Towards a controlled lattice study of the QCD chiral critical point

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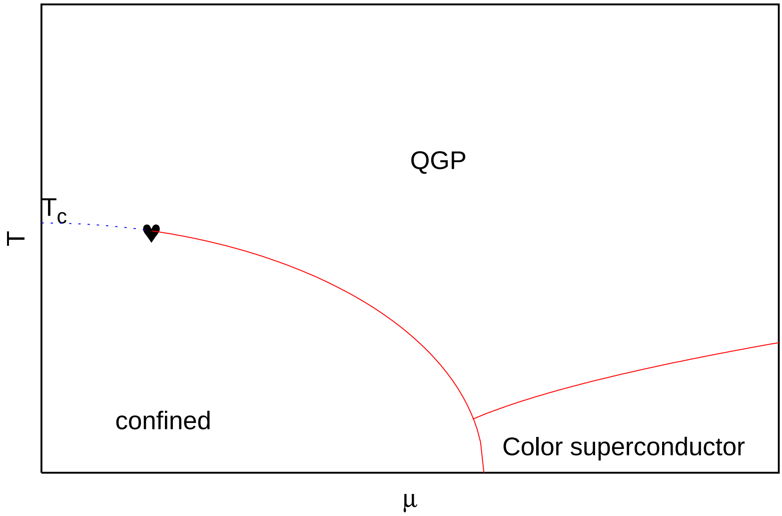
Schematic phase diagram



Can one locate the critical point (μ_E, T_E) ?

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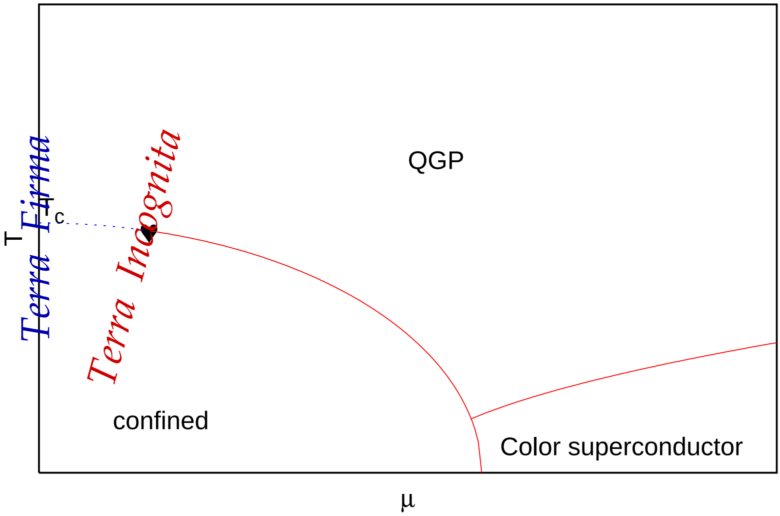

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The sign and overlap problems

- Integrate over fermions: $\det(\not{D} + m + \mu\gamma_0)$ complex *unless* $\mu = 0$ *or* $\mu = i\mu_i$

  $\longrightarrow$ standard importance sampling impossible

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- **Reweighting**: - simulate theory with no sign pb., eg. $\mu = 0$
  - reweight each measurement with $\rho(U) = \frac{\det(U,\mu\neq 0)}{\det(U,\mu=0)}$ complex
  - $\langle \rho(U) \rangle = \frac{Z(\mu\neq 0)}{Z(\mu=0)} \sim \exp(-V\frac{\Delta f(\mu)}{T})$ $\longrightarrow$ large V ?, large μ ?

1. maintain statistical accuracy on $\langle \rho \rangle$: **sign** pb.

2. ensure that $Z(\mu \neq 0)$ is properly sampled: **overlap** pb.

1 and **2** require statistics $\propto$ $\exp(+V)$

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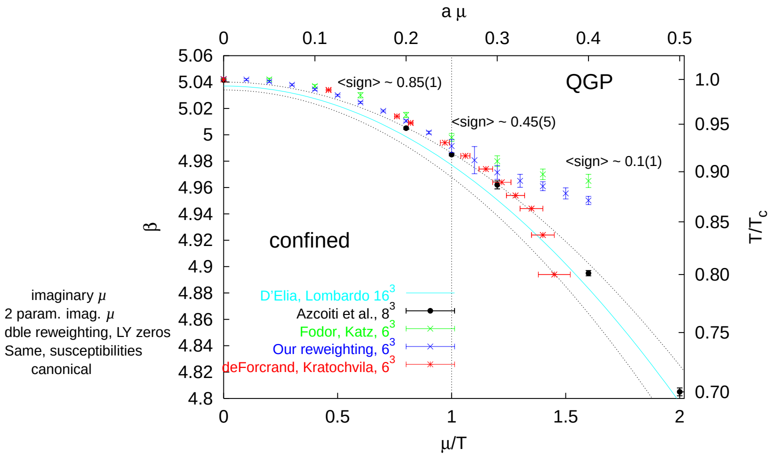
- Measure **derivatives** w.r.t. μ at $\mu = 0$: $\boxed{\langle W(\mu) \rangle = \langle W(\mu = 0) \rangle + \sum_k \mathbf{c_k} \left(\frac{\mu}{\pi T}\right)^k}$
  - directly at $\mu = 0$ MILC, TARO, Bielefeld-Swansea, Gavai-Gupta,..
  - by fitting polynomial to $\mu = i\mu_i$ results D'Elia-Lombardo, PdF-Philipsen,..

