QCD Critical Point Workshop

INT - 11-15 August 2008

Spinodal decomposition:
A tool for seeing the phase transition?

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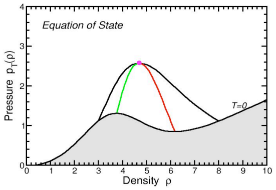
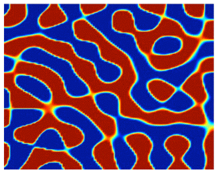
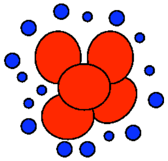
Thermodynamics of phase transitions



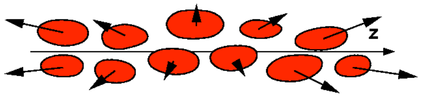
Spinodal decomposition



Nuclear spinodal fragmentation



The deconfinement transition?

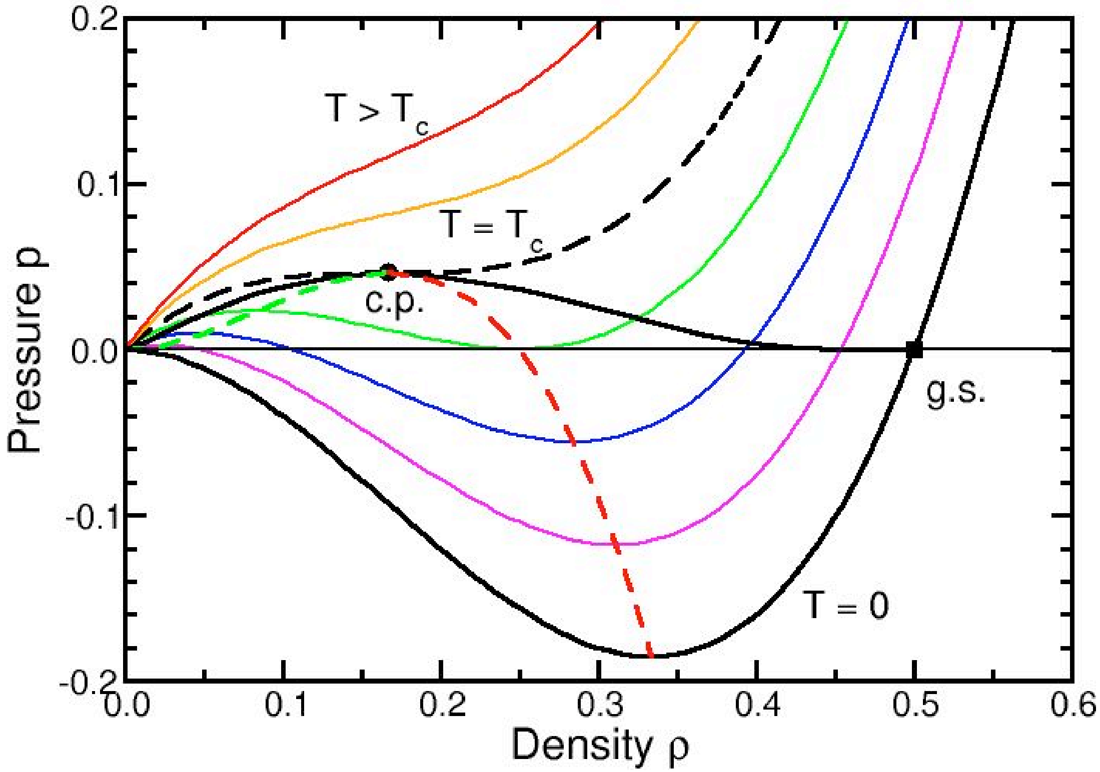


Urgent challenges!

Familiar example: Nuclear matter

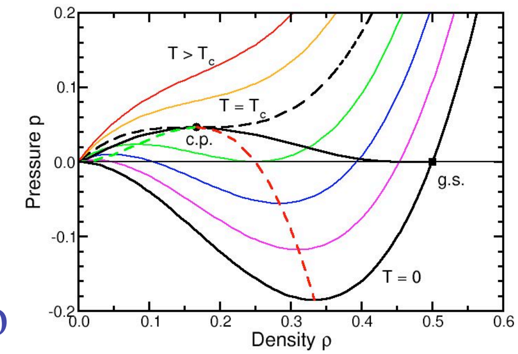
(One conserved charge)

Nuclear equation of state $p_T(\rho)$



Thermodynamics reminder

Statistical equilibrium in bulk matter



Control parameter(s) $\{X\}$:

$$\begin{cases} \text{Energy } E = V\varepsilon \\ \text{Number } N = V\rho \\ \text{Volume } V \to \infty \end{cases}$$

Entropy function $S\{X\}$:

$$\boxed{S(E,N,V) = V\sigma(\varepsilon,\rho)}$$

Derivative(s) $\lambda_X = \partial_X S$:

$$\begin{cases} \beta = 1/T = \partial_E S(E,N,V) = \partial_\varepsilon \sigma(\varepsilon,\rho) \\ \alpha = -\mu/T = \partial_N S(E,N,V) = \partial_\rho \sigma(\varepsilon,\rho) \\ \pi = p/T = \partial_V S(E,N,V) = \sigma - \beta\varepsilon - \alpha\rho \end{cases}$$

Thermodynamic <u>*coexistence*</u>: $T_1 = T_2$ & $\mu_1 = \mu_2$ & $p_1 = p_2$

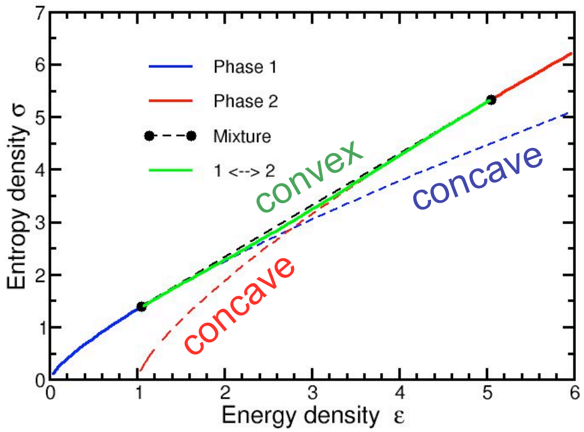
$\delta S_{tot} = 0$ => $(\partial_\chi \sigma)_1 = (\partial_\chi \sigma)_2$ <=> *common tangent!*

