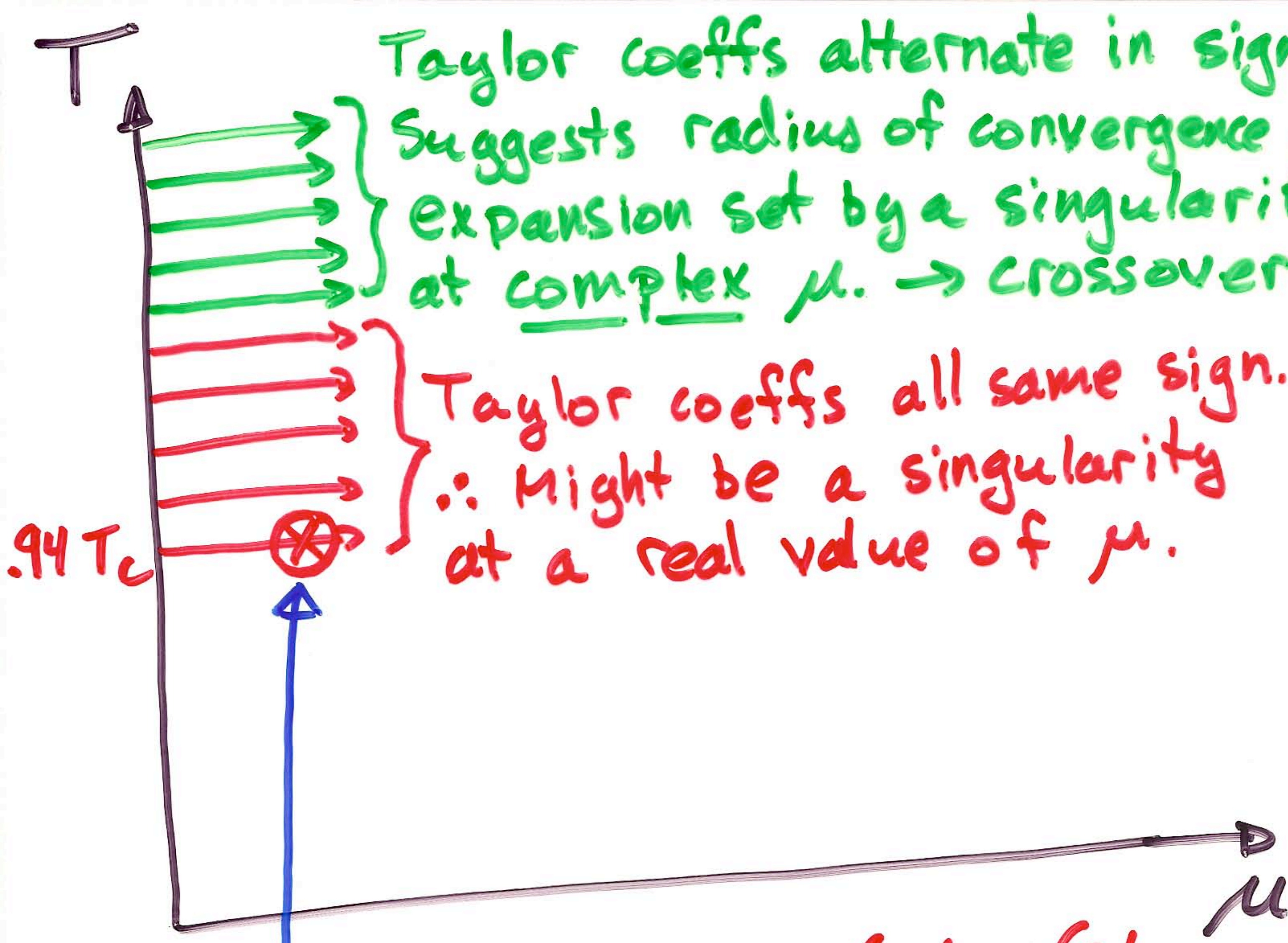
Use fact that Taylor expansion must break down at critical point.