Controlled thermodynamics and continuum limits $\Rightarrow$ **derivatives** only

The good news: curvature of the pseudo-critical line

All with $N_f = 4$ staggered fermions, $am_q = 0.05, N_t = 4 \quad (a \sim 0.3$ fm)

PdF & Kratochvila



Agreement for $\mu/T \lesssim 1$

The good news: curvature of the pseudo-critical line

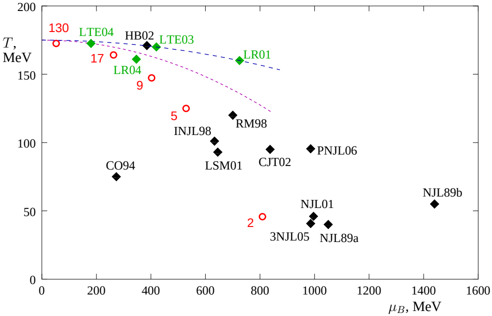
$$\frac{T_c(\mu)}{T_c(\mu=0)} = 1 - \mathbf{t_2} \left(\frac{\mu}{\pi T}\right)^2 + \dots$$

Lattice, all with $N_t = 4$:

N_f	am	N_s	t_2	Action	β-Function	Method
2	0.1	16	0.69(35)	p4	non-pert.	Taylor+Rew.
	0.025	6,8	0.500(34)	stag.	2-loop pert.	Imag.
3	0.1	16	0.247(59)	p4	non-pert.	Taylor+Rew.
	0.026	8,12,16	0.667(6)	stag.	2-loop pert.	Imag.
	0.005	16	1.13(45)	p4	non-pert.	Taylor+Rew.
4	0.05	16	0.93(9)	stag.	2-loop pert.	Imag.
2+1	0.0092,0.25	6-12	0.284(9)	stag.	non-pert.	Rew.

- Compare with **freeze-out**: $t_2 \approx \mathbf{2.05}$ Cleymans et al.
- Extrapolation $m_q \longrightarrow m_{\rm phys}, \; a \longrightarrow 0$ feasible
- Indications (PdF& OP $N_t = 6$; Fodor et al. LAT08): t_2 *decreases* as $a \longrightarrow 0$

The bad news: locating the critical point



M. Stephanov, hep-lat/0701002

- Challenging task:

  detect divergent correlation length (2nd order)

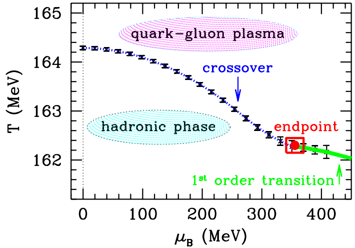
  vs finite but large (crossover, 1rst order)

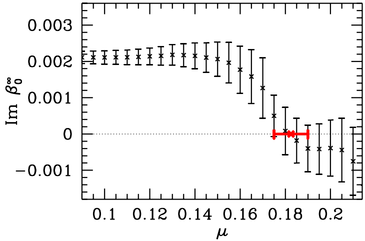
  on small lattice

  Mission impossible?

Critical point already determined, but...

Fodor & Katz: hep-lat/0402006 ($\sim$ physical quark masses)





$$(\mu_E^q, T_E) = (120(13), 162(2)) \text{ MeV}$$

Strategy: reweight from $(\mu = 0, T_c)$ along pseudo-critical line

Legitimate **concerns**:

- Discretization error? $N_t = 4 \implies a \sim 0.3$ fm
- Abrupt qualitative change near μ_E:

  abrupt change of physics or breakdown of algorithm (Splittorff)?

$\longrightarrow$ repeat with conservative approach (**derivative**)

Critical point from radius of convergence?

$$\frac{p}{T^4} = \sum_{n=0}^{\infty} c_{2n}(T) \left(\frac{\mu}{T}\right)^{2n}$$

Singularity $(\mu_E, T_E) \Rightarrow \boxed{\frac{\mu_E}{T_E} = \lim_{n\to\infty} \sqrt{\left|\frac{c_{2n}}{c_{2n+2}}\right|}}$ Karsch et al.

- Need $n \to \infty$, not $n = 1$ or 2; $\sqrt{\left|\frac{c_2}{c_4}\right|}$ is not a lower or upper bound

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- Need $n \to \infty$, not $n = 1$ or 2; $\sqrt{\left|\frac{c_2}{c_4}\right|}$ is not a lower or upper bound
- Other definitions just as good, eg. $\lim_{n\to\infty} \left|\frac{c_0}{c_{2n}}\right|^{\frac{1}{2n}}$

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- Need $n \to \infty$, not $n = 1$ or 2; $\sqrt{\left|\frac{c_2}{c_4}\right|}$ is not a lower or upper bound
- Also $\frac{n_q}{T^3} = \sum_{n=1}^{\infty} 2n\, c_{2n} \left(\frac{\mu}{T}\right)^{2n-1} \to \frac{\mu_E}{T_E} = \lim_{n\to\infty} \sqrt{\left|\frac{2n\, c_{2n}}{(2n+2)c_{2n+2}}\right|}$

  $n = 1 \ \to\ $ factor $1/\sqrt{2}$

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- Need $n \to \infty$, not $n = 1$ or 2; $\sqrt{\left|\frac{c_2}{c_4}\right|}$ is not a lower or upper bound

- Or $\frac{\chi_q}{T^2} = \sum_{n=1}^{\infty} 2n(2n-1)\, c_{2n}\left(\frac{\mu}{T}\right)^{2n-2} \longrightarrow \frac{\mu_E}{T_E} = \lim_{n\to\infty}\sqrt{\left|\frac{2n(2n-1)c_{2n}}{(2n+2)(2n+1)c_{2n+2}}\right|}$

  $n = 1 \;\longrightarrow\;$ factor $1/\sqrt{6}$ Gavai & Gupta

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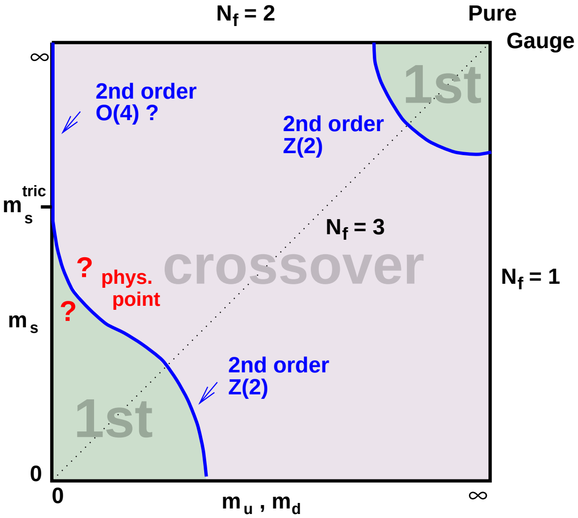
Systematic error uncontrolled

Better strategy?