Thermodynamic (local) <u>*stability*</u>: $\delta^2 S_{tot} < 0$

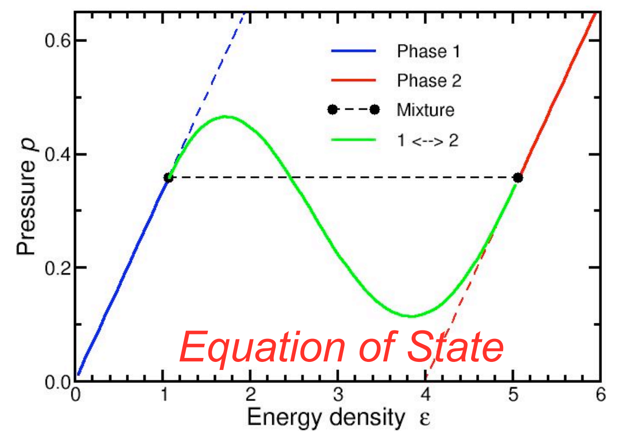
=> $\{\partial_\chi \partial_{\chi'} \sigma\}$ has only *negative* eigenvalues

Simplest example: No conserved charges

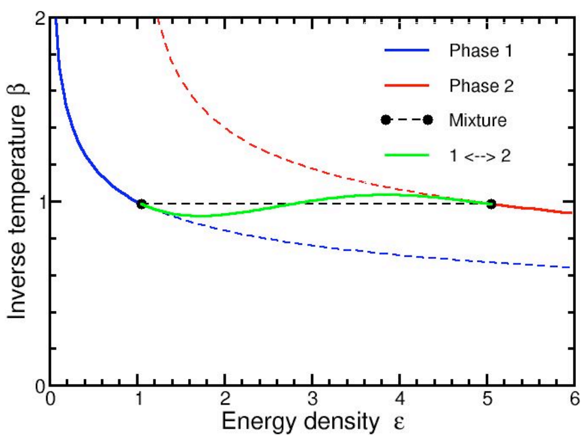
Entropy density: $\sigma(\varepsilon)$



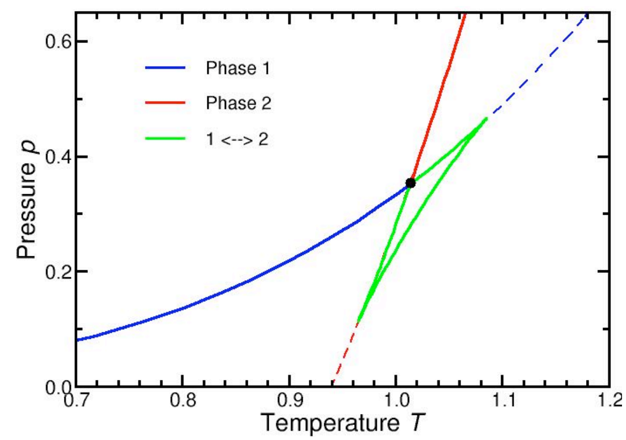
Pressure: $p(\varepsilon) = T\sigma - \varepsilon$



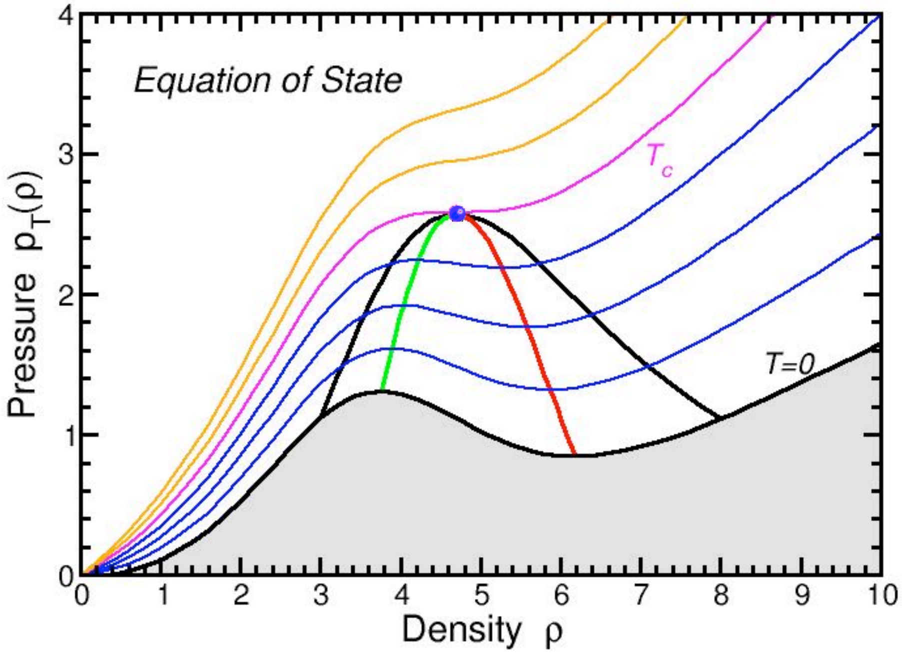
Inverse temperature: $\beta(\varepsilon) = \partial\sigma/\partial\varepsilon$