Bielefeld Swansea ; Gavai Gupta

New results from Gavai + Gupta, June 2008 & earlier this workshop:

- $N_\tau = \underline{\underline{6}}$; $V = 24^3$
- $N_f = 2$; $m_\pi = 230$ MeV
- staggered fermions, so maybe not a bad thing that $N_f = 2$.
- Taylor coefficients $c_0(T)$, $c_2(T)$, $c_4(T)$, $c_6(T)$.

  [ie up to μ^8 term in P]



At this T, find $c_6/c_4 = c_4/c_2 = c_2/c_0$, as would be the case for a pole at real μ. And, as yields a consistent estimate of radius of conv.

Also, at the same T, coeffs have expected finite size scaling (upon comparing $LT = 2$ and 4).

I dentify this T as T_E, and this radius of convergence as μ_E.

Gavai and Gupta find:

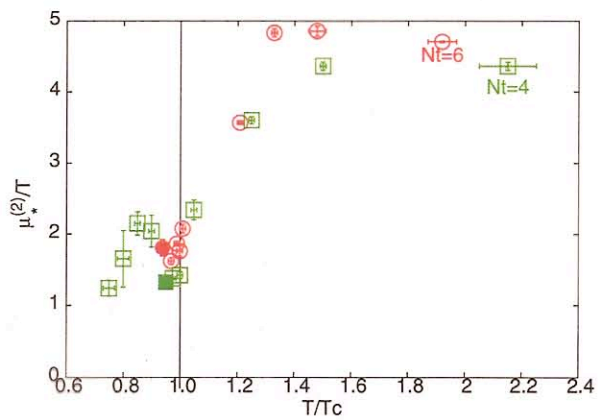
$$\frac{T^E}{T_c} = 0.94 \pm 0.01$$

$$\frac{\mu_E}{T_E} = 1.8 \pm 0.1$$

Issues:

- $N_\tau = 6$. "Crawling towards the continuum limit." Gupta
- $N_f = 2 \longrightarrow N_f = 2+1$
- What is the best estimator of T^E, ie what combination of criteria, given $C_0(T)$, $C_2(T)$, $C_4(T)$, $C_6(T)$?

Radius of convergence

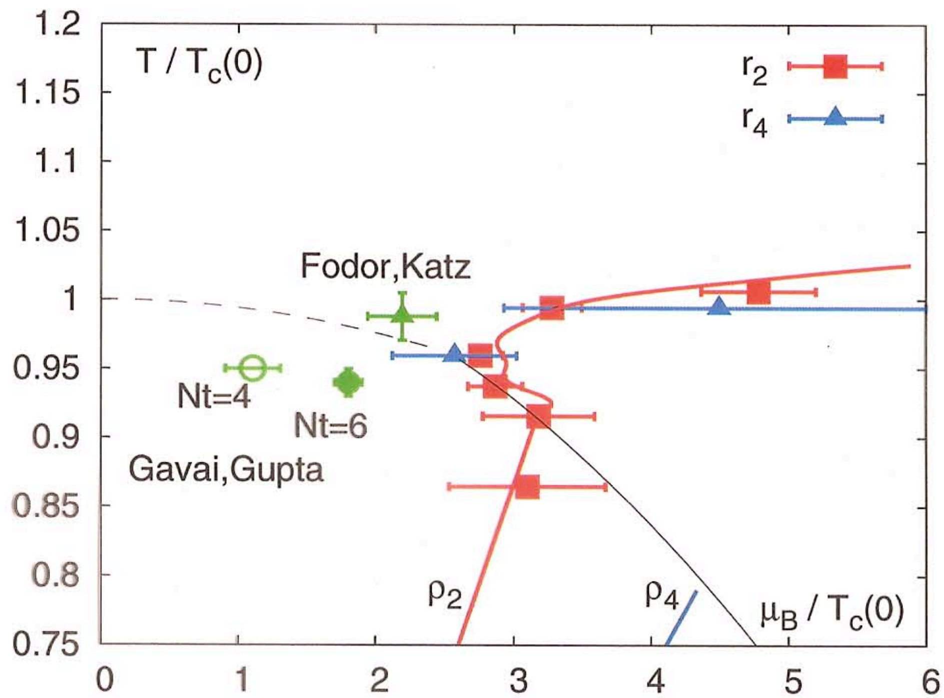


Lattice spacing dependence quantifies possible systematic errors.