Generalize QCD to arbitrary $(m_{u,d}, m_s)$, T: phase diagram

$\mu = 0$



Generalize QCD to arbitrary $(m_{u,d}, m_s)$, T: phase diagram

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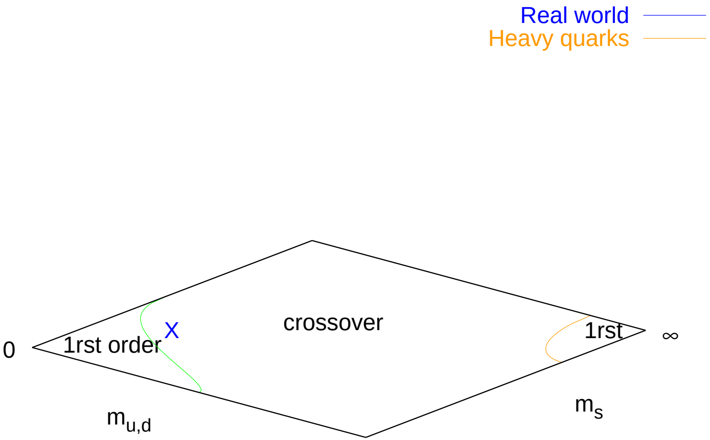
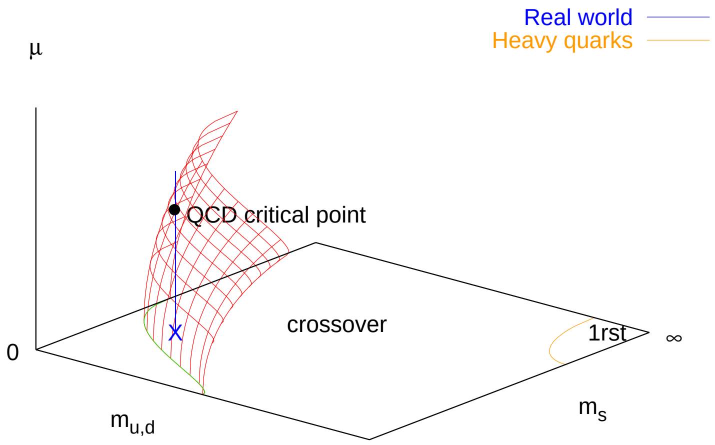


Now turn on μ

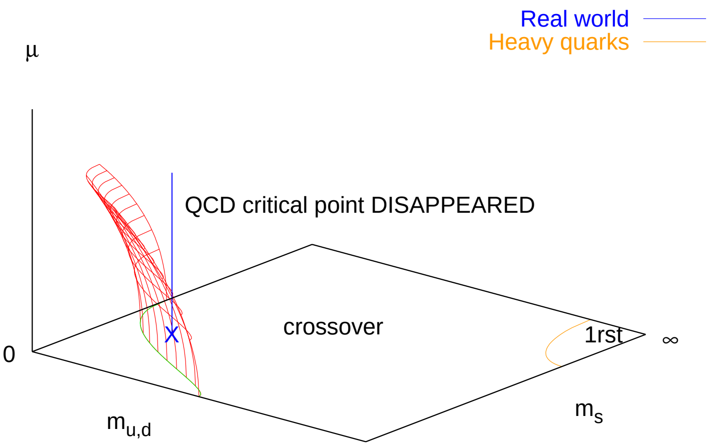
Generalize QCD to arbitrary $(m_{u,d}, m_s)$, T: phase diagram

$\mu \neq 0$



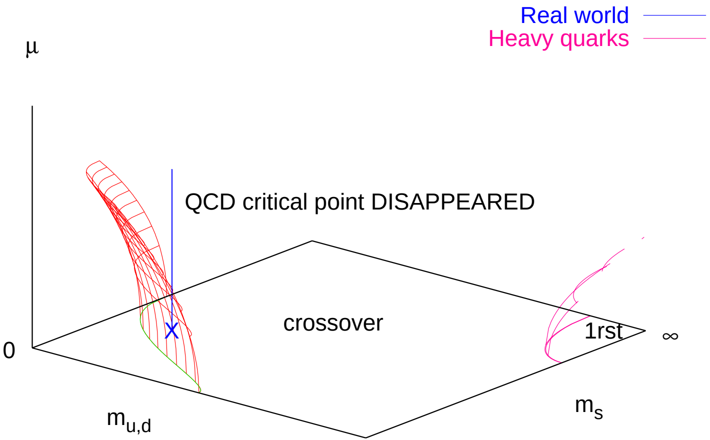
Conventional wisdom: first-order region expands with real $|\mu|$

Generalize QCD to arbitrary $(m_{u,d}, m_s)$, T: phase diagram



Exotic scenario: first-order region shrinks with real $|\mu|$ $\quad \frac{d\ m_c}{d\mu^2}|_{\mu=0} < 0$

Generalize QCD to arbitrary $(m_{u,d}, m_s)$, T: phase diagram



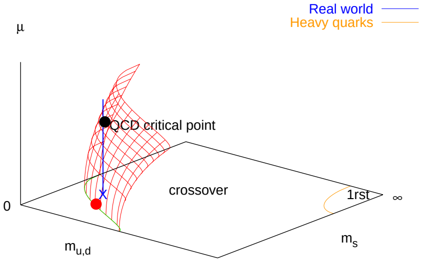
For heavy quarks, first-order region shrinks (PdF, Kim, Takaishi, hep-lat/0510069)