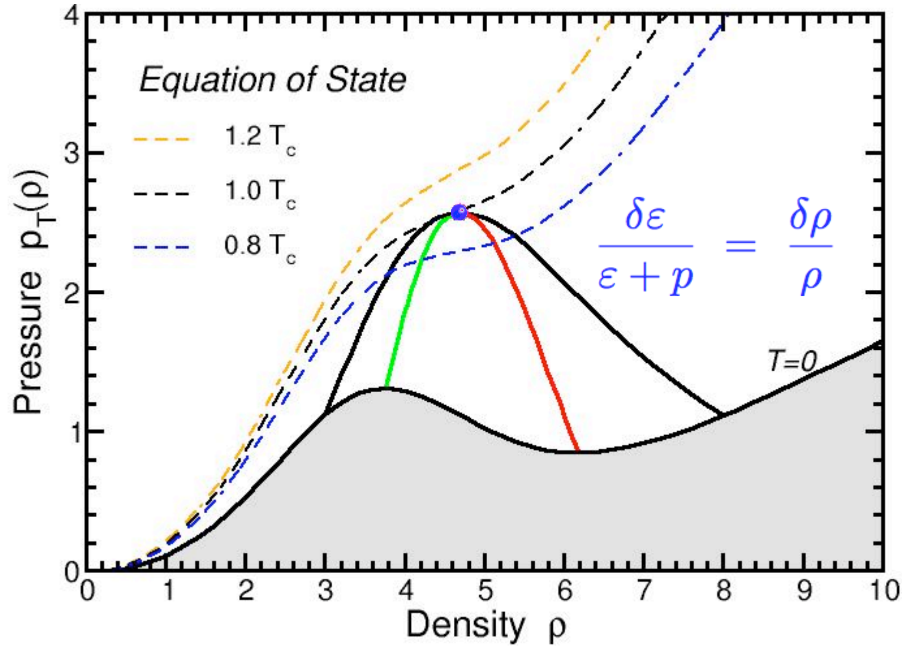
Pressure: $p(T)$



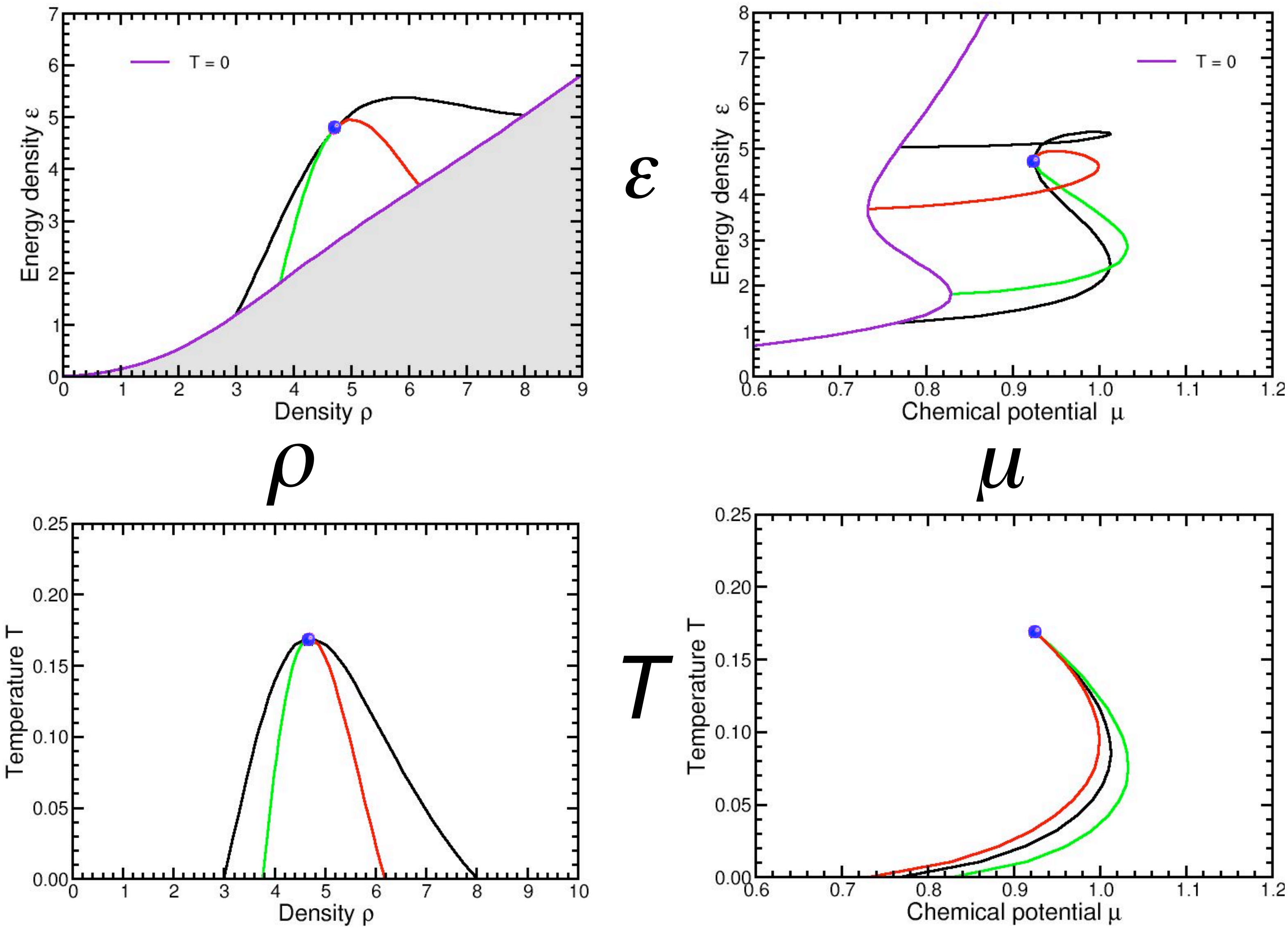
Schematic Equation of State
for Compressed Baryonic Matter