Estimating $T_c(\mu_c)$ and μ_c/T

Status of the RBC-BI project

- calculations for $N_\tau = 4$ and 6; $N_\sigma = 4N_\tau$
- uses an $\mathcal{O}(a^2)$ improved staggered action (p4fat3)
- estimator for μ_c: $$\left(\frac{\mu_c(T)}{T_c(0)}\right)_n \equiv \rho_n = \frac{T}{T_c(0)}\sqrt{\frac{c_n}{c_{n+2}}}$$



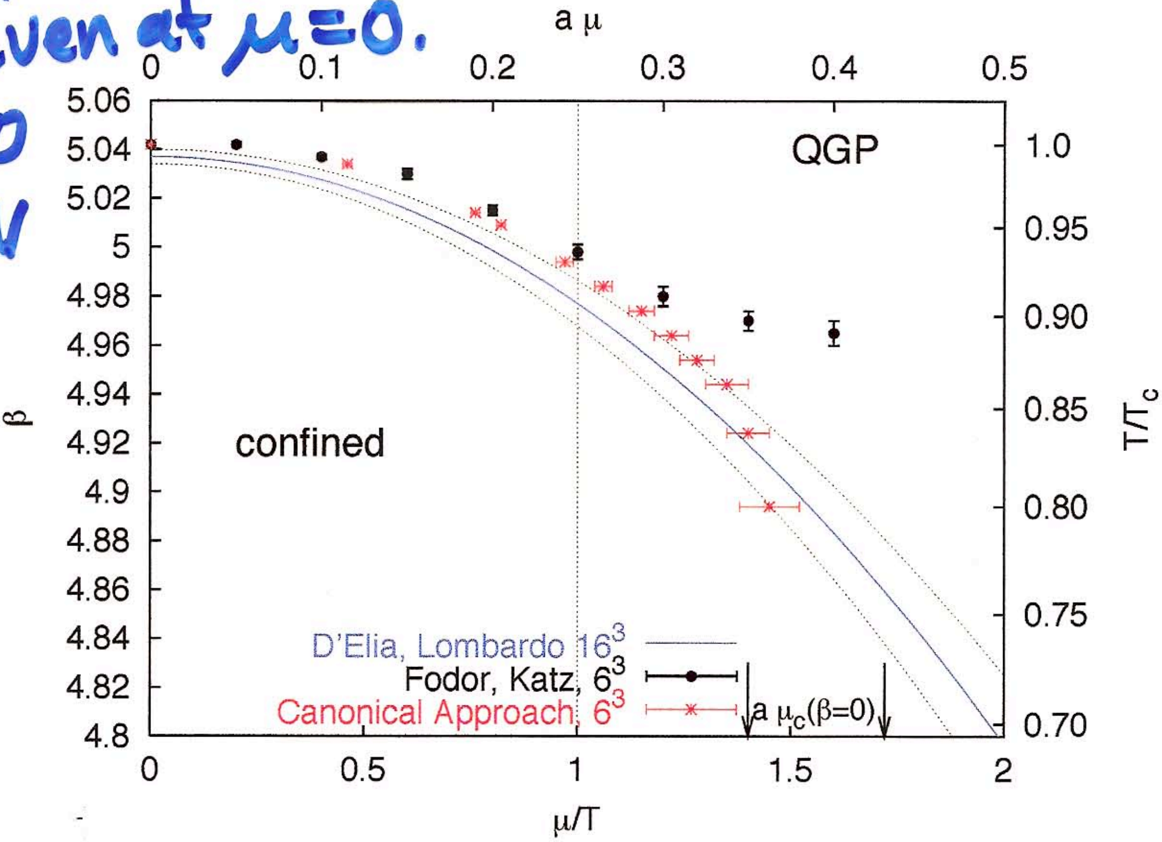
- slight quark mass dependence
- weak cut-off dependence
- $\mathcal{O}(\mu^6)$ requires more statistics