Strategy, with Owe Philipsen

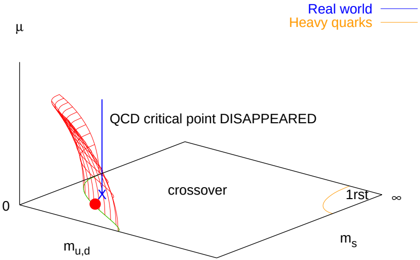
1. Tune quark mass(es) to $m_c(0)$: 2nd order transition at $\mu = 0, T = T_c$
   known universality class: 3d Ising

2. Measure derivatives $\frac{d^k m_c}{d\mu^{2k}}|_{\mu=0}$:

$$\frac{m_c(\mu)}{m_c(0)} = 1 + \sum_{k=1} \mathbf{c_k} \left(\frac{\mu}{\pi T}\right)^{2k}$$



$c_1 > 0$

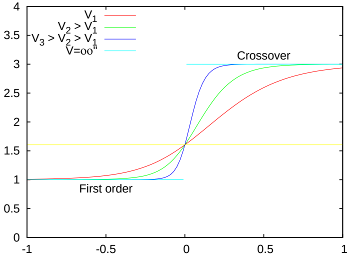


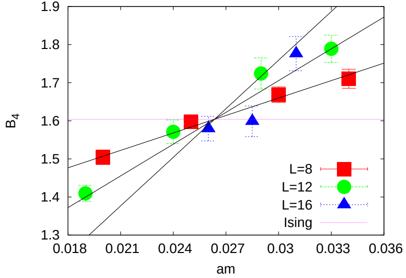
$c_1 < 0$

Observable: Binder cumulant

- Probability distribution of order parameter
  - distinguishes crossover (Gaussian) vs 1rst order (2 peaks)
  - 2nd order: scale-invariant distribution with known Ising exponents
  - encoded in Binder cumulant

- Measure $B_4(\bar{\psi}\psi) \equiv \left.\frac{\langle(\delta\bar{\psi}\psi)^4\rangle}{\langle(\delta\bar{\psi}\psi)^2\rangle^2}\right|_{\langle(\delta\bar{\psi}\psi)^3\rangle=0} = \begin{cases} 3 & \text{crossover} \\ 1 & \text{first}-\text{order} \quad \text{for } V \to \infty \\ 1.604 & \text{3d Ising} \end{cases}$

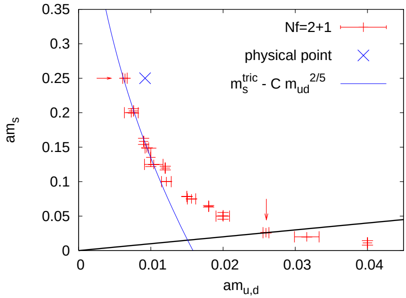


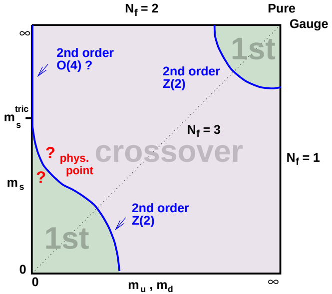


- Finite volume, $\mu = 0$: $B_4(am) = 1.604 + c(L)(am - am_0^c) + \ldots, \; c(L) \propto L^{1/\nu}$

Results: hep-lat/0607017, 0808.1096

1. Line of second-order phase transitions in the quark mass plane $(m_{u,d}, m_s)$ via Binder cumulant $B_4 = \langle(\delta\bar{\psi}\psi)^4\rangle / \langle(\delta\bar{\psi}\psi)^2\rangle^2$





$\mu = 0$:

- data consistent with tricritical point at $m_{u,d} = 0$, $m_s \sim 2.8T_c$
- physical point in crossover region cf. Fodor & Katz

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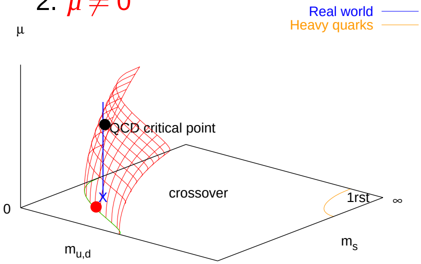
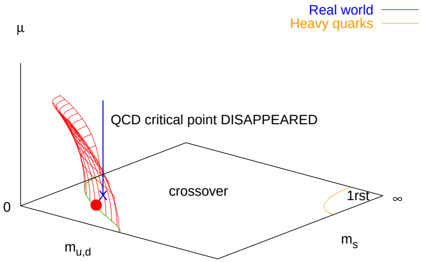
2. $\mu \neq 0$





Strategy: tune m_q for 2nd-order P.T. at $\mu = 0$, then turn on [imaginary] μ

Does the transition become 1rst-order (left) or crossover (right)?

$$B_4(am, a\mu) = 1.604 + \sum_{k,l=1} b_{kl}\,(am - am_0^c)^k (a\mu)^{2l}$$

$$\frac{d\, am^c}{d(a\mu)^2} = -\frac{\partial B_4}{\partial (a\mu)^2} \Big/ \frac{\partial B_4}{\partial am} = -b_{01}/b_{10}, \quad \text{hard / easy}$$

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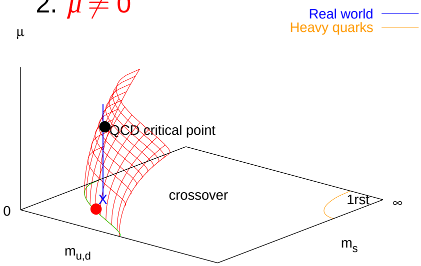

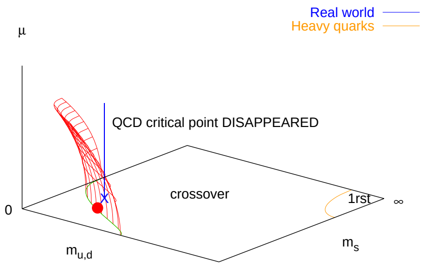

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Answer: very little change ($\rightarrow$ surface almost vertical)

Two methods to measure change in B_4: $\frac{\partial B_4}{\partial (a\mu)^2}$

1. Finite-μ: MC at several $\mu = i\mu_i$, fit $B_4(\mu_i)$ with truncated Taylor series in μ^2
Danger: truncation error?