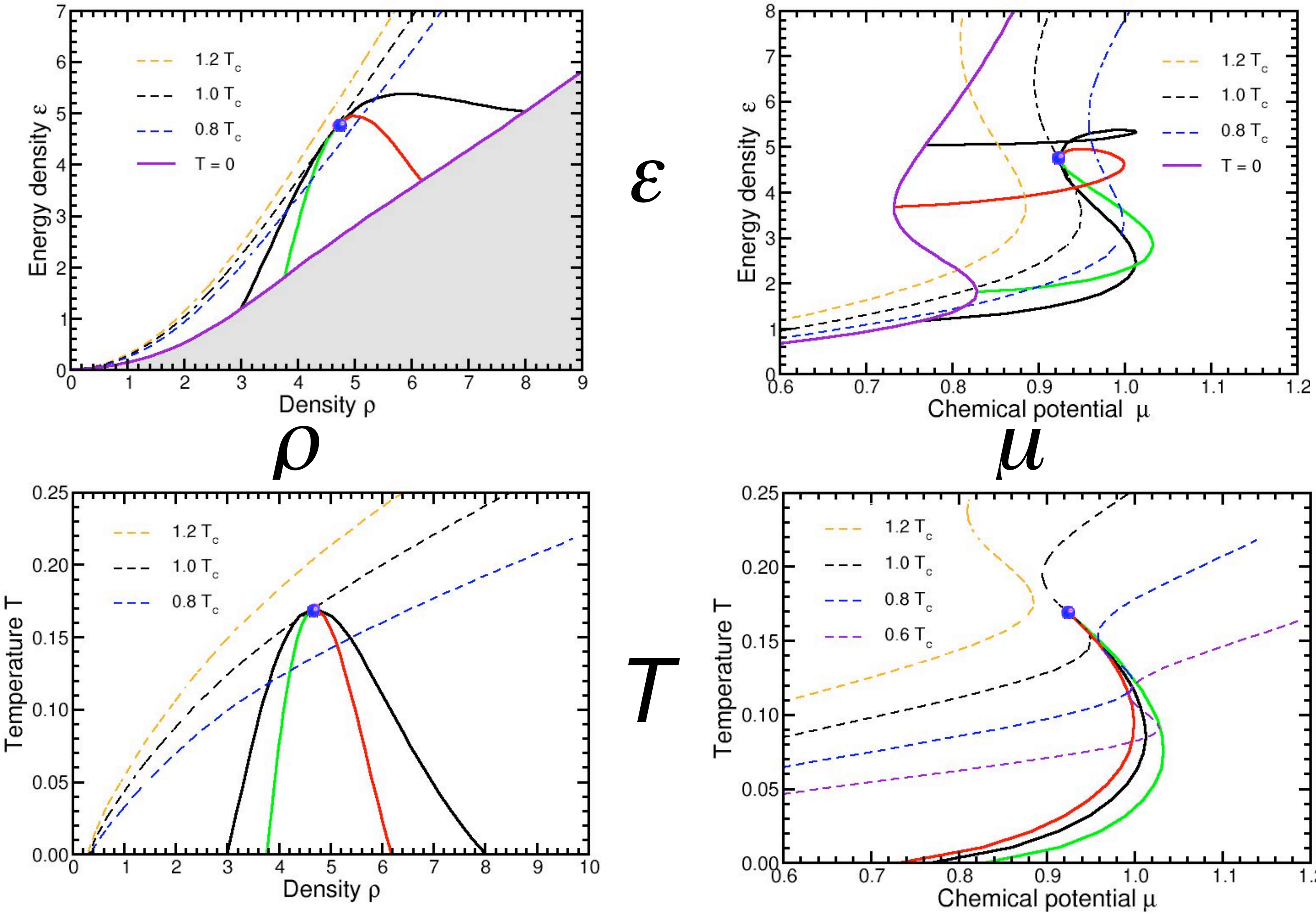
Isentropic expansion: σ/ρ constant



Phase diagram in different representations



Isentropic phase trajectories in different representations



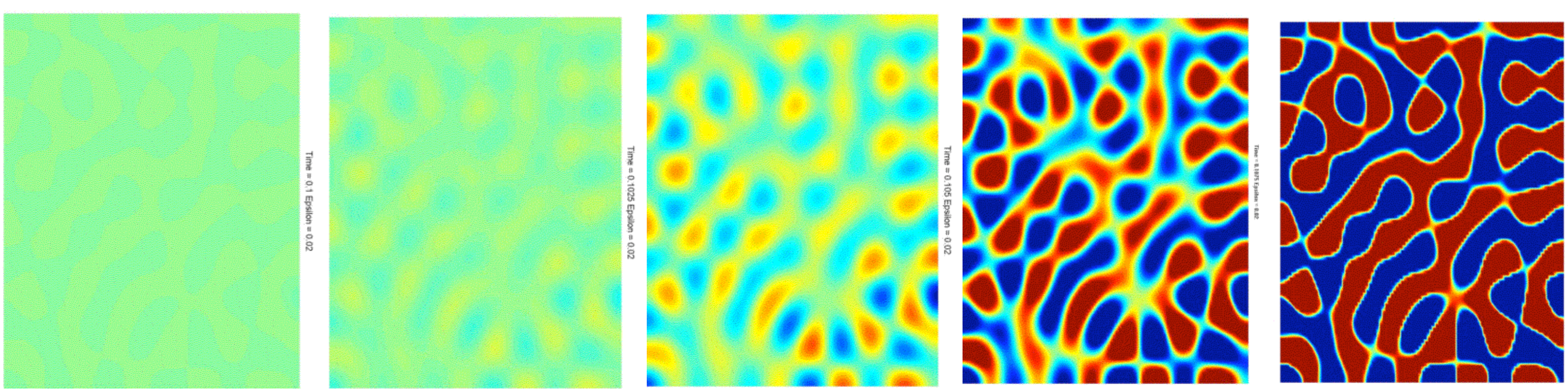
Spinodal decomposition - from Wikipedia, the free encyclopedia

Spinodal decomposition is a process by which a mixture of two materials can separate into distinct regions with different material concentrations. [This differs from *nucleation* in that spinodal phase separation occurs throughout the material, not just at nucleation sites.]

Phase separation may occur whenever a material finds itself in the *thermodynamically unstable region* of the phase diagram. The boundary of this unstable region (the *binodal*) is signaled by a common tangent of the *thermodynamic potential*. Inside the binodal boundary, the *spinodal region* is entered when the curvature of the potential turns negative. The binodal and spinodal meet at the *critical point*. It is when a material is brought into the spinodal phase region that spinodal decomposition can occur.