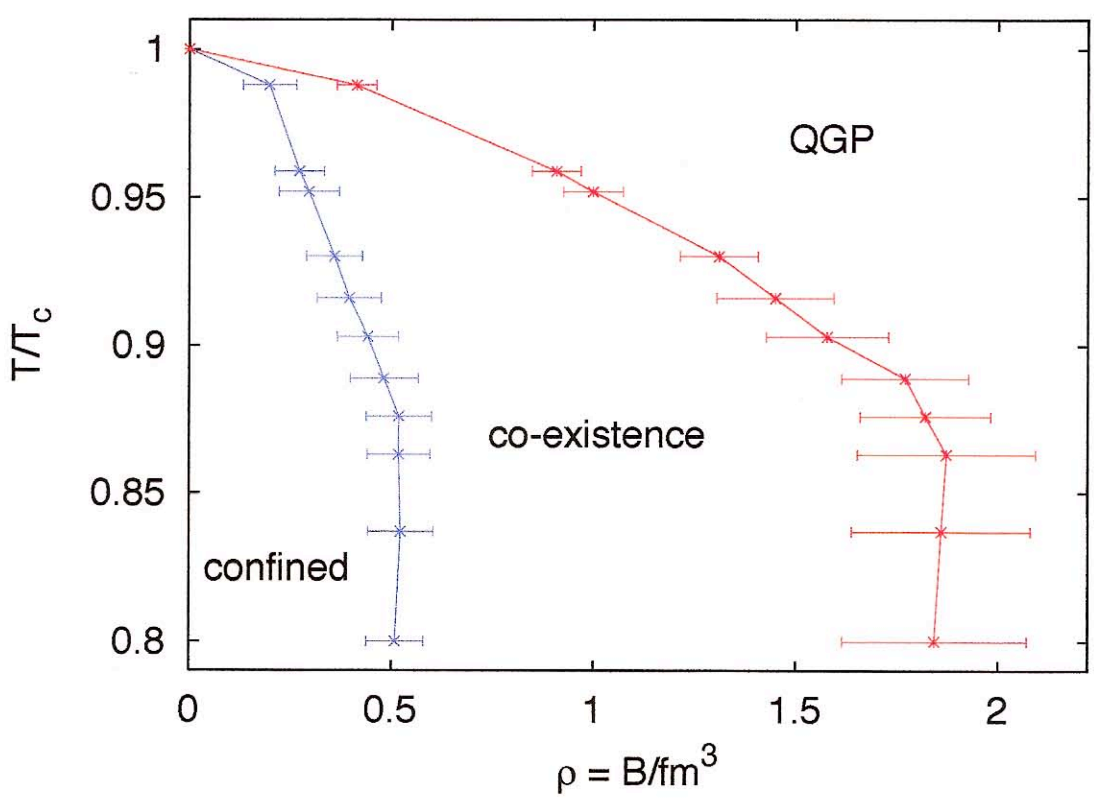
LATTICE CALCULATIONS AT FIXED n_B

$N_f = 4 \to 1^{st}$ order even at $\mu = 0$.

de Forcrand Kratochvila

$m_\pi = 350$ MeV





Calculation on 6^3 lattice, with $0 < B < 30$. $(V \sim (2\,fm)^3)$

- Will be very interesting to see what they find with $N_f = 2+1$.

- Determining the location of the critical point this way will have very different "systematic error" relative to calculations relying on $\mu/T < 1$. (ie reweighting Fodor Katz or Taylor expansion Ejiri et al, Gavai Gupta)

- In principle can be pushed to larger μ/T, but remains to be seen how large a V can be reached at a given μ or n_B.

LOCATING THE CRITICAL POINT

Location still uncertain:

$$\frac{\mu_B^{\text{critical point}}}{T_c(\mu=0)} \sim 2\,,\ > \mathcal{O}(3)\,,\ \sim 1.7\,,\ \gtrsim 2$$

RBC-BI

Fodor Katz, Philipsen deForcrand, Gavai Gupta

- gaining confidence will require "crawling towards the continuum limit", and several methods agreeing.
- If $\mu_B^{c.p.} < 3T$, this will happen
- If $\mu_B^{c.p.} > 3T$, all methods should come to agree on this. But, barring an unforeseen algorithmic breakthrough, unlikely that lattice calculations will locate it with confidence.

In the race between lattice calculations and experimental searches to bcate the critical point, the lattice team is running strongly but not yet threatening to end the race.

So, lets turn to experimental searches ....