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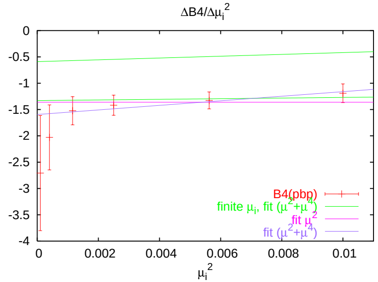
Comparison $8^3 \times 4, N_f = 3$:

- consistent value for $\frac{\partial B_4}{\partial(a\mu)^2}$
- also for NLO $\frac{\partial^2 B_4}{\partial(a\mu)^4}$
- **Derivative** method superior

5 million traj., 2 weeks Grid computing

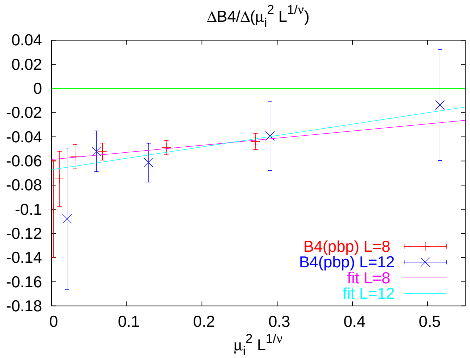


No doubt about sign $\longrightarrow$ non-standard scenario!?

$N_t = 4, N_f = 3$, finite-size scaling

$$\frac{d\,am^c}{d(a\mu)^2} = -\frac{\partial B_4}{\partial (a\mu)^2} \Big/ \frac{\partial B_4}{\partial am};\quad \text{scaling} \longrightarrow \text{each factor} \propto L^{1/\nu},\quad \nu = 0.63$$

Compare $8^3 \times 4$ and $12^3 \times 4$ (Derivative method) ($m_\pi L = 3.4$ and 5.1):

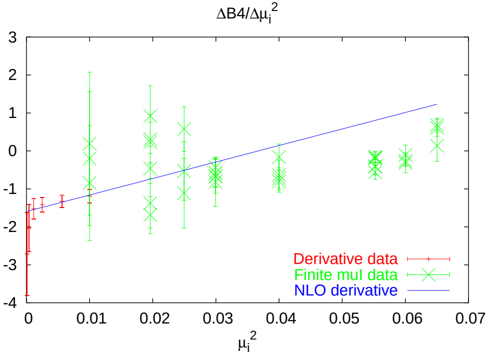


- Consistency of leading and subleading terms
- Subleading term $\sim \left(\frac{\mu}{\pi T}\right)^4$ weakens curvature for imaginary μ

$\Longrightarrow$ reinforces exotic scenario for real μ

$N_t = 4, N_f = 3$: combining the two methods

Methods 1 and 2 cover different ranges of μ_i $\longrightarrow$ combine them



$$\frac{B_4(\mu_i) - B_4(0)}{\mu_i^2} = \underbrace{b_{01}}_{<0} + \underbrace{b_{02}}_{>0} \mu_i^2$$

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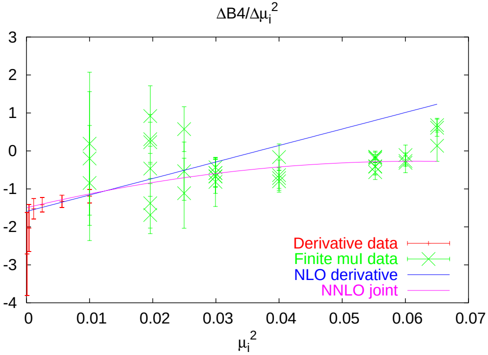

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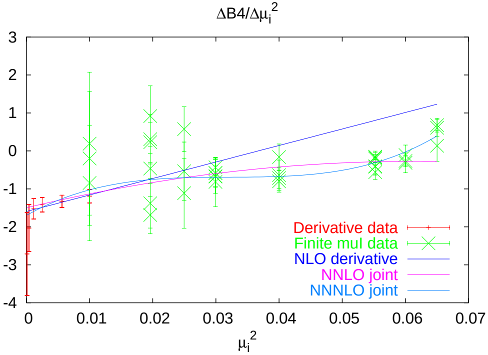

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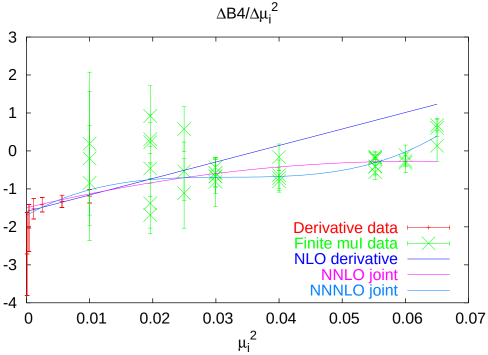

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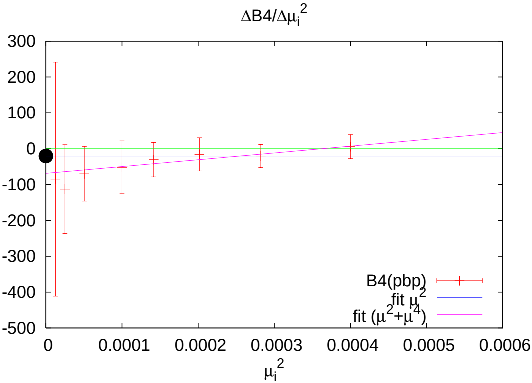
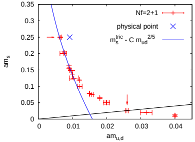
Real μ: $B_4(\mu) = B_4(0) + \underbrace{(-b_{01})}_{>0}\mu^2 + \underbrace{(+b_{02})}_{>0}\mu^4 + \underbrace{(-b_{03})}_{>0}\mu^6 + \underbrace{(+b_{04})}_{>0}\mu^8$

B_4 increases with $\mu \rightarrow$ crossover: all terms reinforce exotic scenario!