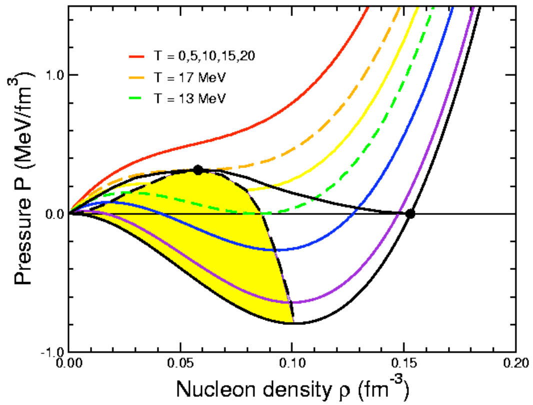
To reach the spinodal region of the phase diagram, the system must be brought through the binodal region where nucleation may occur. For spinodal decomposition to be realized, a very fast transition (a *quench*) is required to evolve the system from the stable region through the meta-stable nucleation region and well into the mechanically unstable spinodal phase region.

In the spinodal phase region, the thermodynamics favors spontaneous separation of the components. But large regions will change their concentrations only slowly due to the amount of material that must be moved, and small regions will shrink away due to the energy cost of the *interface* between the two different component materials. Thus domains of a characteristic *spinodal length* scale will be favored and since the growth is exponential, such domain sizes will come to dominate the morphology in the course of the associated *spinodal time*.



Spinodal Multifragmentation

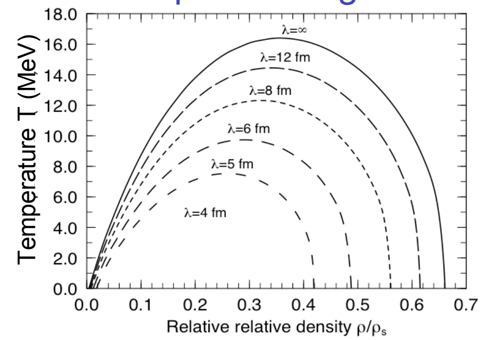
Nuclear EoS:

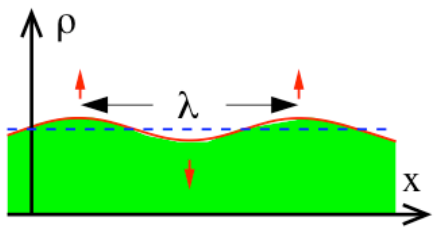


1st order phase transtion

Spinodal instability

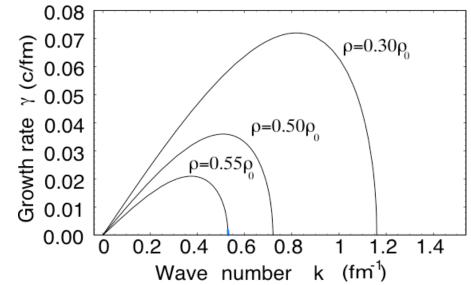
Spinodal region:



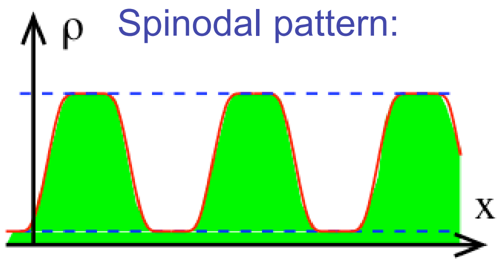


Density undulations may be amplified

Growth rates:



Spinodal pattern:

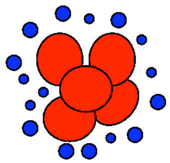


Ph Chomaz, M Colonna, J Randrup
Nuclear Spinodal Fragmentation
Physics Reports 389 (2004) 263

Fragments
≈ *equal!*

Highly non-statistical => Good candidate signature

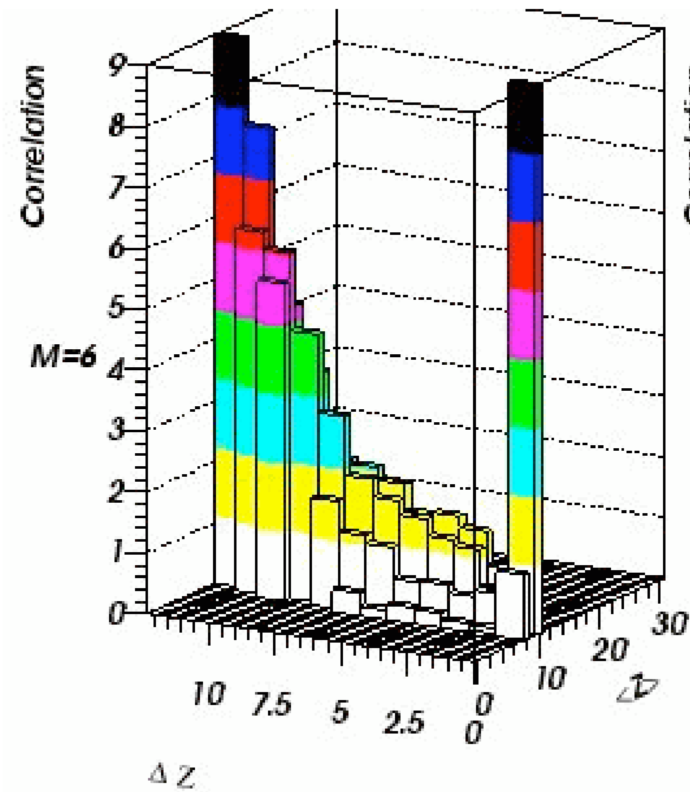
Spinodal decomposition in nuclear multifragmentation