$N_t = 4, N_f = 2+1$: moving along the critical line





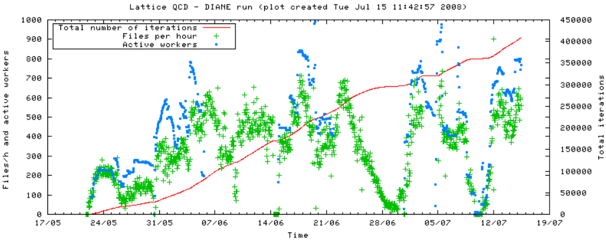
- $16^3 \times 4, am_s = 0.25, am_{u,d} = 0.005$, *lighter than in nature* $(m_\pi L = 3.4)$
  700k trajectories, 2 months of Grid computing
- $b_{01} = -20(9)$ (μ^2 fit) $\longrightarrow$ $\partial am^c/\partial(a\mu^2) = -0.19(8)$

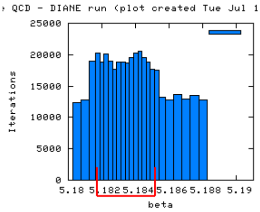
[or $b_{01} = -69(11)$ ($\mu^2 + \mu^4$ fit)]

- $c_1 = -24(11)$, ie. $\frac{m_c(\mu)}{m_c(0)} = 1 \mathbf{-24(11)} \left(\frac{\mu}{\pi T}\right)^2$ not quite conclusive yet

LQCD on the Computing Grid

- 725k trajectories (2 quark masses) in 2 months $\longrightarrow$ 115 CPU years
- on average 700 CPUs active at all times
- 330k files = 3 TB of data transferred
- computing support provided by CERN IT/GS: *thanks a lot!*





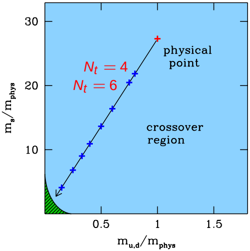
prioritized scheduling

- calculations on EGEE Grid
- resources provided by CERN, CYFRONET (Poland), CSCS (Switzerland), NIKHEF (Holland) + 10 more across Europe

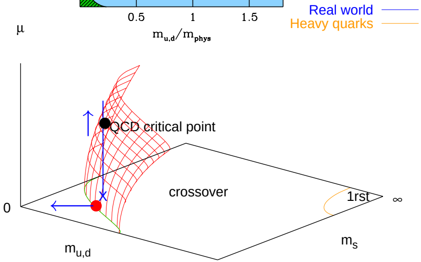
$N_t = 6, N_f = 3$: towards the continuum limit

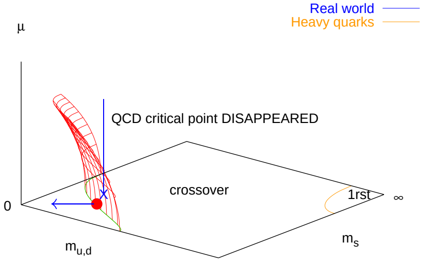
1. $\mu = 0$: re-tune the quark mass for 2nd-order transition at $T = T_c$

$\longrightarrow$ At $T = 0$, $\frac{m_\pi}{T_c} = 0.954(12)$ instead of $1.680(4)$ $(N_t = 4)$



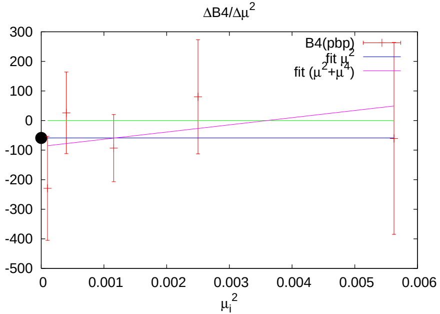
cf. Endrodi, Fodor et al., arXiv:0710.0998





$N_t = 6, N_f = 3$: towards the continuum limit

2. Measure $\frac{\partial B_4}{\partial(am)}$ (easy) and $b_1 \equiv \frac{\partial B_4}{\partial(a\mu)^2}$ (hard)



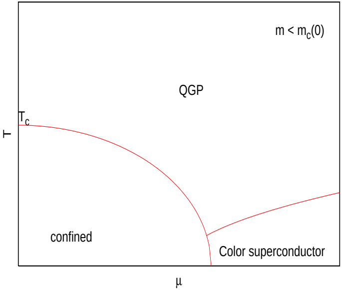
- $18^3 \times 6, am = 0.003$, $m_\pi = 0.95 T_c \sim$ 170 MeV $(m_\pi L = 2.9)$
  120k trajectories, 6 months of SX-8
- $b_{01} = -58(49)$ (μ^2 fit) $\longrightarrow c_1 = -28(23)$, ie. $\frac{m_c(\mu)}{m_c(0)} = 1 \mathbf{-28(23)} \left(\frac{\mu}{\pi T}\right)^2$

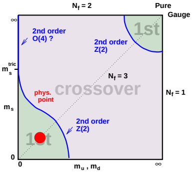
[or $b_{01} = -88(75)$ ($\mu^2 + \mu^4$ fit)]

- Assume $c_1 = +18$, ie. 2 sigmas away; then $\frac{\mu_E}{T_E} = 1 \Rightarrow \frac{m_c(\mu_E)}{m_c(0)} \sim 3$, insufficient to reach physical point

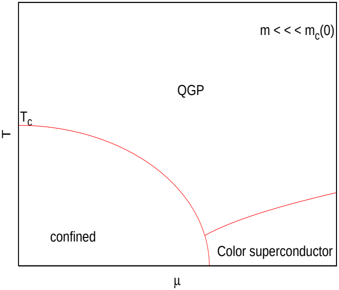
Resulting phase diagram (simplest possibility)

Standard scenario





Exotic scenario



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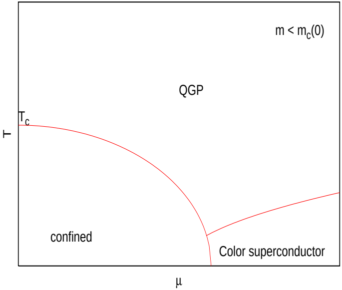

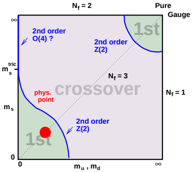

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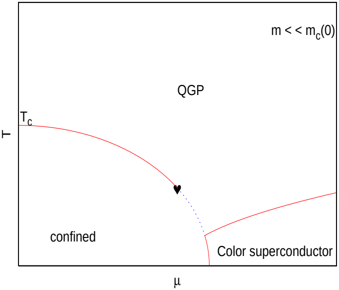

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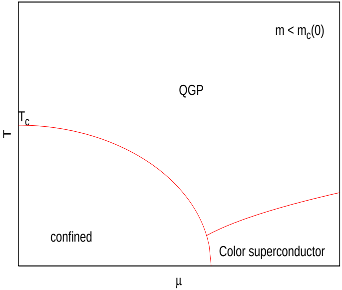
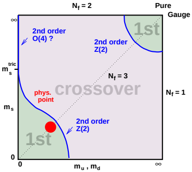
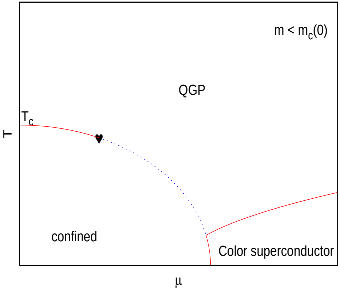




Exotic scenario



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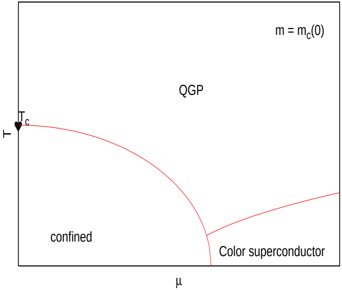

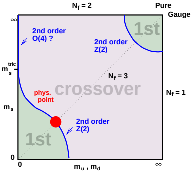

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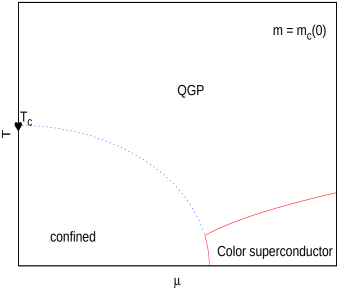

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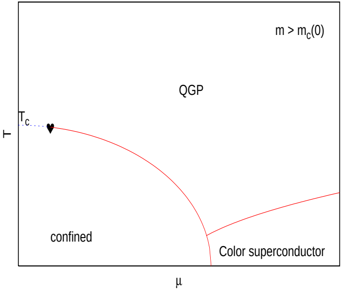

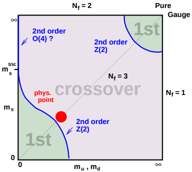

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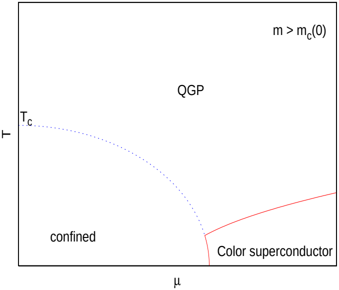

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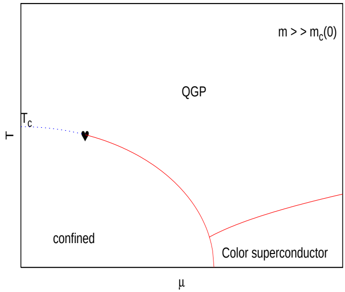

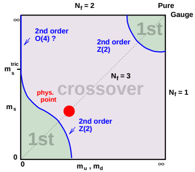

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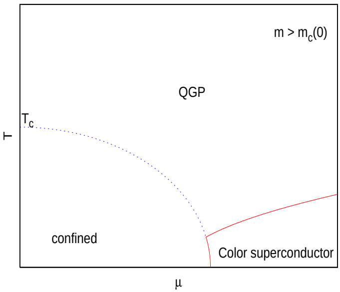

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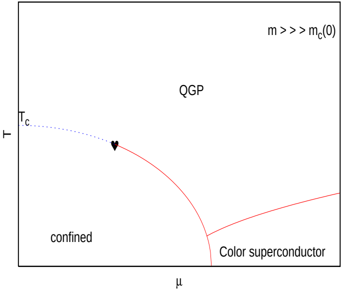
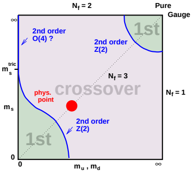
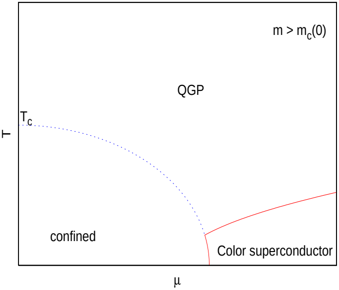


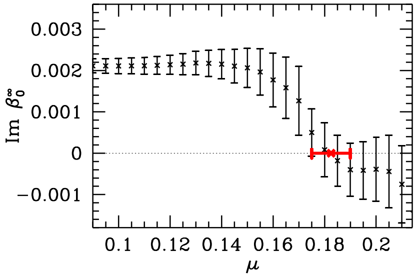


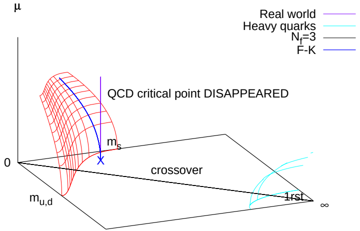
Exotic scenario



Contradiction with other lattice studies?