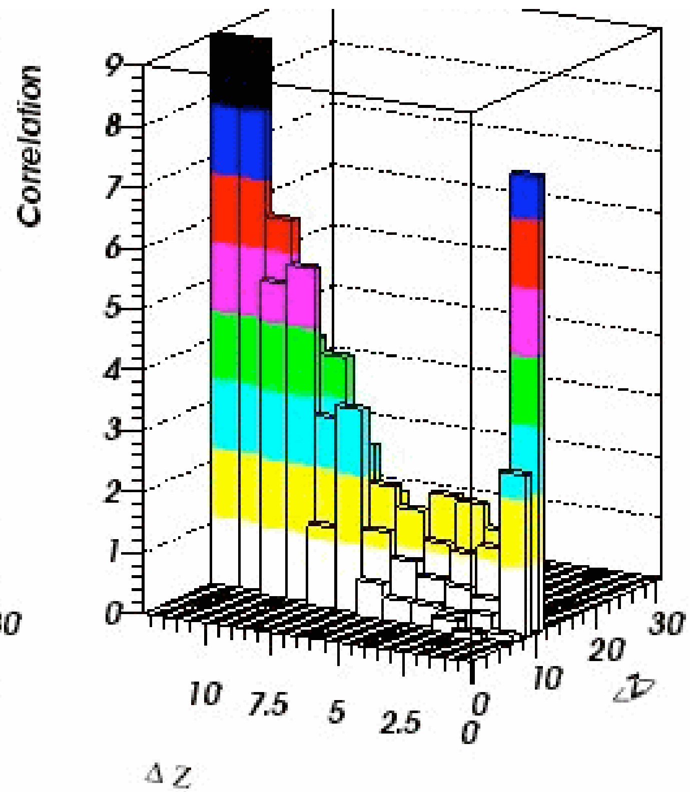


32 MeV/A Xe + Sn (b=0)

(select events with 6 IMFs)

Bin wrt $\begin{cases} \langle Z \rangle : & \text{average IMF charge} \\ \Delta Z : & \text{dispersion in IMF charge} \end{cases}$





Experiment (*INDRA @ GANIL*)

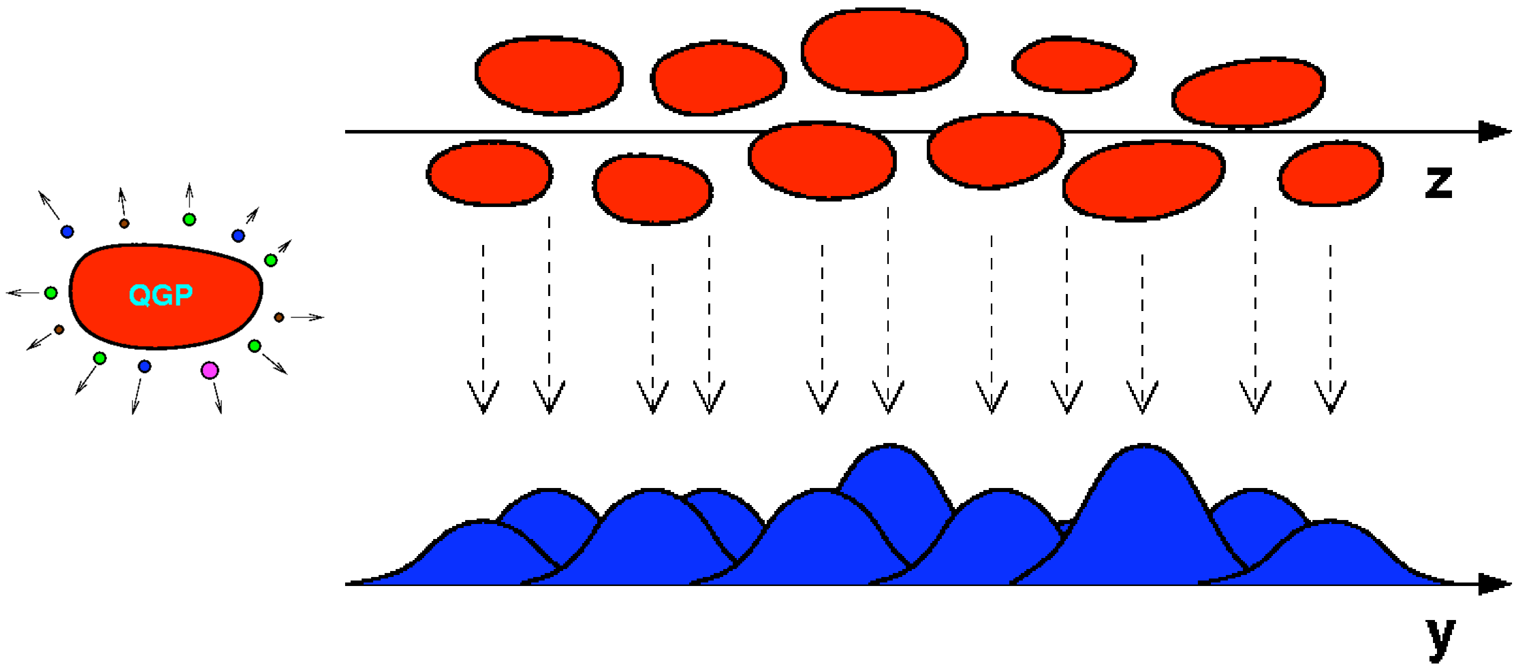
Borderie *et al*, PRL 86 (2001) 3252

Theory (*Boltzmann-Langevin*)

Chomaz, Colonna, Randrup, …

Idealized model for exploration of clumping:

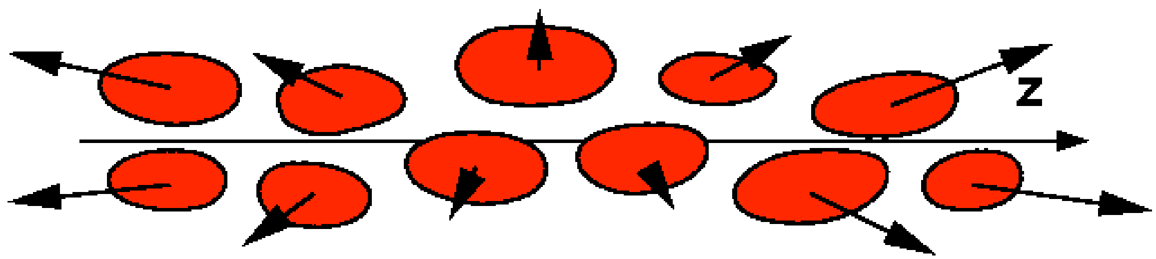
The expanding system decomposes into plasma blobs
which hadronize thermally:

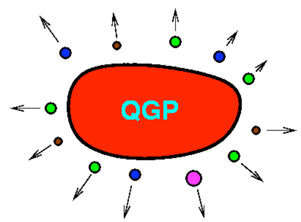


Somewhat similar scenarios have been considered by

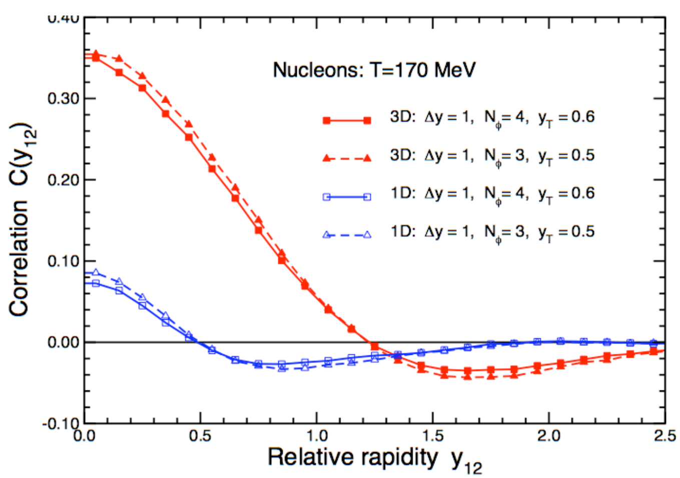
- I.N. Mishustin, Phys Rev Lett 82 (4779) 1999:
  Nonequilibrium phase transition in rapidly expanding matter
- D. Bower & S. Gavin, J Heavy Ion Phys 15 (2002) 269:
  Baryon fluctuations and the QCD phase transition

Rapidity correlations



Each blob hadronizes thermally - 

- in its own flow frame



[J. Heavy Ion Physics 22 (2005) 69]

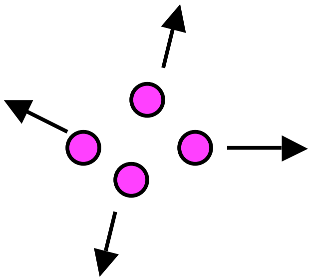
1D: $y_{12}(\mathbf{p}_1,\mathbf{p}_2) = | y_1(\mathbf{p}_1) - y_2(\mathbf{p}_2) |$

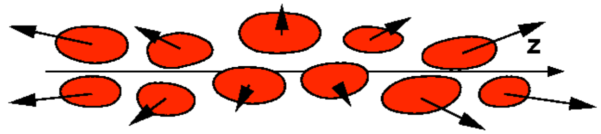
3D:

$$y_{12}(\mathbf{p}_1, \mathbf{p}_2) = \ln[\gamma_{12} + \sqrt{\gamma_{12}^2 - 1}]$$

$$m_1 m_2 \gamma_{12} = E_1 E_2 - \mathbf{p}_1 \cdot \mathbf{p}_2$$

Invariant-mass correlations





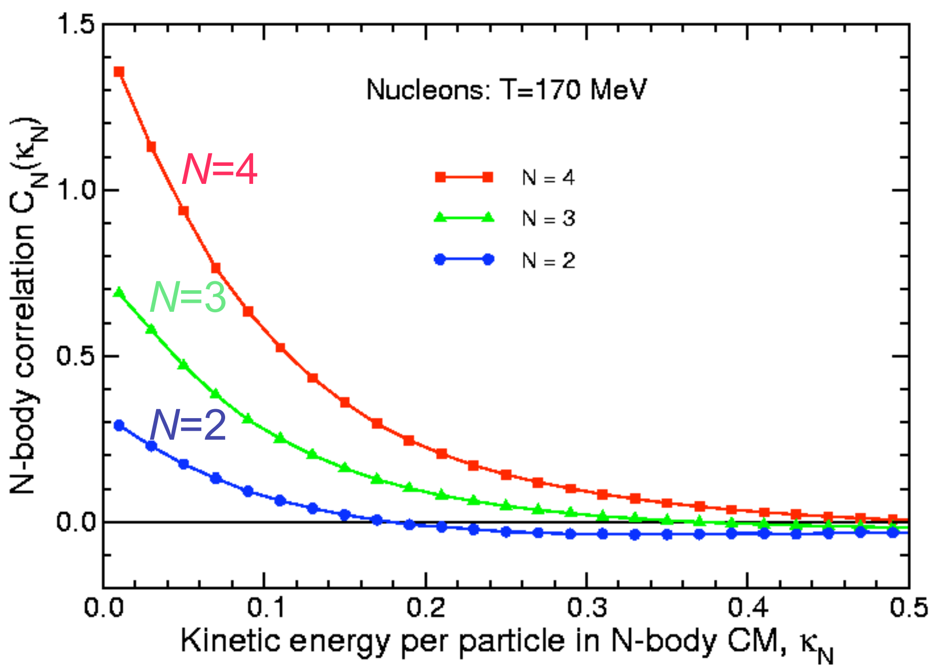
Kinetic energy per particle (in the *N*-body CM frame):

$$\kappa_N\{\boldsymbol{p}_n\} = \frac{1}{N}\left[[P\{\boldsymbol{p}_n\}\cdot P\{\boldsymbol{p}_n\}]^{\frac{1}{2}} - \sum_n m_n \right]$$

Total four-momentum:

$$P\{\boldsymbol{p}_n\} = \sum_n (E_n, \boldsymbol{p}_n)$$

Distribution of κ: $P_N(\kappa) \equiv \prec \delta(\kappa - \kappa_N\{\boldsymbol{p}_n\}) \succ$



[J. Heavy Ion Physics 22 (2005) 69]

Correlation function:

$$C_N(\kappa) \equiv P_N(\kappa) \,/\, P_N^0(\kappa) - 1$$

Same event / *Mixed* events

Higher-order correlations stand out more clearly!