- Fodor & Katz: $(T_E, \mu_E) = (162(2), 120(13))$ MeV ?



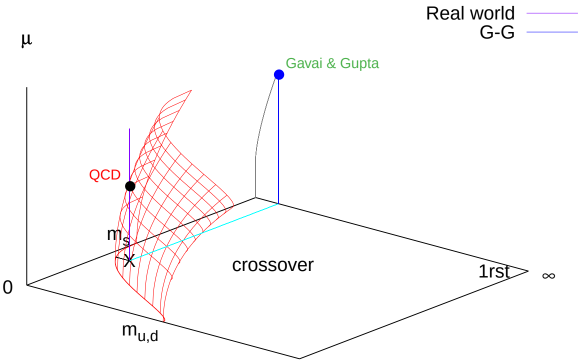


- Very little μ-dependence until $\mu \sim \mu_E \longrightarrow$ need high-degree Taylor expansion
- $m_q a$, ie. $\frac{m_q}{T_c}$ fixed, while $T_c(\mu)$ decreases for $\mu \neq 0 \Rightarrow$ non-const. physics
  Lighter quarks at larger μ favor first-order transition

Contradiction with other lattice studies?

- Gavai & Gupta: $\mu_E / T_E \lesssim 1$?

  different theory $N_f = 2$

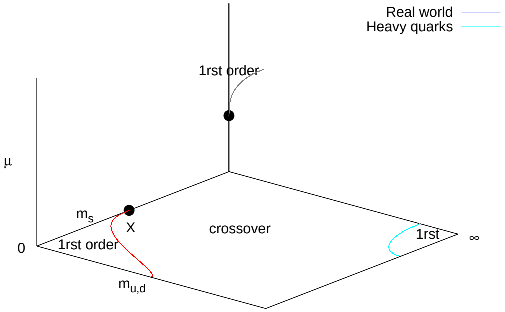


- Agreement with isospin μ Kogut-Sinclair, PdF-Stephanov-Wenger

Arguments for standard wisdom?

- $O(4)$ transition for 2 massless flavors — Pisarski & Wilczek

$\Rightarrow$ tricritical points $(m_{u,d} = 0, m_s = \infty, \mu = \mu^*)$ and $(m_{u,d} = 0, m_s = m_s^*, \mu = 0)$



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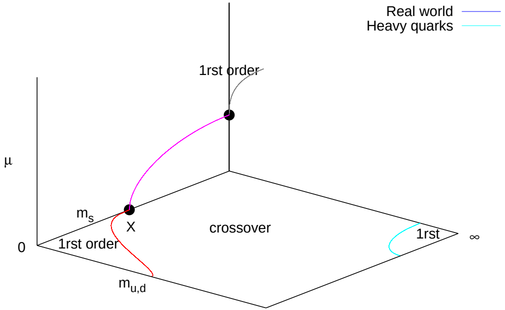
- $N_f = 2$ and $N_f = 2+1$ analytically connected



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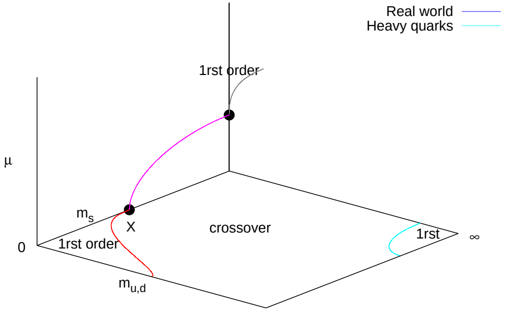

Critique:

- $O(4)$ if strong enough $U_A(1)$ anomaly, otherwise first-order

Chandrasekharan & Mehta

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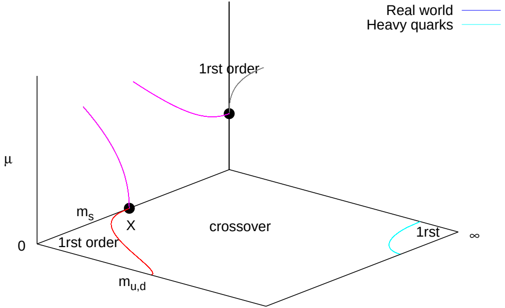


Critique:

- $O(4)$ if strong enough $U_A(1)$ anomaly, otherwise first-order

Chandrasekharan & Mehta

- $N_f = 2$ and $N_f = 2+1$ need not be connected

Conclusions

- Pseudo-critical temperature $\frac{T_c(\mu)}{T_c(\mu=0)} = 1 - t_2 \left(\frac{\mu}{\pi T}\right)^2 + \ldots$ *soon*

  $t_2 < t_2^{\text{freeze-out}}$, factor $\gtrsim 3$?

- Binder cumulant $B_4(X) \equiv \left.\frac{\langle(\delta X)^4\rangle}{\langle(\delta X)^2\rangle^2}\right|_{\langle(\delta X)^3\rangle=0}$ goes from 3 (Gaussian) to 1.604 (crit. pt.) $\longrightarrow$ experimentally measurable? cf. kurtosis

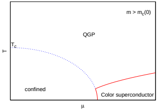
- $\frac{m_c(\mu)}{m_c(0)} = 1 + \mathbf{c_1} \left(\frac{\mu}{\pi T}\right)^2 + \ldots$: *can control systematics*

$N_t = 4$,	$N_f = 3$	LO+NLO	0808.1096
	$N_f = 2+1$	LO	soon
$N_t = 6$,	$N_f = 3$	LO	underway
$\cdots$	**Non-standard scenario $c_1 < 0$ favored**		

- $a \longrightarrow 0$: critical surface *far* from physical point

  $\Longrightarrow$ need $c_1 > 0$ *and large* for $\frac{\mu_E}{T_E} \lesssim 1$, disfavored by data

- **QCD critical point?**



$\mu_E^B \lesssim 500$ MeV unlikely, or non-chiral