(but require larger samples)

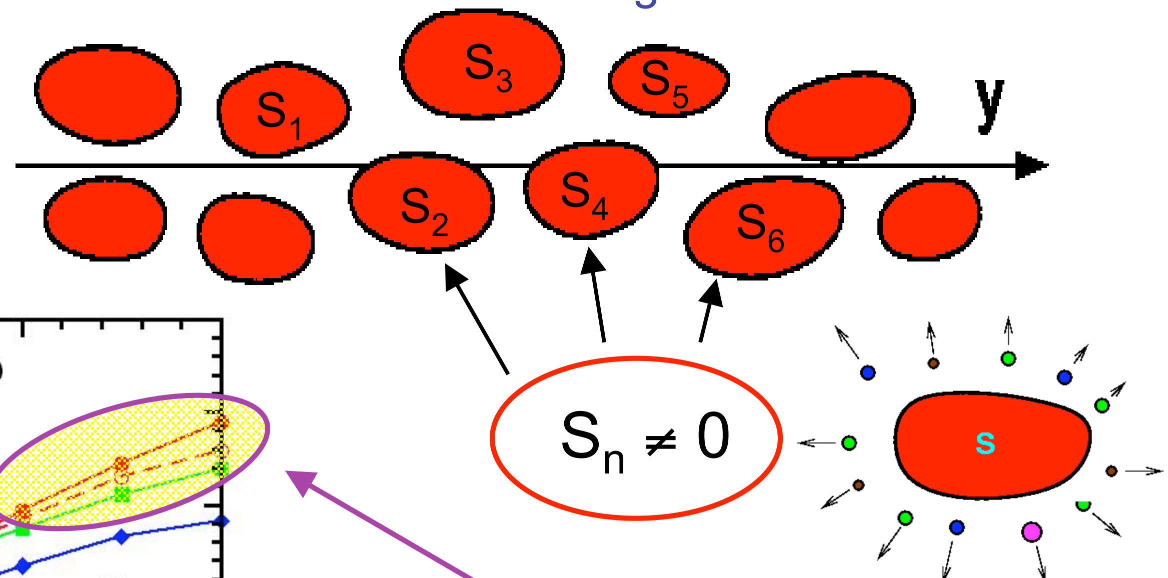
Strangeness trapping

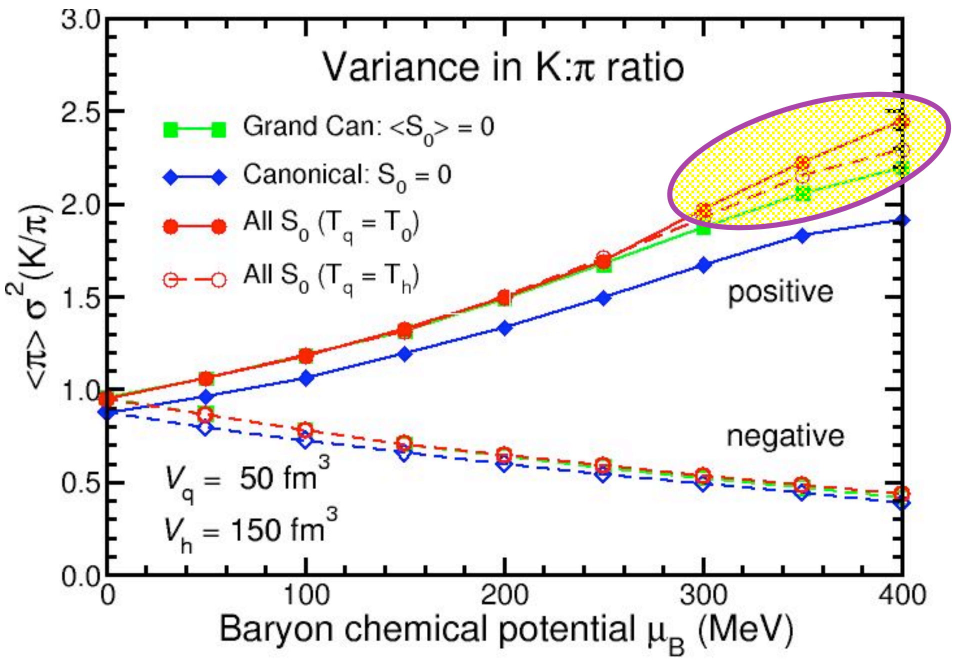
[V. Koch, A. Majumder, J. Randrup, Phys. Rev. C72, 064903 (2005)]

First-order phase transition => spinodal decomposition

The expanding system decomposes into plasma blobs
which each contain a certain amount of strangeness:

The hadronization of
each isolated blob
conserves strangeness!





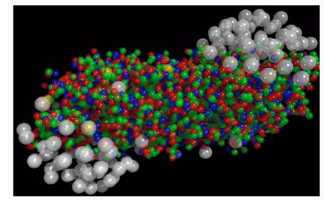
=> *Kaon fluctuations are enhanced!*

Already seen by NA49 at SPS?

More refined transport treatments are needed!

most urgent,
most difficult

What is expected in idealized matter ⟷ What occurs in a nuclear collision



large, stationary, in global equilibrium — *small, fast evolving, out of equilibrum*

Requirements:
(some of them)

- Must describe each phase separately: hadron gas & QGP
- Must have an ``interesting'' EoS: 1st order & critical point
- Must ``work'' in the phase-coexistence region

Microscopic transport models are inadequate:

deconfinement!
EoS is unknown!
no phase transition!

Ordinary fluid dynamics is inapplicable:

no inherent scale!
no interface energy!